Chapter 9 Organizer

Section/Objectives	Standards: National	Standards: State/Local	Lab and Demo Planning
Chapter Opener	See page 14T for a key to the standards.		
Section 9.1 1. **Define** the momentum of an object. 2. **Determine** the impulse given to an object. 3. **Define** the angular momentum of an object.	UCP.2, UCP.3, A.1, A.2, B.4, E.1		**Student Lab:** **Launch Lab,** p. 229: one hollow, plastic ball with cutouts in one hemisphere, one bocce ball **Teacher Demonstration:** **Quick Demo,** p. 231: bed sheet, raw egg
Section 9.2 4. **Relate** Newton's third law to conservation of momentum in collisions and explosions. 5. **Recognize** the conditions under which momentum is conserved. 6. **Solve** conservation of momentum problems in two dimensions.	UCP.1, UCP.2, UCP.3, A.1, A.2, B.4, E.1, G.3		**Student Lab:** **Mini Lab,** p. 239: one small hard rubber ball about the size of a table tennis ball, one large hard rubber ball the size of a tennis ball, meter-stick **Additional Mini Lab,** p. 241: cardboard box, packing material, softball, meterstick, spring scale calibrated in newtons, platform balance **Internet Physics Lab,** pp. 246–247: Internet access required **Teacher Demonstration:** **Quick Demo,** p. 242: rotating stool, two heavy blocks

Differentiated Instruction

L1 Level 1 activities should be appropriate for students with learning difficulties.

L2 Level 2 activities should be within the ability range of all students.

L3 Level 3 activities are designed for above-average students.

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FAST FILE Chapters 6–10 Resources, Chapter 9 Transparency 9-2 Master, p. 131 Transparency 9-3 Master, p. 133 Transparency 9-4 Master, p. 135 Study Guide, pp. 117–122 Reinforcement, p. 125 Enrichment, pp. 127–128 Section 9-2 Quiz, p. 124 Mini Lab Worksheet, p. 111 Physics Lab Worksheet, pp. 113–116 **Teaching Transparency 9-2** **Teaching Transparency 9-3** **Teaching Transparency 9-4** **Connecting Math to Physics** **Laboratory Manual,** pp. 45–48 **Probeware Laboratory Manual,** pp. 21–24	**Interactive Chalkboard CD-ROM:** Section 9.2 Presentation **TeacherWorks™ CD-ROM** **Problem of the Week at physicspp.com** **Mechanical Universe:** Angular Momentum; Conservation of Momentum

Assessment Resources

FAST FILE Chapters 6–10 Resources, Chapter 9

Chapter Assessment, pp. 137–142

Additional Challenge Problems, p. 9
Physics Test Prep, pp. 17–18
Pre-AP/Critical Thinking, pp. 17–18
Supplemental Problems, pp. 17–18

Technology

Interactive Chalkboard CD-ROM:
Chapter 9 Assessment
Exam*View*® Pro Testmaker CD-ROM
Vocabulary PuzzleMaker
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Chapter Overview

Section one discusses changes in motion of an object by considering an object's momentum before and after impulses act on it. Section two considers what is required to conserve the momentum of a system: When no net external forces act on an object or collection of objects, their momentum does not change—it is conserved.

Think About This

The impulse depends on how long the force acts on the bat. In this case the peak force is about 14,700 N. This is discussed further in Using Figure 9-1 on page 230.

▶ Key Terms

impulse, p. 230
momentum, p. 230
impulse-momentum theorem, p. 230
angular momentum, p. 233
angular impulse-angular momentum theorem, p. 234
closed system, p. 236
isolated system, p. 237
law of conservation of momentum, p. 237
law of conservation of angular momentum, p. 243

Chapter 9 Momentum and Its Conservation

What You'll Learn

- You will describe momentum and impulse and apply them to the interactions between objects.
- You will relate Newton's third law of motion to conservation of momentum.
- You will explore the momentum of rotating objects.

Why It's Important

Momentum is the key to success in many sporting events, including baseball, football, ice hockey, and tennis.

Baseball Every baseball player dreams of hitting a home run. When a player hits the ball, at the moment of collision, the ball and the bat are deformed by the collision. The resulting change in momentum determines the batter's success.

Think About This ▶

What is the force on a baseball bat when a home run is hit out of the park?

Physics Online
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LAUNCH Lab

Purpose to determine how both mass and velocity affect the direction in which an object moves after a head-on collision

Materials one hollow plastic ball with cutouts in one hemisphere, one bocce ball

Teaching Strategy

- Encourage students to vary the angles of the collisions and to try the experiment multiple times.

Expected Results The bocce ball should have the dominating effect on the resultant velocities because of its large mass. On occasion, students may be able to roll the hollow plastic ball fast enough that it has the larger effect.

LAUNCH Lab

What happens when a hollow plastic ball strikes a bocce ball?

Question

What direction will a hollow plastic ball and a bocce ball move after a head-on collision?

Procedure

1. Roll a bocce ball and a hollow plastic ball toward each other on a smooth surface.
2. Observe the direction each one moves after the collision.
3. Repeat the experiment, this time keeping the bocce ball stationary, while rolling the hollow plastic ball toward it.
4. Observe the direction each one moves after the collision.
5. Repeat the experiment one more time, but keep the hollow plastic ball stationary, while rolling the bocce ball toward it.
6. Observe the direction each one moves after the collision.

Analysis

What factors affect how fast the balls move after the collision? What factors determine the direction each one moves after the collision?

Critical Thinking What factor(s) would cause the bocce ball to move backward after colliding with the hollow plastic ball?

9.1 Impulse and Momentum

It is always exciting to watch a baseball player hit a home run. The pitcher fires the baseball toward the plate. The batter swings at the baseball and the baseball recoils from the impact of the bat at high speed. Rather than concentrating on the force between the baseball and bat and their resulting accelerations, as in previous chapters, you will approach this collision in a different way in this chapter. The first step in analyzing this type of interaction is to describe what happens before, during, and after the collision between the baseball and bat. You can simplify the collision between the baseball and the bat by making the assumption that all motion is in the horizontal direction. Before the collision, the baseball moves toward the bat. During the collision, the baseball is squashed against the bat. After the collision, however, the baseball moves at a higher velocity away from the bat, and the bat continues in its path, but at a slower velocity.

Objectives

- **Define** the momentum of an object.
- **Determine** the impulse given to an object.
- **Define** the angular momentum of an object.

Vocabulary

impulse
momentum
impulse-momentum theorem
angular momentum
angular impulse-angular momentum theorem

Horizons Companies

Analysis Both mass and velocity affect how fast and in which direction the balls move after the collision. The ball that has more momentum will affect the other ball more. If the two balls have roughly equal momentum, they will most likely both bounce backward. However, if there is a large mass difference or a large velocity difference, then the ball with the larger momentum may continue to move forward after the collision, only at a much smaller speed.

Critical Thinking Velocity would be the single most important factor. If the hollow plastic ball moves with a large velocity and hits the bocce ball head-on, and if the bocce ball's velocity at the time of collision is $v = 0$, then the bocce ball will reverse direction. The hollow plastic ball, which has a much smaller mass in comparison, has to move with a significantly larger velocity to have an effect on the bocce ball.

Section 9.1

1 FOCUS

Bellringer Activity

Collision Force Drop a heavy object, such as this book, onto your desk. Then repeat, but drop it onto a pad on your desk. Have students use their prior knowledge to list what they can and cannot determine about the nature of the collision. They can find the mass of the object and should be able to calculate its velocity just as it hit the desk, as well as understanding that the velocity is zero after hitting. Without knowing the time it took for the object to stop, however, they cannot determine the acceleration of the object as it came to a stop, nor the force of the desk on the object.

L2 Visual-Spatial

Tie to Prior Knowledge

Laws of Motion Newton's second law of motion (Chapter 6) is used to provide the relationship between momentum and impulse. Students may need to review angular velocity (Chapter 8) before they learn about angular momentum. Students will read and interpret a force-time graph.

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2 TEACH

Using Figure 9-1

Have students examine Figure 9-1. Ask them to find the maximum force. 14,700 N (1.47×10^4 N) Ask if it is possible to determine how long the collision lasted? Yes How? The area under the curve is the impulse. Explain that one standard measure is the time interval over which the force was larger than half its maximum value, or 0.8 ms in the figure. Ask students how they can determine the magnitude of impulse. The area under the curve is the impulse. One method is to copy the area under the curve onto graph paper and count squares, find the area of each square and multiply. They could also find the area of a triangle that has an apex at the maximum force and that touches the two half-maximum force points. Have students compare the area of that triangle with the true area, 13.1 N·s. The area by counting squares is 12 N·s. **L2 Visual-Spatial**

Concept Development

- **Momentum and Velocity** Use $\boldsymbol{p} = m\boldsymbol{v}$ to differentiate between momentum and velocity.
- **Momentum and Impulse** Use $F\Delta t = m\Delta\boldsymbol{v} = m(\boldsymbol{v}_f - \boldsymbol{v}_i) = m\boldsymbol{v}_f - m\boldsymbol{v}_i = \boldsymbol{p}_f - \boldsymbol{p}_i$ to differentiate between momentum and change in momentum, or impulse. Emphasize that both momentum and impulse are vector quantities—they both have magnitude and direction. Explain that impulse points in the same direction as the change in momentum.

THE MECHANICAL UNIVERSE
HIGH SCHOOL ADAPTATION

Videotape

Angular Momentum

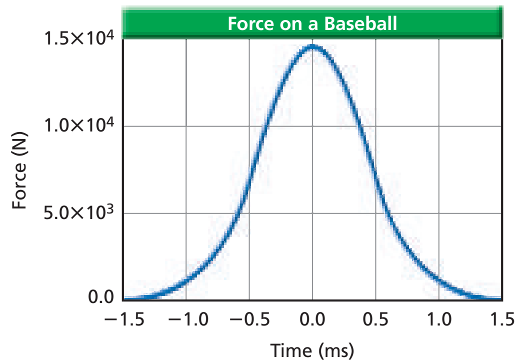

Figure 9-1 The force acting on a baseball increases, then rapidly decreases during a collision, as shown in this force-time graph.

Impulse and Momentum

How are the velocities of the ball, before and after the collision, related to the force acting on it? Newton's second law of motion describes how the velocity of an object is changed by a net force acting on it. The change in velocity of the ball must have been caused by the force exerted by the bat on the ball. The force changes over time, as shown in **Figure 9-1.** Just after contact is made, the ball is squeezed, and the force increases. After the force reaches its maximum, which is more than 10,000 times the weight of the ball, the ball recovers its shape and snaps away from the bat. The force rapidly returns to zero. This whole event takes place within about 3.0 ms. How can you calculate the change in velocity of the baseball?

Impulse Newton's second law of motion, $\boldsymbol{F} = m\boldsymbol{a}$, can be rewritten by using the definition of acceleration as the change in velocity divided by the time needed to make that change. It can be represented by the following equation:

$$\boldsymbol{F} = m\boldsymbol{a} = m\left(\frac{\Delta\boldsymbol{v}}{\Delta t}\right)$$

Multiplying both sides of the equation by the time interval, Δt, results in the following equation:

$$\boldsymbol{F}\Delta t = m\Delta\boldsymbol{v}$$

Impulse, or $\boldsymbol{F}\Delta t$, is the product of the average force on an object and the time interval over which it acts. Impulse is measured in newton-seconds. For instances in which the force varies with time, the magnitude of an impulse is found by determining the area under the curve of a force-time graph, such as the one shown in Figure 9-1.

The right side of the equation, $m\Delta\boldsymbol{v}$, involves the change in velocity: $\Delta\boldsymbol{v} = \boldsymbol{v}_f - \boldsymbol{v}_i$. Therefore, $m\Delta\boldsymbol{v} = m\boldsymbol{v}_f - m\boldsymbol{v}_i$. The product of the object's mass, m, and the object's velocity, $\boldsymbol{v}$, is defined as the **momentum** of the object. Momentum is measured in kg·m/s. An object's momentum, also known as linear momentum, is represented by the following equation.

Momentum $\boldsymbol{p} = m\boldsymbol{v}$

The momentum of an object is equal to the mass of the object times the object's velocity.

Recall the equation $\boldsymbol{F}\Delta t = m\Delta\boldsymbol{v} = m\boldsymbol{v}_f - m\boldsymbol{v}_i$. Because $m\boldsymbol{v}_f = \boldsymbol{p}_f$ and $m\boldsymbol{v}_i = \boldsymbol{p}_i$, this equation can be rewritten as follows: $\boldsymbol{F}\Delta t = m\Delta\boldsymbol{v} = \boldsymbol{p}_f - \boldsymbol{p}_i$. The right side of this equation, $\boldsymbol{p}_f - \boldsymbol{p}_i$, describes the change in momentum of an object. Thus, the impulse on an object is equal to the change in its momentum, which is called the **impulse-momentum theorem.** The impulse-momentum theorem is represented by the following equation.

Impulse-Momentum Theorem $\boldsymbol{F}\Delta t = \boldsymbol{p}_f - \boldsymbol{p}_i$

The impulse on an object is equal to the object's final momentum minus the object's initial momentum.

Color Convention

- Momentum and impulse vectors are **orange.**
- Force vectors are **blue.**
- Acceleration vectors are **violet.**
- Velocity vectors are **red.**
- Displacement vectors are **green.**

9.1 Resource MANAGER

FAST FILE Chapters 6–10 Resources
Transparency 9–1 Master, p. 129
Study Guide, pp. 117–122
Section 9–1 Quiz, p. 123
Teaching Transparency 9-1
Connecting Math to Physics

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If the force on an object is constant, the impulse is the product of the force multiplied by the time interval over which it acts. Generally, the force is not constant, however, and the impulse is found by using an average force multiplied by the time interval over which it acts, or by finding the area under a force-time graph.

Because velocity is a vector, momentum also is a vector. Similarly, impulse is a vector because force is a vector. This means that signs will be important for motion in one dimension.

Using the Impulse-Momentum Theorem

What is the change in momentum of a baseball? From the impulse-momentum theorem, you know that the change in momentum is equal to the impulse acting on it. The impulse on a baseball can be calculated by using a force-time graph. In Figure 9-1, the area under the curve is approximately 13.1 N·s. The direction of the impulse is in the direction of the force. Therefore, the change in momentum of the ball also is 13.1 N·s. Because 1 N·s is equal to 1 kg·m/s, the momentum gained by the ball is 13.1 kg·m/s in the direction of the force acting on it.

Assume that a batter hits a fastball. Before the collision of the ball and bat, the ball, with a mass of 0.145 kg, has a velocity of −38 m/s. Assume that the positive direction is toward the pitcher. Therefore, the baseball's momentum is $p_i = (0.145\ \text{kg})(-38\ \text{m/s}) = -5.5\ \text{kg·m/s}$.

What is the momentum of the ball after the collision? Solve the impulse-momentum theorem for the final momentum: $p_f = p_i + F\Delta t$. The ball's final momentum is the sum of the initial momentum and the impulse. Thus, the ball's final momentum is calculated as follows.

$$\begin{aligned} p_f &= p_i + 13.1\ \text{kg·m/s} \\ &= -5.5\ \text{kg·m/s} + 13.1\ \text{kg·m/s} = +7.6\ \text{kg·m/s} \end{aligned}$$

What is the baseball's final velocity? Because $p_f = mv_f$, solving for $\boldsymbol{v}_f$ yields the following:

$$v_f = \frac{p_f}{m} = \frac{+7.6\ \text{kg·m/s}}{+0.145\ \text{kg·m/s}} = +52\ \text{m/s}$$

A speed of 52 m/s is fast enough to clear most outfield fences if the baseball is hit in the correct direction.

Using the Impulse-Momentum Theorem to Save Lives

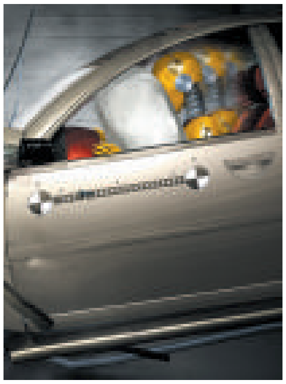
A large change in momentum occurs only when there is a large impulse. A large impulse can result either from a large force acting over a short period of time or from a smaller force acting over a long period of time.

What happens to the driver when a crash suddenly stops a car? An impulse is needed to bring the driver's momentum to zero. According to the impulse-momentum equation, $\boldsymbol{F}\Delta t = \boldsymbol{p}_f - \boldsymbol{p}_i$. The final momentum, $\boldsymbol{p}_f$, is zero. The initial momentum, $\boldsymbol{p}_i$, is the same with or without an air bag. Thus, the impulse, $\boldsymbol{F}\Delta t$, also is the same. An air bag, such as the one shown in **Figure 9-2,** reduces the force by increasing the time interval during which it acts. It also exerts the force over a larger area of the person's body, thereby reducing the likelihood of injuries.

APPLYING PHYSICS

► **Running Shoes** Running is hard on the feet. When a runner's foot strikes the ground, the force exerted by the ground on it is as much as four times the runner's weight. The cushioning in an athletic shoe is designed to reduce this force by lengthening the time interval over which the force is exerted. ◄

Figure 9-2 An air bag is inflated during a collision when the force due to the impact triggers the sensor. The chemicals in the air bag's inflation system react and produce a gas that rapidly inflates the air bag.



Rick Fischer/Masterfile

APPLYING PHYSICS

► Have students examine their running shoes to determine how far the soles compress when a given force is exerted on them. Suggest designing a shoe tester that could exert a measured force on an object the size of the ball of the foot and measure the distance that force compresses the shoe. Forces as large as four times the student's weight should be tested. Have students research what make and model of shoe most reduces the force on the running foot. Students can gather a variety of shoes to test which kind most reduces force on the foot. ◄

QUICK DEMO

Impulse

Estimated Time 5 minutes

Materials bedsheet, raw egg, goggles

Procedure Take your class out of the building or to a safe and easy-to-clean area. Have two assistants hold a bedsheet vertically between them. Ask students to predict whether or not you can break an egg by tossing it with all your might against the bedsheet. Then throw a raw egg with considerable velocity toward the middle of the sheet. The bedsheet usually stops the egg without breaking it. Explain that the bedsheet stops the egg over a much longer time interval (Δt) than a brick wall would. Emphasize that a longer Δt means that the force exerted on the egg is reduced.

Teacher F.Y.I. — CONTENT BACKGROUND

Force Function How can one develop a mathematical function for a force that is varying (like that in Figure 9-1)? One approach is to approximate it as a constant force. Mathematically, this is equivalent to creating a rectangle with the same area as is under the curve of the F-t graph. Of course, the shape of the rectangle does not change the impulse. One choice is to make the average force equal to the maximum force and adjust the time interval to get the correct area. A second is to choose a time interval that best represents the time over which the force acts and adjust the force to get the correct area. Without detailed measurements of the force as a function of time, there is no single correct method.

Identifying Misconceptions

Momentum and Velocity
Momentum is not the same as velocity. In all examples so far, the momentum and velocity are related by a fixed ratio, the mass. For this reason, some students may not see a reason to have a second quantity, and they thus will treat momentum as if it were velocity. The difference will not be obvious until the next section on collisions.

Reinforcement

Vectors Create some subtraction exercises using initial and final momentum vectors. Emphasize that the difference will be the impulse. Include some with either initial or final momentum of zero and with the two momenta in both the same and opposite directions. **L1 Visual-Spatial**

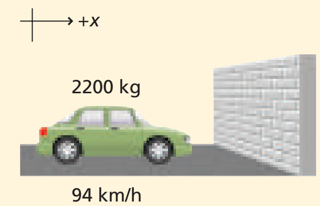
IN-CLASS Example

Question Assume that there is a passenger of 85-kg mass in the vehicle described in the example problem. For both gentle and emergency braking situations, calculate the impulse and average force needed to bring the person to a stop along with the vehicle.

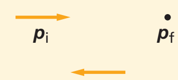
Answer $p_i = (85\ \text{kg})(26\ \text{m/s})$
$p_i = 2.2\times10^3\ \text{kg·m/s}$. $p_f = 0$.
So $F\Delta t = -2.2\times10^3\ \text{kg·m/s}$
When $\Delta t = 21$ s, $F = -1.1\times10^2$ N
When $\Delta t = 3.8$ s, $F = -5.8\times10^2$ N

EXAMPLE Problem 1

Average Force A 2200-kg vehicle traveling at 94 km/h (26 m/s) can be stopped in 21 s by gently applying the brakes. It can be stopped in 3.8 s if the driver slams on the brakes, or in 0.22 s if it hits a concrete wall. What average force is exerted on the vehicle in each of these stops?



Vector diagram



1 Analyze and Sketch the Problem
- Sketch the system.
- Include a coordinate axis and select the positive direction to be the direction of the velocity of the car.
- Draw a vector diagram for momentum and impulse.

Known:		Unknown:
$m = 2200$ kg	$\Delta t_{\text{gentle braking}} = 21$ s	$F_{\text{gentle braking}} = ?$
$v_i = +26$ m/s	$\Delta t_{\text{hard braking}} = 3.8$ s	$F_{\text{hard braking}} = ?$
$v_f = +0.0$ m/s	$\Delta t_{\text{hitting a wall}} = 0.22$ s	$F_{\text{hitting a wall}} = ?$

2 Solve for the Unknown

Determine the initial momentum, p_i.

$$p_i = mv_i$$
$$= (2200\ \text{kg})(+26\ \text{m/s}) \quad \text{Substitute } m = 2200\ \text{kg},\ v_i = +26\ \text{m/s}$$
$$= +5.7\times10^4\ \text{kg·m/s}$$

Determine the final momentum, p_f.

$$p_f = mv_f$$
$$= (2200\ \text{kg})(+0.0\ \text{m/s}) \quad \text{Substitute } m = 2200\ \text{kg},\ v_f = +0.0\ \text{m/s}$$
$$= +0.0\ \text{kg·m/s}$$

Apply the impulse-momentum theorem to obtain the force needed to stop the vehicle.

$$F\Delta t = p_f - p_i$$
$$F\Delta t = (+0.0\ \text{kg·m/s}) - (5.7\times10^4\ \text{kg·m/s}) \quad \text{Substitute } p_f = 0.0\ \text{kg·m/s},\ p_i = 5.7\times10^4\ \text{kg·m/s}$$
$$= -5.7\times10^4\ \text{kg·m/s}$$
$$F = \frac{-5.7\times10^4\ \text{kg·m/s}}{\Delta t}$$

$$F_{\text{gentle braking}} = \frac{-5.7\times10^4\ \text{kg·m/s}}{21\ \text{s}} \quad \text{Substitute } \Delta t_{\text{gentle braking}} = 21\ \text{s}$$
$$= -2.7\times10^3\ \text{N}$$
$$F_{\text{hard braking}} = \frac{-5.7\times10^4\ \text{kg·m/s}}{3.8\ \text{s}} \quad \text{Substitute } \Delta t_{\text{hard braking}} = 3.8\ \text{s}$$
$$= -1.5\times10^4\ \text{N}$$
$$F_{\text{hitting a wall}} = \frac{-5.7\times10^4\ \text{kg·m/s}}{0.22\ \text{s}} \quad \text{Substitute } \Delta t_{\text{hitting a wall}} = 0.22\ \text{s}$$
$$= -2.6\times10^5\ \text{N}$$

Math Handbook
Operations with Significant Digits
pages 835–836

3 Evaluate the Answer
- **Are the units correct?** Force is measured in newtons.
- **Does the direction make sense?** Force is exerted in the direction opposite to the velocity of the car and thus, is negative.
- **Is the magnitude realistic?** People weigh hundreds of newtons, so it is reasonable that the force needed to stop a car would be in the thousands of newtons. The impulse is the same for all three stops. Thus, as the stopping time is shortened by more than a factor of 10, the force is increased by more than a factor of 10.

PHYSICS PROJECT Activity

Seat Belts and Airbags Have students explore how airbags work to reduce forces exerted in automobile collisions. Have them research and download film clips that show how airbag inflation can "soften the blow" to the crash dummies. Students can each develop a demo using a laboratory cart, a clay "passenger," and various strategies to protect the passenger when the cart crashes into a barrier. The sharp edge of the cart can be covered with rubber to model a padded dashboard. Ribbons can model seat belts, and a soft balloon can be the airbag. Some students may even opt to equip their carts with foam bumpers. **L2 Kinesthetic**

PRACTICE Problems

Additional Problems, Appendix B

1. A compact car, with mass 725 kg, is moving at 115 km/h toward the east. Sketch the moving car.

a. Find the magnitude and direction of its momentum. Draw an arrow on your sketch showing the momentum.

b. A second car, with a mass of 2175 kg, has the same momentum. What is its velocity?

2. The driver of the compact car in the previous problem suddenly applies the brakes hard for 2.0 s. As a result, an average force of 5.0×10^3 N is exerted on the car to slow it down.

a. What is the change in momentum; that is, the magnitude and direction of the impulse, on the car?

b. Complete the "before" and "after" sketches, and determine the momentum and the velocity of the car now.

3. A 7.0-kg bowling ball is rolling down the alley with a velocity of 2.0 m/s. For each impulse, shown in **Figures 9-3a** and **9-3b,** find the resulting speed and direction of motion of the bowling ball.

4. The driver accelerates a 240.0-kg snowmobile, which results in a force being exerted that speeds up the snowmobile from 6.00 m/s to 28.0 m/s over a time interval of 60.0 s.

a. Sketch the event, showing the initial and final situations.

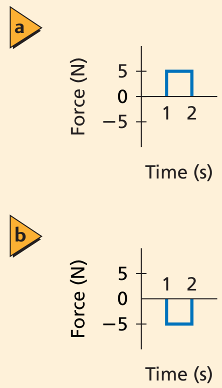
b. What is the snowmobile's change in momentum? What is the impulse on the snowmobile?

c. What is the magnitude of the average force that is exerted on the snowmobile?

5. Suppose a 60.0-kg person was in the vehicle that hit the concrete wall in Example Problem 1. The velocity of the person equals that of the car both before and after the crash, and the velocity changes in 0.20 s. Sketch the problem.

a. What is the average force exerted on the person?

b. Some people think that they can stop their bodies from lurching forward in a vehicle that is suddenly braking by putting their hands on the dashboard. Find the mass of an object that has a weight equal to the force you just calculated. Could you lift such a mass? Are you strong enough to stop your body with your arms?



■ **Figure 9-3**

Angular Momentum

As you learned in Chapter 8, the angular velocity of a rotating object changes only if torque is applied to it. This is a statement of Newton's law for rotational motion, $\tau = I\Delta\omega/\Delta t$. This equation can be rearranged in the same way as Newton's second law of motion was, to produce $\tau\Delta t = I\Delta\omega$.

The left side of this equation, $\tau\Delta t$, is the angular impulse of the rotating object. The right side can be rewritten as $\Delta\omega = \omega_f - \omega_i$. The product of a rotating object's moment of inertia and angular velocity is called **angular momentum,** which is represented by the symbol L. The angular momentum of an object can be represented by the following equation.

Angular Momentum $L = I\omega$

The angular momentum of an object is equal to the product of the object's moment of inertia and the object's angular velocity.

TEACHER TO TEACHER

Activity

Egg-Drop Physics Have students construct containers for their eggs that will be dropped from a significant height; for example, the top of a stairwell in your school. Regulate the mass of each container so that the change in momentum or impulse of impact is the same for each container. The goal is to reduce the force of impact on the egg so that the egg will not break. Students can do this by using air drag to reduce the maximum velocity of the container. Or they can increase the time of impact of the egg inside the container to decrease the force. **L2 Kinesthetic**

— Stephen Bailey • *Center for Discovery Learning* • Lakewood, Colorado

PRACTICE Problems

1. See Solutions Manual.

a. 2.32×10^4 kg·m/s eastward

b. 38.4 km/h eastward

2. a. 1.0×10^4 N·s directed westward

b. See Solutions Manual. 1.3×10^4 kg·m/s eastward, 65 km/h eastward

3. a. 2.7 m/s in the same direction as the original velocity

b. 1.3 m/s in the same direction as the original velocity

4. a. See Solutions Manual.

b. 5.28×10^3 kg·m/s

c. 88.0 N

5. See Solutions Manual.

a. 7.8×10^3 N opposite to the direction of motion

b. 8.0×10^2 kg; such a mass is too heavy to lift. You cannot safely stop yourself with your arms.

ACTIVITY

■ **Angular Impulse** Illustrate angular momentum either by using a stool that rotates freely or by spinning a bicycle wheel with a weighted tire and an extended axle for easy handling. If using the stool, it may be best if you sit on it while twirling. In either case, ask students how to get you or the wheel rotating. Apply a torque, such as shoving off the side of a desk with your hand if you are on a stool, or in the case of the wheel, by pushing off the axle. Ask students what they would do to increase the rotation speed. Increase rotation speed by applying a larger torque over a longer period—in other words, by increasing the angular impulse. **L2 Logical-Mathematical**

Concept Development

Angular Momentum versus Angular Velocity Ask students to define both angular momentum and angular velocity. Angular momentum, the momentum of an object rotating about an axis, is the product of an object's moment of inertia and its angular velocity. Angular velocity is an object's rate of rotation around an axis.

Discussion

Question How does angular momentum differ from angular velocity?

Answer The two differ by the object's moment of inertia. L2

Critical Thinking

Rotating Systems Have students brainstorm examples of rotating systems that have angular momentum. Have students draw examples from everyday life and from nature—consider, for example, the fields of astronomy, meteorology, and sports. Examples of rotating systems include galaxies, solar systems, Earth and other planets, storms like hurricanes and tornadoes, high- and low-pressure areas, whirlpools and eddies, spinning balls, swinging bats, gymnasts, dancers, divers, spinning wheels, rotating doors, drills, and rotary saw blades. L3

Angular momentum is measured in kg·m^2/s. Just as the linear momentum of an object changes when an impulse acts on it, the angular momentum of an object changes when an angular impulse acts on it. Thus, the angular impulse on the object is equal to the change in the object's angular momentum, which is called the **angular impulse-angular momentum theorem.** The angular impulse-angular momentum theorem is represented by the following equation.

Angular Impulse-Angular Momentum Theorem $\tau\Delta t = L_f - L_i$

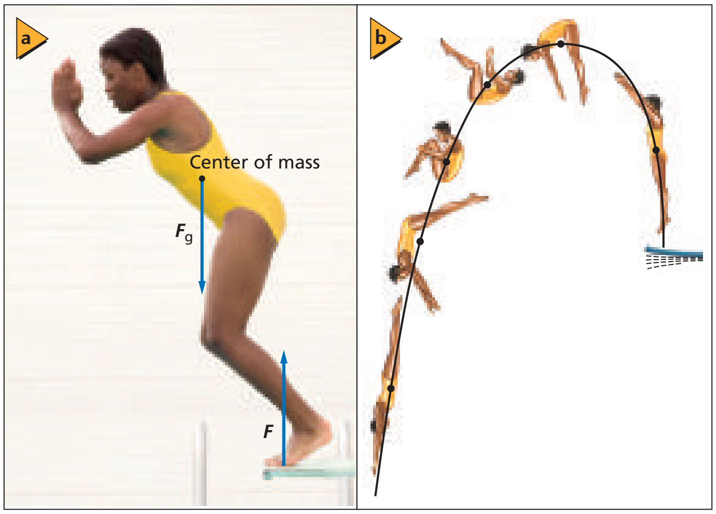
The angular impulse on an object is equal to the object's final angular momentum minus the object's initial angular momentum.

If there are no forces acting on an object, its linear momentum is constant. If there are no torques acting on an object, its angular momentum is also constant. Because an object's mass cannot be changed, if its momentum is constant, then its velocity is also constant. In the case of angular momentum, however, the object's angular velocity does not remain constant. This is because the moment of inertia depends on the object's mass and the way it is distributed about the axis of rotation or revolution. Thus, the angular velocity of an object can change even if no torques are acting on it.

Astronomy Connection

Consider, for example, a planet orbiting the Sun. The torque on the planet is zero because the gravitational force acts directly toward the Sun. Therefore, the planet's angular momentum is constant. When the distance between the planet and the Sun decreases, however, the planet's moment of inertia of revolution in orbit about the Sun also decreases. Thus, the planet's angular velocity increases and it moves faster. This is an explanation of Kepler's second law of planetary motion, based on Newton's laws of motion.

Figure 9-4 The diver's center of mass is in front of her feet as she gets ready to dive **(a).** As the diver changes her moment of inertia by moving her arms and legs to increase her angular momentum, the location of the center of mass changes, but the path of the center of mass remains a parabola **(b).**



Tim Fuller

CHALLENGE Activity

Applications of Rotation Have students research various models of sport-utility vehicles, jeeps, and other passenger vehicles to assess them for safety. How well do they handle sharp turns when traveling at high or even moderate speeds? Ask students to choose the vehicle they believe is best designed to tilt at large angles. They also can choose a poor design. Have them determine which, if any, of these models is prone to rollover. Ask students to prepare and present a safety report with a model showing the vehicle's center of mass, base, point of support, and axis of rotation. Have students explain to the class how their vehicle is designed for safety and how it maintains stability when tipped or tilted. L2 **Visual-Spatial**

Consider the diver in **Figure 9-4.** How does she start rotating her body? She uses the diving board to apply an external torque to her body. Then, she moves her center of mass in front of her feet and uses the board to give a final upward push to her feet. This torque acts over time, Δt, and thus increases the angular momentum of the diver.

Before the diver reaches the water, she can change her angular velocity by changing her moment of inertia. She may go into a tuck position, grabbing her knees with her hands. By moving her mass closer to the axis of rotation, the diver decreases her moment of inertia and increases her angular velocity. When she nears the water, she stretches her body straight, thereby increasing the moment of inertia and reducing the angular velocity. As a result, she goes straight into the water.

An ice-skater uses a similar method to spin. To begin rotating on one foot, the ice-skater applies an external torque to her body by pushing a portion of the other skate into the ice, as shown in **Figure 9-5.** If she pushes on the ice in one direction, the ice will exert a force on her in the opposite direction. The force results in a torque if the force is exerted some distance away from the pivot point, and in a direction that is not toward it. The greatest torque for a given force will result if the push is perpendicular to the lever arm.

The ice-skater then can control his angular velocity by changing her moment of inertia. Both arms and one leg can be extended from the body to slow the rotation, or pulled in close to the axis of rotation to speed it up. To stop spinning, another torque must be exerted by using the second skate to create a way for the ice to exert the needed force.

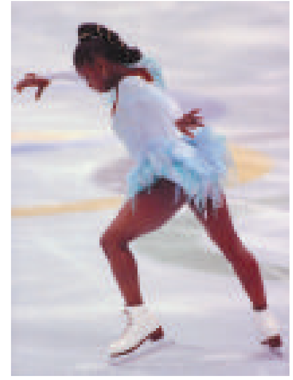


Figure 9-5 To spin on one foot, an ice-skater extends one leg and pushes on the ice. The ice exerts an equal and opposite force on her body and produces an external torque.

9.1 Section Review

6. **Momentum** Is the momentum of a car traveling south different from that of the same car when it travels north at the same speed? Draw the momentum vectors to support your answer.

7. **Impulse and Momentum** When you jump from a height to the ground, you let your legs bend at the knees as your feet hit the floor. Explain why you do this in terms of the physics concepts introduced in this chapter.

8. **Momentum** Which has more momentum, a supertanker tied to a dock or a falling raindrop?

9. **Impulse and Momentum** A 0.174-kg softball is pitched horizontally at 26.0 m/s. The ball moves in the opposite direction at 38.0 m/s after it is hit by the bat.
   a. Draw arrows showing the ball's momentum before and after the bat hits it.
   b. What is the change in momentum of the ball?
   c. What is the impulse delivered by the bat?
   d. If the bat and softball are in contact for 0.80 ms, what is the average force that the bat exerts on the ball?

10. **Momentum** The speed of a basketball as it is dribbled is the same when the ball is going toward the floor as it is when the ball rises from the floor. Is the basketball's change in momentum equal to zero when it hits the floor? If not, in which direction is the change in momentum? Draw the basketball's momentum vectors before and after it hits the floor.

11. **Angular Momentum** An ice-skater spins with his arms outstretched. When he pulls his arms in and raises them above his head, he spins much faster than before. Did a torque act on the ice-skater? If not, how could his angular velocity have increased?

12. **Critical Thinking** An archer shoots arrows at a target. Some of the arrows stick in the target, while others bounce off. Assuming that the masses of the arrows and the velocities of the arrows are the same, which arrows produce a bigger impulse on the target? *Hint: Draw a diagram to show the momentum of the arrows before and after hitting the target for the two instances.*

Physics Online physicspp.com/self_check_quiz

3 ASSESS

Check for Understanding

Bounce Impulse Bounce a ball on the floor. Have students diagram the initial and final momenta and the impulse. Ask students what provides the impulse. The floor provides the impulse. Have students compare the impulses on two balls of different mass. Both balls hit the floor with the same velocity, but the ball with the larger mass has the larger momentum, so it will have the higher impulse. L2

Extension

Specific Impulse Have students interested in model rocketry explain to the class how impulse and momentum are involved in the operation of rockets. The specific impulse of a rocket propellant is a rough measure of how fast the propellant is ejected out of the back of the rocket. Because space travel is concerned with acceleration, and acceleration is determined by thrust, the larger the exhaust velocity, the larger the specific impulse. In designing chemical-propellant rockets, the goal is not to minimize the amount of fuel but to maximize the thrust (force) per unit of fuel burned.
L3 **Logical-Mathematical**

9.1 Section Review

6. Yes, momentum is a vector quantity and the momenta of the two cars are in opposite directions. See Solutions Manual.
7. You reduce the force by increasing the length of time it takes to stop the motion of your body.
8. The raindrop has more momentum, because a supertanker at rest has zero momentum.
9. a. See Solutions Manual.
   b. 11.1 kg·m/s
   c. 11.1 N·s
   d. 1.4×10^4 N
10. No, the change in momentum is upward. Before the ball hits the floor, its momentum vector is downward. After the ball hits the floor, its momentum vector is upward.
11. No torque acted on him. Drawing his arms in decreased his moment of inertia. The angular momentum did not change; his angular velocity increased.
12. The ones that bounce off give larger impulses because they end up with some momentum in the reverse direction.

1 FOCUS

Bellringer Activity

Newton's Cradle Obtain a Newton's cradle apparatus that has six or seven steel balls suspended from two parallel bars. Pull all but two steel balls out of the way. Pull back one of the two steel balls and let it collide into the other one. Have students describe the collision. Repeat, but this time let one steel ball collide into three steel balls. Before you release the steel ball, ask students to predict the outcome of the collision on the system.
L1 Visual-Spatial

Tie to Prior Knowledge

Newton's Laws of Motion Newton's first and third laws of motion (Chapter 6) are related to conservation of momentum. Two-dimensional collisions will require vector addition. Students will use what they know about accelerated motion (Chapter 3), angular velocity and angular momentum (previous section), and rotational dynamics (Chapter 8) to understand the law of conservation of angular momentum.

2 TEACH

Identifying Misconceptions

System of Objects In Section 9.1, systems were limited to single bodies. Ask students if it is true that a system can only be a single object or body. No. A system can have more than one object, even if the objects are not attached to each other. Describe the Earth-Moon system and how it moves as a single system in orbit around the Sun, even though Earth and the Moon are not physically attached and move relative to each other. The Earth-Moon system is not a closed system, because of the role of the Sun.

9.2 Conservation of Momentum

▶ Objectives
- **Relate** Newton's third law to conservation of momentum.
- **Recognize** the conditions under which momentum is conserved.
- **Solve** conservation of momentum problems.

▶ Vocabulary

closed system
isolated system
law of conservation of momentum
law of conservation of angular momentum

In the first section of this chapter, you learned how a force applied during a time interval changes the momentum of a baseball. In the discussion of Newton's third law of motion, you learned that forces are the result of interactions between two objects. The force of a bat on a ball is accompanied by an equal and opposite force of the ball on the bat. Does the momentum of the bat, therefore, also change?

Two-Particle Collisions

The bat, the hand and arm of the batter, and the ground on which the batter is standing are all objects that interact when a batter hits the ball. Thus, the bat cannot be considered a single object. In contrast to this complex system, examine for a moment the much simpler system shown in **Figure 9-6,** the collision of two balls.

During the collision of the two balls, each one briefly exerts a force on the other. Despite the differences in sizes and velocities of the balls, the forces that they exert on each other are equal and opposite, according to Newton's third law of motion. These forces are represented by the following equation: $\boldsymbol{F}_{\text{D on C}} = -\boldsymbol{F}_{\text{C on D}}$

How do the impulses imparted by both balls compare? Because the time intervals over which the forces are exerted are the same, the impulses must be equal in magnitude but opposite in direction. How did the momenta of the balls change as a result of the collision?

According to the impulse-momentum theorem, the change in momentum is equal to the impulse. Compare the changes in the momenta of the two balls.

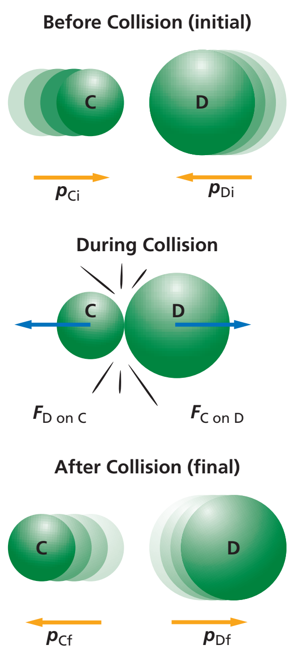
$$\text{For ball C: } \boldsymbol{p}_{\text{Cf}} - \boldsymbol{p}_{\text{Ci}} = \boldsymbol{F}_{\text{D on C}}\Delta t$$

$$\text{For ball D: } \boldsymbol{p}_{\text{Df}} - \boldsymbol{p}_{\text{Di}} = \boldsymbol{F}_{\text{C on D}}\Delta t$$

Because the time interval over which the forces were exerted is the same, the impulses are equal in magnitude, but opposite in direction. According to Newton's third law of motion, $-\boldsymbol{F}_{\text{C on D}} = \boldsymbol{F}_{\text{D on C}}$. Thus,

$$\boldsymbol{p}_{\text{Cf}} - \boldsymbol{p}_{\text{Ci}} = -(\boldsymbol{p}_{\text{Df}} - \boldsymbol{p}_{\text{Di}}), \text{ or } \boldsymbol{p}_{\text{Cf}} + \boldsymbol{p}_{\text{Df}} = \boldsymbol{p}_{\text{Ci}} + \boldsymbol{p}_{\text{Di}}.$$

This equation states that the sum of the momenta of the balls is the same before and after the collision. That is, the momentum gained by ball D is equal to the momentum lost by ball C. If the system is defined as the two balls, the momentum of the system is constant, and therefore, momentum is conserved for the system.



■ **Figure 9-6** When two balls collide, they exert forces on each other that change their momenta.

Momentum in a Closed, Isolated System

Under what conditions is the momentum of the system of two balls conserved? The first and most obvious condition is that no balls are lost and no balls are gained. Such a system, which does not gain or lose mass, is said to be a **closed system.** The second condition required to conserve the momentum of a system is that the forces involved are internal forces; that is, there are no forces acting on the system by objects outside of it.

9.2 Resource MANAGER

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- Transparency 9–3 Master, p. 133
- Transparency 9–4 Master, p. 135
- Study Guide, pp. 117–122
- Reinforcement, p. 125
- Enrichment, pp. 127–128
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Teaching Transparency 9-2
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When the net external force on a closed system is zero, the system is described as an **isolated system.** No system on Earth can be said to be absolutely isolated, however, because there will always be some interactions between a system and its surroundings. Often, these interactions are small enough to be ignored when solving physics problems.

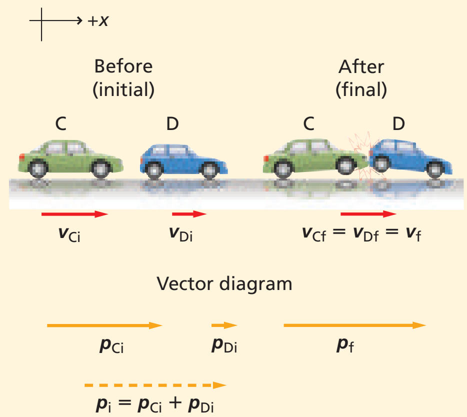
Systems can contain any number of objects, and the objects can stick together or come apart in a collision. Under these conditions, the **law of conservation of momentum** states that the momentum of any closed, isolated system does not change. This law will enable you to make a connection between conditions, before and after an interaction, without knowing any of the details of the interaction.

▶ EXAMPLE Problem 2

Speed A 1875-kg car going 23 m/s rear-ends a 1025-kg compact car going 17 m/s on ice in the same direction. The two cars stick together. How fast do the two cars move together immediately after the collision?

1 Analyze and Sketch the Problem

- Define the system.
- Establish a coordinate system.
- Sketch the situation showing the "before" and "after" states.
- Draw a vector diagram for the momentum.



Known:	Unknown:
$m_C = 1875$ kg	$v_f = ?$
$v_{Ci} = +23$ m/s	
$m_D = 1025$ kg	
$v_{Di} = +17$ m/s	

2 Solve for the Unknown

Momentum is conserved because the ice makes the total external force on the cars nearly zero.

$$p_i = p_f$$

$$p_{Ci} + p_{Di} = p_{Cf} + p_{Df}$$

$$m_C v_{Ci} + m_D v_{Di} = m_C v_{Cf} + m_D v_{Df}$$

Because the two cars stick together, their velocities after the collision, denoted as v_f, are equal.

$$v_{Cf} = v_{Df} = v_f$$

$$m_C v_{Ci} + m_D v_{Di} = (m_C + m_D) v_f$$

Solve for v_f.

$$v_f = \frac{(m_C v_{Ci} + m_D v_{Di})}{(m_C + m_D)}$$

$$= \frac{(1875\ \text{kg})(+23\ \text{m/s}) + (1025\ \text{kg})(+17\ \text{m/s})}{(1875\ \text{kg} + 1025\ \text{kg})}$$

Substitute $m_C = 1875$ kg, $v_{Ci} = +23$ m/s, $m_D = 1025$ kg, $v_{Di} = +17$ m/s

$$= +21\ \text{m/s}$$

Math Handbook
Order of Operations
page 843

3 Evaluate the Answer

- **Are the units correct?** Velocity is measured in m/s.
- **Does the direction make sense?** v_i and v_f are in the positive direction; therefore, v_f should be positive.
- **Is the magnitude realistic?** The magnitude of v_f is between the initial speeds of the two cars, but closer to the speed of the more massive one, so it is reasonable.

Discussion

Question How do Newton's laws of motion relate to a closed, isolated system of two objects that collide?

Answer The two objects will follow Newton's third law of motion when they collide. Each object exerts an equal and opposite force on the other during the collision. The two objects will continue to move together. If you could calculate the center of mass of the system, you would see that it moves with a constant velocity before, during, and after the collision, according to Newton's first law of motion. L2

▶ IN-CLASS Example

Question A 1875-kg car going 23 m/s strikes a 1025-kg car going 17 m/s in the opposite direction. The two cars, traveling on ice, stick together. How fast do they move after the collision?

Answer Let $v_{Di} = -17$ m/s. Then

$$v_f = \frac{(1875\ \text{kg})(23\ \text{m/s}) + \cdots}{(1875\ \text{kg} + \cdots}\ \frac{(1025\ \text{kg})(-17\ \text{m/s})}{1025\ \text{kg})}$$

$v_f = 8.9$ m/s. Note that this is slower than the rear-end type of collision in the example.

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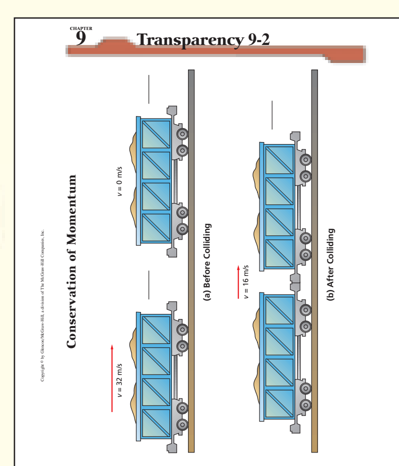


TEACHER TO TEACHER Activity

Force Shield Place a massive object on your hand, then hit it with a small hammer. Show the students that this does not hurt your hand. Due to conservation of momentum, the large mass of the object in your hand compared to the mass of the hammer causes the high velocity of the hammer to be converted into a low velocity of the object—so low that it does not crush your hand. L2 Logical-Mathematical

— **Paul Tiffany** • *Underwood Public School* • Underwood, Minnesota

PRACTICE Problems

13. 1.1 m/s

14. 0.034 m/s

15. 1.2×10^3 m/s

16. 2.8 m/s

17. 6.7 m/s

18. 2.0 m/s in the opposite direction

Identifying Misconceptions

Rocket Propulsion When Robert Goddard began his rocket experiments, a prominent newspaper said he was doomed to failure because, as any high school student knew, the rocket could move only if the gases that were expelled pushed on air, and there was no air in space. How, then, do rockets move? The newspaper had it all wrong: Gases expelled from a rocket do not push on the air. The gases push on the rocket itself. It is the forward push of the gases that accelerates the rocket. L2

THE MECHANICAL UNIVERSE
HIGH SCHOOL ADAPTATION

Videotape

Angular Momentum; Conservation of Momentum

PRACTICE Problems

Additional Problems, Appendix B

13. Two freight cars, each with a mass of 3.0×10^5 kg, collide and stick together. One was initially moving at 2.2 m/s, and the other was at rest. What is their final speed?

14. A 0.105-kg hockey puck moving at 24 m/s is caught and held by a 75-kg goalie at rest. With what speed does the goalie slide on the ice?

15. A 35.0-g bullet strikes a 5.0-kg stationary piece of lumber and embeds itself in the wood. The piece of lumber and bullet fly off together at 8.6 m/s. What was the original speed of the bullet?

16. A 35.0-g bullet moving at 475 m/s strikes a 2.5-kg bag of flour that is on ice, at rest. The bullet passes through the bag, as shown in **Figure 9-7,** and exits it at 275 m/s. How fast is the bag moving when the bullet exits?

17. The bullet in the previous problem strikes a 2.5-kg steel ball that is at rest. The bullet bounces backward after its collision at a speed of 5.0 m/s. How fast is the ball moving when the bullet bounces backward?

18. A 0.50-kg ball that is traveling at 6.0 m/s collides head-on with a 1.00-kg ball moving in the opposite direction at a speed of 12.0 m/s. The 0.50-kg ball bounces backward at 14 m/s after the collision. Find the speed of the second ball after the collision.

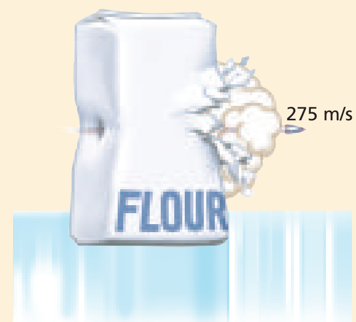


Figure 9-7

Recoil

It is very important to define a system carefully. The momentum of a baseball changes when the external force of a bat is exerted on it. The baseball, therefore, is not an isolated system. On the other hand, the total momentum of two colliding balls within an isolated system does not change because all forces are between the objects within the system.

Can you find the final velocities of the two in-line skaters in **Figure 9-8?** Assume that they are skating on a smooth surface with no external forces. They both start at rest, one behind the other.

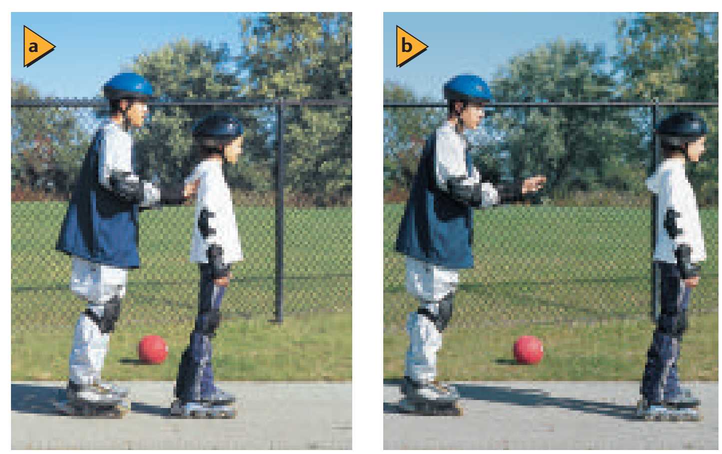


Figure 9-8 The internal forces exerted by Skater C, the boy, and Skater D, the girl, cannot change the total momentum of the system.

Laura Sifferlin

Teacher F.Y.I.

REAL-LIFE PHYSICS

Sweet Spot Tennis rackets are designed to maximize the velocity given the ball and to help the player control the direction of the ball. The design also helps reduce the forces exerted by the racket on the player's hand. Players use the term *sweet spot* to denote the location on the racket where it feels good to hit the ball. When the ball is hit at the sweet spot the high-frequency vibrations of the racket are minimized. It also is the region where the coefficient of restitution (COR) is high. The COR is measured by dropping a ball on a rigidly held racket. It is defined as the ratio of the magnitude of the velocity of the ball leaving the racket to magnitude of the velocity just before it is hit.

Skater C, the boy, gives skater D, the girl, a push. Now, both skaters are moving, making this situation similar to that of an explosion. Because the push was an internal force, you can use the law of conservation of momentum to find the skaters' relative velocities. The total momentum of the system was zero before the push. Therefore, it must be zero after the push.

Before		After
$\boldsymbol{p}_{Ci} + \boldsymbol{p}_{Di}$	=	$\boldsymbol{p}_{Cf} + \boldsymbol{p}_{Df}$
0	=	$\boldsymbol{p}_{Cf} + \boldsymbol{p}_{Df}$
$\boldsymbol{p}_{Cf}$	=	$-\boldsymbol{p}_{Df}$
$m_C\boldsymbol{v}_{Cf}$	=	$-m_D\boldsymbol{v}_{Df}$

The coordinate system was chosen so that the positive direction is to the left. The momenta of the skaters after the push are equal in magnitude but opposite in direction. The backward motion of skater C is an example of recoil. Are the skaters' velocities equal and opposite? The last equation shown above, for the velocity of skater C, can be rewritten as follows:

$$\boldsymbol{v}_{Cf} = \left(\frac{-m_D}{m_C}\right)\boldsymbol{v}_{Df}$$

The velocities depend on the skaters' relative masses. If skater C has a mass of 68.0 kg and skater D's mass is 45.4 kg, then the ratio of their velocities will be 68.0 : 45.4, or 1.50. The less massive skater moves at the greater velocity. Without more information about how hard skater C pushed skater D, however, you cannot find the velocity of each skater.

Propulsion in Space

How does a rocket in space change its velocity? The rocket carries both fuel and oxidizer. When the fuel and oxidizer combine in the rocket motor, the resulting hot gases leave the exhaust nozzle at high speed. If the rocket and chemicals are the system, then the system is a closed system. The forces that expel the gases are internal forces, so the system is also an isolated system. Thus, objects in space can accelerate by using the law of conservation of momentum and Newton's third law of motion.

A NASA space probe, called *Deep Space 1*, performed a flyby of an asteroid a few years ago. The most unusual of the 11 new technologies on board was an ion engine that exerts as much force as a sheet of paper resting on a person's hand. The ion engine shown in **Figure 9-9,** operates differently from a traditional rocket engine. In a traditional rocket engine, the products of the chemical reaction taking place in the combustion chamber are released at high speed from the rear. In the ion engine, however, xenon atoms are expelled at a speed of 30 km/s, producing a force of only 0.092 N. How can such a small force create a significant change in the momentum of the probe? Instead of operating for only a few minutes, as the traditional chemical rockets do, the ion engine can run continuously for days, weeks, or months. Therefore, the impulse delivered by the engine is large enough to increase the momentum of the 490-kg spacecraft until it reaches the speed needed to complete its mission.

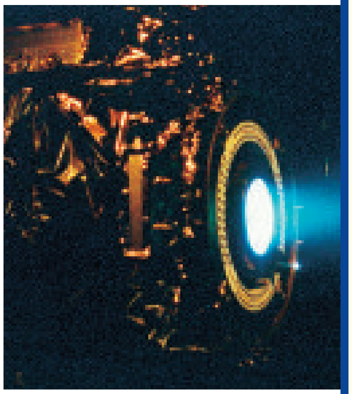


Figure 9-9 The xenon atoms in the ion engine are ionized by bombarding them with electrons. Then, the positively charged xenon ions are accelerated to high speeds.

MINI LAB

Rebound Height

An object's momentum is the product of its mass and velocity.

1. Drop a large rubber ball from about 15 cm above a table.
2. Measure and record the ball's rebound height.
3. Repeat steps 1–2 with a small rubber ball.
4. Hold the small rubber ball on top of, and in contact with, the large rubber ball.
5. Release the two rubber balls from the same height, so that they fall together.
6. Measure the rebound heights of both rubber balls.

Analyze and Conclude

7. Describe the rebound height of each rubber ball dropped by itself.
8. Compare and contrast the rebound heights from number 7 with those from number 6.
9. Explain your observations.

NASA

MINI LAB

Rebound Height

See page 109 of **FAST FILE Chapters 6–10 Resources** *for the accompanying Mini Lab Worksheet.*

Purpose Students will observe, measure, compare, and contrast the rebound heights of rubber balls dropped individually and together.

Materials one small hard rubber ball, about the size of a table-tennis ball; one large hard rubber ball, the size of a tennis ball; meterstick

CAUTION: When balls are dropped together, be sure that students are out of the way of the rebounding balls.

Expected Results The rebound heights will vary depending on the type of rubber balls used. However, rebounds of 75–80 percent are typical for individual rubber balls. When dropped together, the small rubber ball will rebound much higher, possibly as much as four times as high, while the large rubber ball will rebound to a much lower height.

Analyze and Conclude

7. Answers will vary. The large rubber ball and the small rubber ball rebound to about 80 percent of the heights from which they were dropped.

8. Answers will vary. The large rubber ball rebounds to a much lower height, while the small rubber ball rebounds much higher. The large ball only rebounds to about 3 cm, from 15 cm, while the small ball rebounds to about 60 cm.

9. Momentum was transferred from the large rubber ball to the small rubber ball, causing the large ball to rebound to a lower height. The small ball, having a smaller mass, rebounds much higher. Thus momentum is conserved during the interaction.

DIFFERENTIATED INSTRUCTION — Activity

Visually Impaired Allow students to feel the recoil. Blow up a balloon and hand it to a student. Make sure the student keeps the neck closed tightly to prevent the air from escaping. Have the student place the opposite end of the balloon on the palm of his or her hand. Then, ask the student to open the neck of the balloon to release the air. The student should feel the force of the air on the balloon on his or her hand. The balloon is pushed forward by the force of air on the front of the balloon which is no longer balanced by the force of the air on the back of the balloon because the air there is escaping through the neck. **L2 Kinesthetic**

▶ IN-CLASS Example

Question What if the same astronaut in Example Problem 3 had a mass of only 62 kg? What would be the final speed of the astronaut?

Answer The same analysis and final equation can be used, but with a different mass:

$$v_{Cf} = \left(\frac{-m_D v_{Df}}{m_C}\right)$$

$$= \frac{-(0.035\text{ kg})(-875\text{ m/s})}{62\text{ kg}}$$

$$v_{Cf} = -0.49\text{ m/s}$$

▶ PRACTICE Problems

19. 7.91 m/s

20. 9.0 cm/s to the right

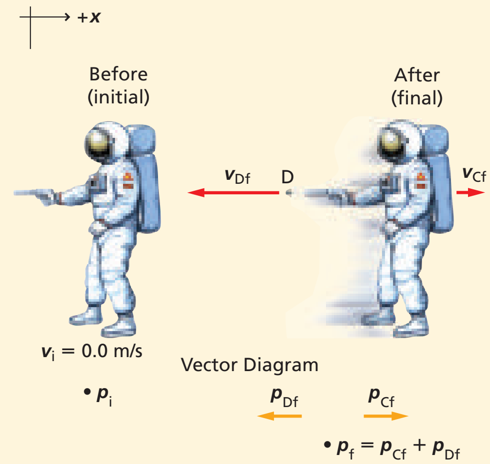
21. 2.8 m/s in the opposite direction

Using Models

Multi-Stage Rocket Model a multi-stage rocket with four or more students on skateboards or wearing inline skates. *CAUTION: Students must wear approved safety equipment. The instructor must clear the area of obstacles and carefully monitor the activity.* The students should be of varying weights. Place them in a line with the lightest person in front. Each should place his or her hands on the shoulders of the person in front, with elbows bent. The person at the back pushes off first, modeling the gases expelled by the first stage. The remaining people will move slowly forward. After a short interval, the person remaining at the back pushes off. This process continues until all have pushed away. The last and smallest person models the payload and should end up moving quite fast. **L1 Kinesthetic**

▶ EXAMPLE Problem 3

Speed An astronaut at rest in space fires a thruster pistol that expels 35 g of hot gas at 875 m/s. The combined mass of the astronaut and pistol is 84 kg. How fast and in what direction is the astronaut moving after firing the pistol?



1 Analyze and Sketch the Problem

- Define the system.
- Establish a coordinate axis.
- Sketch the "before" and "after" conditions.
- Draw a vector diagram showing momenta.

Known:	**Unknown:**
m_C = 84 kg	v_{Cf} = ?
m_D = 0.035 kg	
$v_{Ci} = v_{Di}$ = +0.0 m/s	
v_{Df} = −875 m/s	

2 Solve for the Unknown

The system is the astronaut, the gun, and the chemicals that produce the gas.

$p_i = p_{Ci} + p_{Di} = +0.0\text{ kg·m/s}$ — Before the pistol is fired, all parts of the system are at rest; thus, the initial momentum is zero.

Use the law of conservation of momentum to find p_f.

$$p_i = p_f$$

$$+0.0\text{ kg·m/s} = p_{Cf} + p_{Df}$$

$$p_{Cf} = -p_{Df}$$

The momentum of the astronaut is equal in magnitude, but opposite in direction to the momentum of the gas leaving the pistol.

Solve for the final velocity of the astronaut, v_{Cf}.

$$m_C v_{Cf} = -m_D v_{Df}$$

$$v_{Cf} = \left(\frac{-m_D v_{Df}}{m_C}\right)$$

$$= \frac{-(0.035\text{ kg})(-875\text{ m/s})}{84\text{ kg}}$$

$$= +0.36\text{ m/s}$$

Substitute m_D = 0.035 kg, v_{Df} = −875 m/s, m_C = 84 kg

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Isolating a Variable
page 845

3 Evaluate the Answer

- **Are the units correct?** The velocity is measured in m/s.
- **Does the direction make sense?** The velocity of the astronaut is in the opposite direction to that of the expelled gas.
- **Is the magnitude realistic?** The astronaut's mass is much larger than that of the gas, so the velocity of the astronaut is much less than that of the expelled gas.

▶ PRACTICE Problems

Additional Problems, Appendix B

19. A 4.00-kg model rocket is launched, expelling 50.0 g of burned fuel from its exhaust at a speed of 625 m/s. What is the velocity of the rocket after the fuel has burned? *Hint: Ignore the external forces of gravity and air resistance.*

20. A thread holds a 1.5-kg cart and a 4.5-kg cart together. After the thread is burned, a compressed spring pushes the carts apart, giving the 1.5-kg cart a speed of 27 cm/s to the left. What is the velocity of the 4.5-kg cart?

21. Carmen and Judi dock a canoe. 80.0-kg Carmen moves forward at 4.0 m/s as she leaves the canoe. At what speed and in what direction do the canoe and Judi move if their combined mass is 115 kg?

HELPING STRUGGLING STUDENTS — Activity

Graphic and Algebraic Approaches Two-dimensional collisions give students who have had trouble with forces in two dimensions a second chance to learn how to handle vectors. Most students will be tempted to add momenta as if they were scalar quantities. Some will find the graphical approach easier to understand while others, who are not visual learners, will find the algebraic approach easier. Pick the approach that fits the student's learning style. Pair two students who have different strengths and have them work on the same problem, compare answers, and teach each other how to solve it their way. **L1**

Two-Dimensional Collisions

Up until now, you have looked at momentum in only one dimension. The law of conservation of momentum holds for all closed systems with no external forces. It is valid regardless of the directions of the particles before or after they interact. But what happens in two or three dimensions? **Figure 9-10** shows the result of billiard ball C striking stationary billiard ball D. Consider the two billiard balls to be the system. The original momentum of the moving ball is $\boldsymbol{p}_{\mathrm{Ci}}$ and the momentum of the stationary ball is zero. Therefore, the momentum of the system before the collision is equal to $\boldsymbol{p}_{\mathrm{Ci}}$.

After the collision, both billiard balls are moving and have momenta. As long as the friction with the tabletop can be ignored, the system is closed and isolated. Thus, the law of conservation of momentum can be used. The initial momentum equals the vector sum of the final momenta, so $\boldsymbol{p}_{\mathrm{Ci}} = \boldsymbol{p}_{\mathrm{Cf}} + \boldsymbol{p}_{\mathrm{Df}}$.

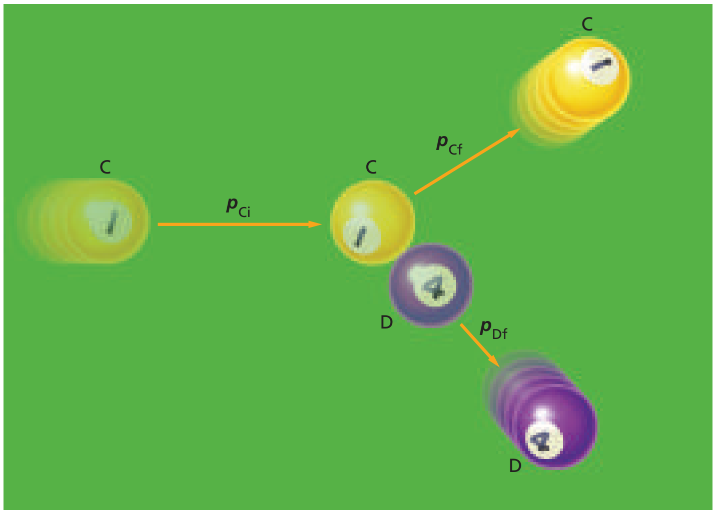
The equality of the momenta before and after the collision also means that the sum of the components of the vectors before and after the collision must be equal. Suppose the *x*-axis is defined to be in the direction of the initial momentum, then the *y*-component of the initial momentum is equal to zero. Therefore, the sum of the final *y*-components also must be zero:

$$p_{\mathrm{Cf},\,y} + p_{\mathrm{Df},\,y} = 0$$

The *y*-components are equal in magnitude but are in the opposite direction and, thus, have opposite signs. The sum of the horizontal components also is equal:

$$p_{\mathrm{Ci}} = p_{\mathrm{Cf},\,x} + p_{\mathrm{Df},\,x}$$

Figure 9-10 The law of conservation of momentum holds for all isolated, closed systems, regardless of the directions of objects before and after a collision.



Using Figure 9-10

Have students draw free-body diagrams of the collision. They can transfer Figure 9-10 onto a worksheet and then measure the lengths and angles of the momentum vectors. Explain that vectors can be moved around the page as long as their lengths and directions are not changed. Have students check to see if the vector sum of the two final momenta equals the initial momentum of ball C (because ball D starts at rest). **L2 Visual-Spatial**

Additional MINI LAB

Impact and Momentum

Purpose Use the law of conservation of momentum to determine velocity.

Materials cardboard box, packing material, softball, meterstick, spring scale calibrated in newtons, platform balance

Procedure

1. Pack a cardboard box loosely with packing material so that when you toss in a softball, it will stay there. Prepare a data table.

2. Measure and record the mass of the box and packing material. Place the box on a smooth surface and mark its starting position.

3. Toss a softball into the box. Measure and record the distance the box and ball moved.

4. Measure the force of friction by using the spring scale to pull the box and ball over the surface at constant velocity. Record data.

5. Measure and record the mass of the box with packing and ball.

Assessment Have students use the law of conservation of momentum and equations of motion (Chapter 3) to calculate the softball's velocity just before it hit the box. Use $F_{\mathrm{f}} = ma$ to calculate negative acceleration of the combined box and ball as they slid across the surface. Use $v_2^2 = 0 = v_1^2 + 2ad$ to calculate the initial velocity of the combined box and ball. *Hint: The momentum of the ball just before impact is equal to the momentum of the combined box, packing material, and ball after impact.*

Teacher F.Y.I. — REAL-LIFE CAREERS

Accident Reconstruction Expert Investigating automobile accidents requires an understanding of collisions, friction, and Newton's laws of motion. Accident reconstruction experts work in a variety of ways to determine the cause of automobile accidents using evidence such as tire tracks. Some of their techniques are illustrated in the practice problems in this section. They frequently serve as expert witnesses in court cases. In recent years, specialized data-collecting equipment and computer software have made their job easier. These experts have formed at least 20 professional organizations that help them improve their skills and exchange information. Both community colleges and universities have short training courses for investigators.

▶ IN-CLASS Example

Question A 975-kg car, C, moving south at 22.5 m/s, collides with a 2165-kg truck, D, moving west at 17.5 m/s. They stick together. In what direction and with what speed do they move after the collision?

Answer $m_C = 975$ kg, $m_D = 2165$ kg, $v_{Ci,y} = -22.5$ m/s, $v_{Di,x} = -17.5$ m/s. $p_{f,y} = p_{i,y} = m_C v_{Ci,y} = (975\text{ kg})(-22.5\text{ m/s}) = -2.19\times10^4$ kg·m/s.

$p_{f,x} = p_{i,x} = m_D v_{Di,x} = (216\text{ kg})(-17.5\text{ m/s}) = -3.78\times10^4$ kg·m/s.

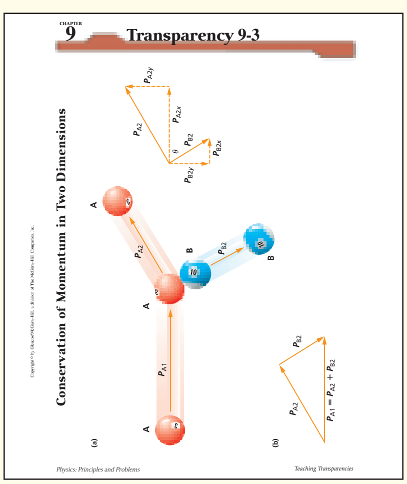
So $p_f = 4.37\times10^4$ kg·m/s. $v_f = p_f/(m_C + m_D) = 13.9$ m/s.

$\theta = \tan^{-1}$.

$$\frac{p_{f,x}}{p_{f,y}} = \left(\frac{-3.78\times10^4\text{ kg·m/s}}{-2.19\times10^4\text{ kg·m/s}}\right)$$

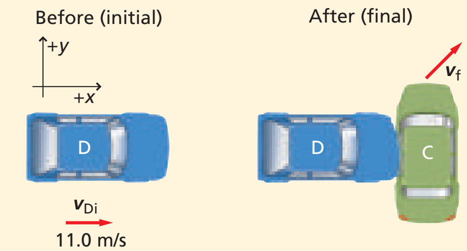
$\theta = 30.1°$.

Page 131, FAST FILE Chapters 6–10 Resources



▶ EXAMPLE Problem 4

Speed A 1325-kg car, C, moving north at 27.0 m/s, collides with a 2165-kg car, D, moving east at 11.0 m/s. The two cars are stuck together. In what direction and with what speed do they move after the collision?

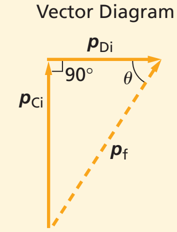


1 Analyze and Sketch the Problem

- Define the system.
- Sketch the "before" and "after" states.
- Establish the coordinate axis with the *y*-axis north and the *x*-axis east.
- Draw a momentum-vector diagram.

Known:	Unknown:
$m_C = 1325$ kg	$v_{f,x} = ?$
$m_D = 2165$ kg	$v_{f,y} = ?$
$v_{Ci,y} = 27.0$ m/s	$\theta = ?$
$v_{Di,x} = 11.0$ m/s	

Vector Diagram



2 Solve for the Unknown

Determine the initial momenta of the cars and the momentum of the system.

$$p_{Ci} = m_C v_{Ci,y}$$
$$= (1325\text{ kg})(27.0\text{ m/s}) \quad \text{Substitute } m_C = 1325\text{ kg}, v_{Ci,y} = 27.0\text{ m/s}$$
$$= 3.58\times10^4\text{ kg·m/s (north)}$$

$$p_{Di} = m_D v_{Di,x}$$
$$= (2165\text{ kg})(11.0\text{ m/s}) \quad \text{Substitute } m_D = 2165\text{ kg}, v_{Di,x} = 11.0\text{ m/s}$$
$$= 2.38\times10^4\text{ kg·m/s (east)}$$

Use the law of conservation of momentum to find p_f.

$$p_{f,x} = p_{i,x} = 2.38\times10^4\text{ kg·m/s} \quad \text{Substitute } p_{i,x} = p_{Di} = 2.38\times10^4\text{ kg·m/s}$$
$$p_{f,y} = p_{i,y} = 3.58\times10^4\text{ kg·m/s} \quad \text{Substitute } p_{i,y} = p_{Ci} = 3.58\times10^4\text{ kg·m/s}$$

Use the diagram to set up equations for $p_{f,x}$ and $p_{f,y}$.

$$p_f = \sqrt{(p_{f,x})^2 + (p_{f,y})^2}$$
$$= \sqrt{(2.38\times10^4\text{ kg·m/s})^2 + (3.58\times10^4\text{ kg·m/s})^2} \quad \text{Substitute } p_{f,x} = 2.38\times10^4\text{ kg·m/s}, p_{f,y} = 3.58\times10^4\text{ kg·m/s}$$
$$= 4.30\times10^4\text{ kg·m/s}$$

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Inverses of Sine, Cosine, and Tangent
page 856

Solve for θ.

$$\theta = \tan^{-1}\left(\frac{p_{f,y}}{p_{f,x}}\right)$$
$$= \tan^{-1}\left(\frac{3.58\times10^4\text{ kg·m/s}}{2.38\times10^4\text{ kg·m/s}}\right) \quad \text{Substitute } p_{f,y} = 3.58\times10^4\text{ kg·m/s}, p_{f,x} = 2.38\times10^4\text{ kg·m/s}$$
$$= 56.4°$$

Determine the final speed.

$$v_f = \frac{p_f}{(m_C + m_D)}$$
$$= \frac{4.30\times10^4\text{ kg·m/s}}{(1325\text{ kg} + 2165\text{ kg})} \quad \text{Substitute } p_f = 4.30\times10^4\text{ kg·m/s}, m_C = 1325\text{ kg}, m_D = 2165\text{ kg}$$
$$= 12.3\text{ m/s}$$

3 Evaluate the Answer

- **Are the units correct?** The correct unit for speed is m/s.
- **Do the signs make sense?** Answers are both positive and at the appropriate angles.
- **Is the magnitude realistic?** The cars stick together, so v_f must be smaller than v_{Ci}.

QUICK DEMO

Changing Moment of Inertia

Estimated Time 10 minutes

Materials rotating stool, two heavy blocks

Procedure Sit on the stool holding the blocks close to your body. Have a student gently spin you. Extend your arms and bring them back in. Discuss what happens in terms of conservation of angular momentum and changes in moment of inertia. How does that affect ω, angular velocity? When the moment of inertia (I) decreases—arms in closer—angular velocity (ω) increases and you spin faster. Moment of inertia increases when arms are extended; angular velocity decreases, and you spin more slowly.

PRACTICE Problems

Additional Problems, Appendix B

22. A 925-kg car moving north at 20.1 m/s collides with a 1865-kg car moving west at 13.4 m/s. The two cars are stuck together. In what direction and at what speed do they move after the collision?

23. A 1383-kg car moving south at 11.2 m/s is struck by a 1732-kg car moving east at 31.3 m/s. The cars are stuck together. How fast and in what direction do they move immediately after the collision?

24. A stationary billiard ball, with a mass of 0.17 kg, is struck by an identical ball moving at 4.0 m/s. After the collision, the second ball moves 60.0° to the left of its original direction. The stationary ball moves 30.0° to the right of the moving ball's original direction. What is the velocity of each ball after the collision?

25. A 1345-kg car moving east at 15.7 m/s is struck by a 1923-kg car moving north. They are stuck together and move with an initial velocity of 14.5 m/s at $\theta = 63.5°$. Was the north-moving car exceeding the 20.1 m/s speed limit?

Conservation of Angular Momentum

Like linear momentum, angular momentum can be conserved. The **law of conservation of angular momentum** states that if no net external torque acts on an object, then its angular momentum does not change. This is represented by the following equation.

Law of Conservation of Angular Momentum $L_1 = L_2$

An object's initial angular momentum is equal to its final angular momentum.

For example, Earth spins on its axis with no external torques. Its angular momentum is constant. Thus, Earth's angular momentum is conserved. As a result, the length of a day does not change. A spinning ice-skater also demonstrates conservation of angular momentum. **Figure 9-11a** shows an ice-skater spinning with his arms extended. When he pulls in his arms, as shown in **Figure 9-11b,** he begins spinning faster. Without an external torque, his angular momentum does not change; that is, $L = I\omega$ is constant. Thus, the ice-skater's increased angular velocity must be accompanied by a decreased moment of inertia. By pulling his arms close to his body, the ice-skater brings more mass closer to the axis of rotation, thereby decreasing the radius of rotation and decreasing his moment of inertia. You can calculate changes in angular velocity using the law of conservation of angular momentum.

$$L_i = L_f$$

$$\text{thus, } I_i\omega_i = I_f\omega_f$$

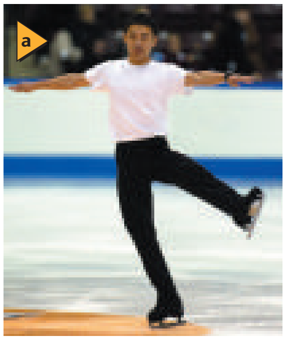
$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f}$$

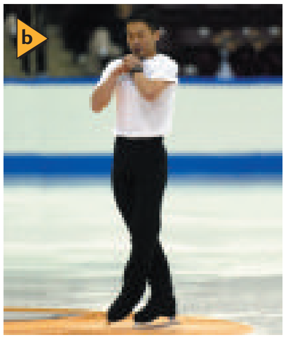
Because frequency is $f = \omega/2\pi$, the above equation can be rewritten as follows:

$$\frac{2\pi(f_f)}{2\pi(f_i)} = \frac{I_i}{I_f}$$

$$\text{thus, } \frac{f_f}{f_i} = \frac{I_i}{I_f}$$

Figure 9-11 When the ice-skater's arms are extended, the moment of inertia increases and his angular velocity decreases **(a).** When his arms are closer to his body the moment of inertia decreases and results in an increased angular velocity **(b).**





F. Scott Grant/IMAGE Communications

PRACTICE Problems

22. 11.2 m/s, 36.6° north of west

23. 18.1 m/s, 15.9° south of east

24. 3.5 m/s, 30.0° to the right and 2.0 m/s, 60.0° to the left

25. 22.1 m/s; Yes, it was exceeding the speed limit.

Using an Analogy

Clay and Inelastic Collisions Clay balls can model moving objects sticking and moving together. Construct balls to have masses that have the same ratio of masses as the objects in Example Problem 4. Use 1 g to represent 100 kg. Provide students with several examples of collisions, having them define the system, sketch the "before" and "after" states, and determine an unknown. **L2 Visual-Spatial**

Concept Development

Conservation of Angular Momentum Angular motion is different from linear motion in that the angular speed of a rotating object can be changed without changing its mass or angular momentum. Moment of inertia can be changed. Use the demonstration on page 242 to illustrate the concept. But in all discussions and demonstrations be sure to define the system carefully. Make sure that torques cannot be exerted between the system and the external world. For example, a spinning skater will eventually slow and stop because of the torque exerted on her by the ice via friction. **L2**

Teacher F.Y.I. CONTENT BACKGROUND

Orbiting versus Spinning Objects Angular momentum is one characteristic used to describe an object in motion about an axis. Because angular momentum is a vector quantity, a complete description includes both a magnitude and a direction. The angular momentum of an orbiting object is *mvr*, where *m* is the object's mass, *v* is its linear velocity, and *r* is the perpendicular distance from the center of rotation that passes through the object's center of gravity. In contrast, the angular momentum of a spinning object is represented by $I\omega$, where *I* is the moment of inertia and ω is the angular velocity.

Reinforcement

Conservation of Angular Momentum Ask students to write a description of angular momentum in their own words. Their descriptions must relate angular momentum to the moment of inertia and angular speed. Their descriptions should also explain how angular momentum is conserved. Example answer: Angular momentum depends on the angular speed and moment of inertia. Angular momentum is conserved as long as there is no net external torque acting on any object in the system. **L1 Linguistic**

Critical Thinking

Adding Mass to a Rotating System Ask students to imagine they are conducting a conservation of momentum laboratory. They are using a disk rotating at 25 rad/s that has a moment of inertia of 2.5 kg·m^2. They drop a metal ring on the rotating disk and observe that the new rotational speed is 18 rad/s. Assuming that angular momentum is conserved and that the metal ring initially was not rotating, ask students what its moment of inertia is. The initial and final angular momentum is (2.5 kg·m^2)(25 rad/s) = 63 kg·m^2/s. Because you know the final angular velocity, you find $I = L/\omega$ = (63 kg·m^2/s)/(18 rad/s) = 3.5 kg·m^2. Thus the added moment of inertia is 1.0 kg·m^2. **L3 Visual-Spatial**

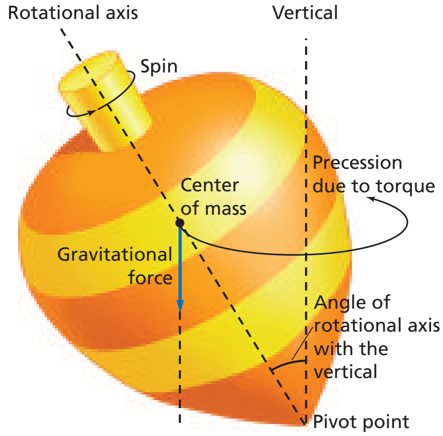


Figure 9-12 The upper end of the top precesses due to the torque acting on the top.

Notice that because f, ω, and I appear as ratios in these equations, any units may be used, as long as the same unit is used for both values of the quantity.

If a torque-free object starts with no angular momentum, it must continue to have no angular momentum. Thus, if part of an object rotates in one direction, another part must rotate in the opposite direction. For example, if you switch on a loosely held electric drill, the drill body will rotate in the direction opposite to the rotation of the motor and bit.

Consider a ball thrown at a weather vane. The ball, moving in a straight line, can start the vane rotating. Consider the ball and vane to be a system. With no external torques, angular momentum is conserved. The vane spins faster if the ball has a large mass, m, a large velocity, v, and hits at right angles as far as possible from the pivot of the vane. The angular momentum of a moving object, such as the ball, is given by $L = mvr$, where r is the perpendicular distance from the axis of rotation.

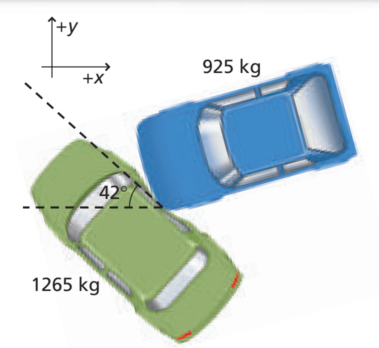
Tops and Gyroscopes

Because of the conservation of angular momentum, the direction of rotation of a spinning object can be changed only by applying a torque. If you played with a top as a child, you may have spun it by pulling the string wrapped around its axle. When a top is vertical, there is no torque on it, and the direction of its rotation does not change. If the top is tipped, as shown in **Figure 9-12,** a torque tries to rotate it downward. Rather than tipping over, however, the upper end of the top revolves, or precesses slowly about the vertical axis. Because Earth is not a perfect sphere, the Sun exerts a torque on it, causing it to precess. It takes about 26,000 years for Earth's rotational axis to go through one cycle of precession.

CHALLENGE PROBLEM

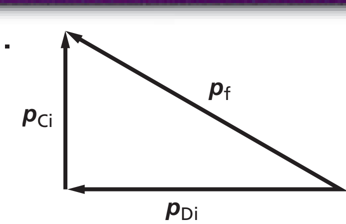
Your friend was driving her 1265-kg car north on Oak Street when she was hit by a 925-kg compact car going west on Maple Street. The cars stuck together and slid 23.1 m at 42° north of west. The speed limit on both streets is 22 m/s (50 mph). Assume that momentum was conserved during the collision and that acceleration was constant during the skid. The coefficient of kinetic friction between the tires and the pavement is 0.65.

1. Your friend claims that she wasn't speeding, but that the driver of other car was. How fast was your friend driving before the crash?
2. How fast was the other car moving before the crash? Can you support your friend's case in court?



CHALLENGE PROBLEM

1.



2.0×10^1 m/s

2. 3.0×10^1 m/s
The friend was not exceeding the 22 m/s speed limit. The other car was exceeding the limit.

file photo

A gyroscope, such as the one shown in **Figure 9-13,** is a wheel or disk that spins rapidly around one axis while being free to rotate around one or two other axes. The direction of its large angular momentum can be changed only by applying an appropriate torque. Without such a torque, the direction of the axis of rotation does not change. Gyroscopes are used in airplanes, submarines, and spacecraft to keep an unchanging reference direction. Giant gyroscopes are used in cruise ships to reduce their motion in rough water. Gyroscopic compasses, unlike magnetic compasses, maintain direction even when they are not on a level surface.

A football quarterback uses the gyroscope effect to make an accurate forward pass. As he throws, he spins, or spirals the ball. If the quarterback throws the ball in the direction of its spin axis of rotation, the ball keeps its pointed end forward, thereby reducing air resistance. Thus, the ball can be thrown far and accurately. If its spin direction is slightly off, the ball wobbles. If the ball is not spun, it tumbles end over end.

The flight of a plastic disk also is stabilized by spin. A well-spun plastic disk can fly many meters through the air without wobbling. You are able to perform tricks with a yo-yo because its fast rotational speed keeps it rotating in one plane.

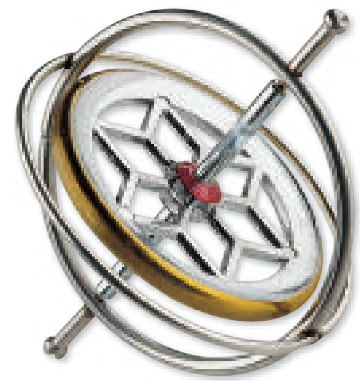


Figure 9-13 Because the orientation of the spin axis of the gyroscope does not change even when it is moved, the gyroscope can be used to fix direction.

9.2 Section Review

26. Angular Momentum The outer rim of a plastic disk is thick and heavy. Besides making it easier to catch, how does this affect the rotational properties of the plastic disk?

27. Speed A cart, weighing 24.5 N, is released from rest on a 1.00-m ramp, inclined at an angle of 30.0° as shown in **Figure 9-14.** The cart rolls down the incline and strikes a second cart weighing 36.8 N.

a. Calculate the speed of the first cart at the bottom of the incline.

b. If the two carts stick together, with what initial speed will they move along?

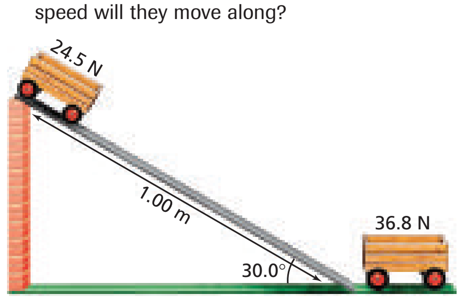


Figure 9-14

28. Conservation of Momentum During a tennis serve, the racket of a tennis player continues forward after it hits the ball. Is momentum conserved in the collision? Explain, making sure that you define the system.

29. Momentum A pole-vaulter runs toward the launch point with horizontal momentum. Where does the vertical momentum come from as the athlete vaults over the crossbar?

30. Initial Momentum During a soccer game, two players come from opposite directions and collide when trying to head the ball. They come to rest in midair and fall to the ground. Describe their initial momenta.

31. Critical Thinking You catch a heavy ball while you are standing on a skateboard, and then you roll backward. If you were standing on the ground, however, you would be able to avoid moving while catching the ball. Explain both situations using the law of conservation of momentum. Explain which system you use in each case.

Physics Online physicspp.com/self_check_quiz

file photo

3 ASSESS

Check for Understanding

Momentum in Proportion Using the principle of conservation of momentum, have students estimate the amount of linear momentum that Earth acquires when a person jumps into the air. If a person jumps up 0.80 m, the velocity when leaving the ground is 4.0 m/s. If the mass is 60.0 kg, the momentum is 240 kg·m/s. Ask students the following questions. What change would that make in Earth's velocity? Earth's mass is 6.0×10^{24} kg, so its velocity would be 4.0×10^{-23} m/s. What if one million people in New York City jumped simultaneously? The change in velocity would be 4.0×10^{-17} m/s. **L2 Logical-Mathematical**

Extension

The Slowing Earth Have students research how and why the angular momentum of Earth due to its rotation is changing with time. Be aware that it is a very complex problem. Changes are due to interactions with the atmosphere, oceans, melting snow, and solar flares. Students should also consider other factors affecting Earth's slowing; for example, the bulge at Earth's equator is decreasing and Earth's molten core rotates slightly faster than Earth. **L3 Linguistic**

9.2 Section Review

26. Most of the mass of the disk is located at the rim, thereby increasing the moment of inertia. Therefore, when the disk is spinning, its angular momentum is larger than it would be if more mass were near the center. With the larger angular momentum, the disk flies through the air with more stability.

27. a. 3.13 m/s **b.** 1.25 m/s

28. No, because the mass of the racket is much larger than that of the ball, only a small change in its velocity is required. In addition, it is being held by a massive, moving arm that is attached to a body in contact with Earth. Thus, the racket and ball do not comprise an isolated system.

29. The vertical momentum comes from the impulsive force of Earth against the pole.

30. Because their final momenta are zero, their initial momenta were equal and opposite.

31. In the case of the skateboard, the ball, the skateboard, and you are an isolated system, and the momentum of the ball is shared. In the second case, unless Earth is included, there is an external force, so momentum is not conserved.

PHYSICSLAB • Internet

Time Allotment
one laboratory period; two laboratory periods if the students collect the data themselves

Process Skills explain, measure in SI, interpret data, analyze, draw conclusions, make and use graphs

Safety Precautions If the students conduct the experiment, remind them to keep clear of falling masses.

Alternative Materials As an alternative to conducting this experiment via the Internet, students can perform the experiment in the lab. You will need four laboratory carts, 50 g of modeling clay, and either a CBL/HC with a motion probe or a meterstick and a video camera.

Teaching Strategies
- **Go through example problems with students to practice solving for momentum.**
- **Stress that momentum is conserved in an ideal world in which no energy is lost from the system. In this system, however, energy may be lost due to friction or deformation.**

Sample Data

Cart #	Mass (g)
1	4.35×10^2
2	4.25×10^2
3	3.83×10^2
4	4.16×10^2

Distance Covered in Approach (cm)	Approach Velocity (cm/s)	Mass of Approaching Cart(s) (g)	Intial Momentum (g·cm/s)
9.00	90.0	4.35×10^2	4.00×10^4
9.00	90.0	8.18×10^2	7.00×10^4
10.0	1.00×10^2	4.35×10^2	4.30×10^4
9.00	90.0	8.51×10^2	8.00×10^4

Distance Covered in Departure (cm)	Departing Velocity (cm/s)	Mass of Departing Cart(s) (g)	Final Momentum (g·cm/s)
5.00	50.0	8.60×10^2	4.00×10^4
6.00	60.0	1.24×10^3	7.00×10^4
3.00	30.0	1.24×10^3	4.00×10^4
5.00	50.0	1.65×10^3	8.00×10^4

PHYSICSLAB • Internet

Alternate CBL instructions can be found on the Web site.
physicspp.com

Sticky Collisions

In this activity, one moving cart will strike a stationary cart. During the collision, the two carts will stick together. You will measure mass and velocity, both before and after the collision. You then will calculate the momentum both before and after the collision.

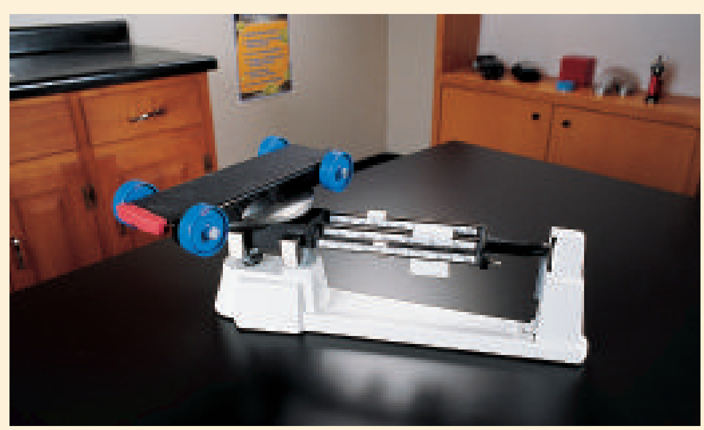
QUESTION
How is the momentum of a system affected by a sticky collision?

Objectives
- **Describe** how momentum is transferred during a collision.
- **Calculate** the momenta involved.
- **Interpret data** from a collision.
- **Draw conclusions** that support the law of conservation of momentum.

Safety Precautions

Materials
Internet access required

Procedure
1. View Chapter 9 lab video clip 1 at **physicspp.com/internet_lab** to determine the mass of the carts.
2. Record the mass of each cart.
3. Watch video clip 2: Cart 1 strikes Cart 2.
4. In the video, three frames represent 0.1 s. Record in the data table the distance Cart 1 travels in 0.1 s before the collision.
5. Observe the collision. Record in the data table the distance the Cart 1-Cart 2 system travels in 0.1 s after the collision.
6. Repeat steps 3–5 for video clip 3: Carts 1 and 3 strike Cart 2.
7. Repeat steps 3–5 for video clip 4: Carts 1, 3, and 4 strike Cart 2.
8. Repeat steps 3-5 for video clip 5: Carts 1 and 3 strike Carts 2 and 4.
9. Repeat steps 3–5 for video clip 6: Cart 1 strikes Carts 2, 3, and 4.



Horizons Companies

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Data Tables

Cart	Mass (kg)
1	
2	
3	
4	

Time of Approach (s)	Distance Covered in Approach (cm)	Initial Velocity (cm/s)	Mass of Approaching Cart(s) (g)	Initial Momentum (g·cm/s)	Time of Departure (s)	Distance Covered in Departure (cm)	Final Velocity (cm/s)	Mass of Departing Cart(s) (g)	Final Momentum (g·cm/s)
0.1					0.1				
0.1					0.1				
0.1					0.1				
0.1					0.1				
0.1					0.1				

Analyze

1. Calculate the initial and final velocities for each of the cart systems.
2. Calculate the initial and final momentum for each of the cart systems.
3. **Make and Use Graphs** Make a graph showing final momentum versus initial momentum for all the video clips.

Conclude and Apply

1. What is the relationship between the initial momentum and the final momentum of the cart systems in a sticky collision?
2. In theory, what should be the slope of the line in your graph?
3. The initial and final data numbers may not be the same due to the precision of the instruments, friction, and other variables. Is the initial momentum typically greater or less than the final momentum? Explain.

Going Further

1. Describe what the velocity and momentum data might look like if the carts did not stick together, but rather, bounced off of each other?
2. Design an experiment to test the impact of friction during the collision of the cart systems. Predict how the slope of the line in your graph will change with your experiment, and then try your experiment.

Real-World Physics

1. Suppose a linebacker collides with a stationary quarterback and they become entangled. What will happen to the velocity of the linebacker-quarterback system if momentum is conserved?
2. If a car rear-ends a stationary car so that the two cars become attached, what will happen to the velocity of the first car? The second car?

Share Your Data

Interpret Data Visit physicspp.com/internet_lab to post your findings from the experiment testing the impact of friction during the collisions of the cart systems. Examine the data and graph of the final momentum versus initial momentum on the Web site. Notice how close to or far off the slope is from 1.00.

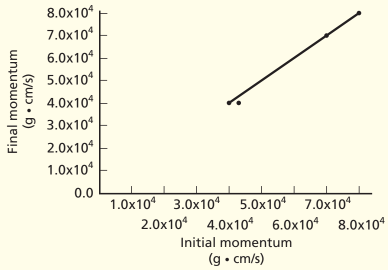
Physics Online

To find out more about momentum, visit the Web site: physicspp.com

ALTERNATIVE INQUIRY LAB

To Make this Lab an Inquiry Lab: Ask students to each design and conduct an experiment that tests the law of conservation of momentum. Students can investigate collisions that stick, collisions that do not stick, and collisions in two dimensions. Have students each develop a procedure for finding a solution to the question "Is momentum conserved in a collision between two bodies?" Have them extend their investigations by seeing if they can apply their findings to a collision between three bodies. Students should discuss their plans and any necessary precautions with you before starting the investigations.

Analyze

1. Cart 1: 90.0 cm/s; Cart 2: 90.0 cm/s; Cart 3: 100.0 cm/s; Cart 4: 90.0 cm/s.
2. Cart 1: initial $= 4.0\times10^4$ g·cm/s, final $= 4.0\times10^4$ g·cm/s
   Cart 2: initial $= 7.0\times10^4$ g·cm/s, final $= 7.0\times10^4$ g·cm/s
   Cart 3: initial $= 4.3\times10^4$ g·cm/s, final $= 4.0\times10^4$ g·cm/s
   Cart 4: initial $= 8.0\times10^4$ g·cm/s, final $= 8.0\times10^4$ g·cm/s
3. 

Conclude and Apply

1. Initial momentum and final momentum are equal to within the uncertainty of the measurements.
2. The slope of the line in a final momentum v. initial momentum graph should be 1.00.
3. The initial momentum would typically be slightly larger than the final momentum because the momentum of this system may be lost due to external forces on the system, such as friction on the cart's axles.

Going Further

The approaching cart might have no velocity or a negative velocity after the collision and the departing cart might be of equal speed or faster depending on the sizes of the two carts.

Real-World Physics

1. The velocity of the linebacker–quarterback system will be slower than the linebacker was by himself.
2. When a car strikes a stationary car from behind, the approaching car will slow down after the collision, while the car that is hit will speed up after the collision.

Background

Efforts are under way to launch the first solar sail, *Cosmos 1*, from a Russian nuclear submarine. It is the first international, privately funded space mission in history. Solar sails have been written about in many science fiction books and envisioned by scientists, but no solar sail has been built or launched until now.

The purpose of the mission is to conduct the first solar sail flight beginning a journey that may one day lead to interstellar flight. Solar sails use a technology that will allow deep-space travel and the exploration of distant stars. Laser photons will be used for those future missions rather than solar photons because the craft will be too far from radiant sunlight.

The mission will be considered a success if, in controlled flight, the craft is able to increase its momentum while in orbit by using sunlight pressure—the momentum imparted by reflecting photons. This would be a small step perhaps, but the first step from Earth to the stars.

Teaching Strategies

- Ask your students to design prototype solar sails. Students can work in small groups to investigate state-of-the-art solar designs and then apply their findings to develop their own designs. Ask students to make models or diagrams to present to the class. Students should be prepared to answer questions from the audience.
- Have interested students write science fiction stories that incorporate space travel via solar sails.

Discussion

Solar Sailing and Solar Wind Point out that it is a common misconception that solar sailing involves gliding along on the solar wind. In fact, the solar sail operates on sunlight pressure; it uses the momentum imparted by reflecting photons. Solar wind, on the other hand, is made up of fast-moving electrons and protons emitted by the Sun as it burns up hydrogen. These particles stream away from the Sun throughout the solar system. Sunlight pressure is 1000 to 10,000 times greater than pressure of the solar wind.

Future Technology

Solar Sailing

Nearly 400 years ago, Johannes Kepler observed that comet tails appeared to be blown by a solar breeze. He suggested that ships would be able to travel in space with sails designed to catch this breeze. Thus, the idea for solar sails was born.

How Does a Solar Sail Work? A solar sail is a spacecraft without an engine. A solar sail works like a giant fabric mirror that is free to move. Solar sails usually are made of 5-micron-thick aluminized polyester film or polyimide film with a 100-nm-thick aluminum layer deposited on one side to form the reflective surface.

Reflected sunlight, rather than rocket fuel, provides the force. Sunlight is made up of individual particles called photons. Photons have momentum, and when a photon bounces off a solar sail, it transfers its momentum to the sail, which propels the spacecraft along.

The force of impacting photons is small in comparison to the force rocket fuel can supply. So, small sails experience only a small amount of force from sunlight, while larger sails experience a greater force. Thus, solar sails may be a kilometer or so across.

What speeds can a solar sail achieve? This depends on the momentum transferred to the sail by photons, as well as the sail's mass. To travel quickly through the solar system, a sail and the spacecraft should be lightweight.

Photons supplied by the Sun are constant. They impact the sail every second of every hour of every day during a space flight. The Sun's continuous supply of photons over time allows the sail to build up huge velocities and enables the spacecraft to travel great distances within a convenient time frame. Rockets require enormous amounts of fuel to move large masses, but solar sails only require photons from the Sun. Thus, solar sails may be a superior way to move large masses over great distances in outer space.

This artist's rendering shows *Cosmos 1*, the first solar sail scheduled for launch in the near future.

Future Journeys The *Cosmos 1*, the first solar-sail prototype, is scheduled for a launch in the very near future. The *Cosmos-1* mission is an international, privately funded venture. The spacecraft looks like a flower with eight huge, solar-sail petals. Being the first solar sail, goals are modest. The mission will be considered successful if the *Cosmos 1* operates for just a few days, accelerating under sunlight pressure.

Solar sails are important, not only for travel, but also for creating new types of space and Earth weather monitoring stations. These stations would be able to provide greater coverage of Earth and more advanced warning of solar storms that cause problems to communication and electric power grids. It is hoped that in the next few decades, solar sails will be used as interplanetary shuttles because of their ability to travel great distances in convenient time frames. Vast distances could someday be traversed by vehicles that do not consume any fuel.

Going Further

1. **Research** how solar sails can help provide advanced warning of solar storms.
2. **Critical Thinking** A certain solar-sail model is predicted to take more time to reach Mars than a rocket-propelled spacecraft would, but less time to go to Pluto than a rocket-propelled spacecraft would. Explain why this is so.

Going Further

1. Encourage students to create small models of Earth weather-monitoring stations powered by solar sails.
2. The solar sail's momentum increases at a small but constant rate, so it takes a long time before the solar sail's speed is faster than that of a chemical-rocket propelled ship.

Chapter 9

Study Guide

9.1 Impulse and Momentum

Vocabulary

- impulse (p. 230)
- momentum (p. 230)
- impulse-momentum theorem (p. 230)
- angular momentum (p. 233)
- angular impulse-angular momentum theorem (p. 234)

Key Concepts

- When doing a momentum problem, first examine the system before and after the event.
- The momentum of an object is the product of its mass and velocity and is a vector quantity.

$$\boldsymbol{p} = m\boldsymbol{v}$$

- The impulse on an object is the average net force exerted on the object multiplied by the time interval over which the force acts.

$$\text{Impluse} = \boldsymbol{F}\Delta t$$

- The impulse on an object is equal to the change in momentum of the object.

$$\boldsymbol{F}\Delta t = \boldsymbol{p}_{\mathrm{f}} - \boldsymbol{p}_{\mathrm{i}}$$

- The angular momentum of a rotating object is the product of its moment of inertia and its angular velocity.

$$L = I\omega$$

- The angular impulse-angular momentum theorem states that the angular impulse on an object is equal to the change in the object's angular momentum.

$$\tau\Delta t = L_{\mathrm{f}} - L_{\mathrm{i}}$$

9.2 Conservation of Momentum

Vocabulary

- closed system (p. 236)
- isolated system (p. 237)
- law of conservation of momentum (p. 237)
- law of conservation of angular momentum (p. 243)

Key Concepts

- According to Newton's third law of motion and the law of conservation of momentum, the forces exerted by colliding objects on each other are equal in magnitude and opposite in direction.
- Momentum is conserved in a closed, isolated system.

$$\boldsymbol{p}_{\mathrm{f}} = \boldsymbol{p}_{\mathrm{i}}$$

- The law of conservation of momentum can be used to explain the propulsion of rockets.
- Vector analysis is used to solve momentum-conservation problems in two dimensions.
- The law of conservation of angular momentum states that if there are no external torques acting on a system, then the angular momentum is conserved.

$$L_{\mathrm{f}} = L_{\mathrm{i}}$$

- Because angular momentum is conserved, the direction of rotation of a spinning object can be changed only by applying a torque.

Physics Online physicspp.com/vocabulary_puzzlemaker

Key Concepts
Summary statements can be used by students to review the major concepts of the chapter.

Physics Online
Visit physicspp.com
/self_check_quiz
/vocabulary_puzzlemaker
/chapter_test
/standardized_test

VOCABULARY PuzzleMaker

For additional help with vocabulary, have students access the Vocabulary PuzzleMaker online.
physicspp.com/vocabulary_puzzlemaker

Concept Mapping

32. See Solutions Manual.

Mastering Concepts

33. Yes, for a bullet to have the same momentum as a truck, it must have a higher velocity because the two masses are not the same.

$m_{bullet} v_{bullet} = m_{truck} v_{truck}$

34. a. The pitcher and the catcher exert the same amount of impulse on the ball, but the two impulses are in opposite directions.

b. The catcher exerts the larger force on the ball because the time interval over which the force is exerted is smaller.

35. No net force on the system means no net impulse on the system and no net change in momentum. However, individual parts of the system may have a change in momentum as long as the net change in momentum is zero.

36. Cars are made with bumpers that retract during a crash to increase the time of a collision, thereby reducing the force.

37. a. by applying an external torque

b. by changing the moment of inertia

38. An isolated system has no external forces acting on it.

39. Momentum is conserved. The change in momentum of gases in one direction must be balanced by an equal change in momentum of the spacecraft in the opposite direction.

40. The eight ball must be moving with the same velocity that the cue ball had just before the collision.

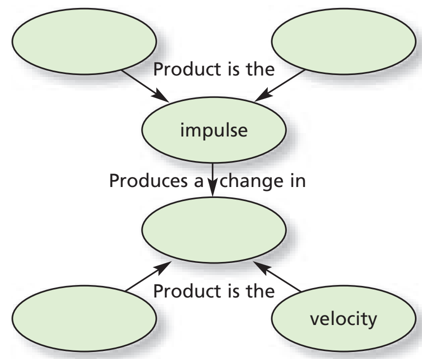
41. a. The momentum of a falling ball is not conserved because a net external force, gravity, is acting on it.

b. One such system in which total momentum is conserved includes the ball plus Earth.

Chapter 9 Assessment

Concept Mapping

32. Complete the following concept map using the following terms: *mass, momentum, average force, time over which the force is exerted.*



Mastering Concepts

33. Can a bullet have the same momentum as a truck? Explain. (9.1)

34. A pitcher throws a curve ball to the catcher. Assume that the speed of the ball doesn't change in flight. (9.1)

a. Which player exerts the larger impulse on the ball?

b. Which player exerts the larger force on the ball?

35. Newton's second law of motion states that if no net force is exerted on a system, no acceleration is possible. Does it follow that no change in momentum can occur? (9.1)

36. Why are cars made with bumpers that can be pushed in during a crash? (9.1)

37. An ice-skater is doing a spin. (9.1)

a. How can the skater's angular momentum be changed?

b. How can the skater's angular velocity be changed without changing the angular momentum?

38. What is meant by "an isolated system?" (9.2)

39. A spacecraft in outer space increases its velocity by firing its rockets. How can hot gases escaping from its rocket engine change the velocity of the craft when there is nothing in space for the gases to push against? (9.2)

40. A cue ball travels across a pool table and collides with the stationary eight ball. The two balls have equal masses. After the collision, the cue ball is at rest. What must be true regarding the speed of the eight ball? (9.2)

41. Consider a ball falling toward Earth. (9.2)

a. Why is the momentum of the ball not conserved?

b. In what system that includes the falling ball is the momentum conserved?

42. A falling basketball hits the floor. Just before it hits, the momentum is in the downward direction, and after it hits the floor, the momentum is in the upward direction. (9.2)

a. Why isn't the momentum of the basketball conserved even though the bounce is a collision?

b. In what system is the momentum conserved?

43. Only an external force can change the momentum of a system. Explain how the internal force of a car's brakes brings the car to a stop. (9.2)

44. Children's playgrounds often have circular-motion rides. How could a child change the angular momentum of such a ride as it is turning? (9.2)

Applying Concepts

45. Explain the concept of impulse using physical ideas rather than mathematics.

46. Is it possible for an object to obtain a larger impulse from a smaller force than it does from a larger force? Explain.

47. Foul Ball You are sitting at a baseball game when a foul ball comes in your direction. You prepare to catch it bare-handed. To catch it safely, should you move your hands toward the ball, hold them still, or move them in the same direction as the moving ball? Explain.

48. A 0.11-g bullet leaves a pistol at 323 m/s, while a similar bullet leaves a rifle at 396 m/s. Explain the difference in exit speeds of the two bullets, assuming that the forces exerted on the bullets by the expanding gases have the same magnitude.

49. An object initially at rest experiences the impulses described by the graph in **Figure 9-15.** Describe the object's motion after impulses A, B, and C.

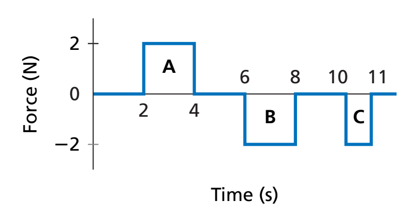


Figure 9-15

 For more problems, go to Additional Problems, Appendix B.

42. a. The floor is outside the system, so it exerts an external force, and therefore an impulse on the ball.

b. Momentum is conserved in the system of ball plus Earth.

43. The external force of a car's brakes can bring the car to a stop by stopping the wheels and allowing the external frictional force of the road against the tires to stop the car. If there is no friction—for example, if the road is icy—then there is no external force and the car does not stop.

44. The child would have to exert a torque on it. He or she could stand next to it and exert a force tangential to the circle on the handles as they go past. He or she could also run at the side and jump onboard.

50. During a space walk, the tether connecting an astronaut to the spaceship breaks. Using a gas pistol, the astronaut manages to get back to the ship. Use the language of the impulse-momentum theorem and a diagram to explain why this method was effective.

51. **Tennis Ball** As a tennis ball bounces off a wall, its momentum is reversed. Explain this action in terms of the law of conservation of momentum. Define the system and draw a diagram as a part of your explanation.

52. Imagine that you command spaceship *Zeldon,* which is moving through interplanetary space at high speed. How could you slow your ship by applying the law of conservation of momentum?

53. Two trucks that appear to be identical collide on an icy road. One was originally at rest. The trucks are stuck together and move at more than half the original speed of the moving truck. What can you conclude about the contents of the two trucks?

54. Explain, in terms of impulse and momentum, why it is advisable to place the butt of a rifle against your shoulder when first learning to shoot.

55. **Bullets** Two bullets of equal mass are shot at equal speeds at blocks of wood on a smooth ice rink. One bullet, made of rubber, bounces off of the wood. The other bullet, made of aluminum, burrows into the wood. In which case does the block of wood move faster? Explain.

Mastering Problems

9.1 Impulse and Momentum

56. **Golf** Rocío strikes a 0.058-kg golf ball with a force of 272 N and gives it a velocity of 62.0 m/s. How long was Rocío's club in contact with the ball?

57. A 0.145-kg baseball is pitched at 42 m/s. The batter hits it horizontally to the pitcher at 58 m/s.
   a. Find the change in momentum of the ball.
   b. If the ball and bat are in contact for 4.6×10^{-4} s, what is the average force during contact?

58. **Bowling** A force of 186 N acts on a 7.3-kg bowling ball for 0.40 s. What is the bowling ball's change in momentum? What is its change in velocity?

59. A 5500-kg freight truck accelerates from 4.2 m/s to 7.8 m/s in 15.0 s by the application of a constant force.
   a. What change in momentum occurs?
   b. How large of a force is exerted?

60. In a ballistics test at the police department, Officer Rios fires a 6.0-g bullet at 350 m/s into a container that stops it in 1.8 ms. What is the average force that stops the bullet?

61. **Volleyball** A 0.24-kg volleyball approaches Tina with a velocity of 3.8 m/s. Tina bumps the ball, giving it a speed of 2.4 m/s but in the opposite direction. What average force did she apply if the interaction time between her hands and the ball was 0.025 s?

62. **Hockey** A hockey player makes a slap shot, exerting a constant force of 30.0 N on the hockey puck for 0.16 s. What is the magnitude of the impulse given to the puck?

63. **Skateboarding** Your brother's mass is 35.6 kg, and he has a 1.3-kg skateboard. What is the combined momentum of your brother and his skateboard if they are moving at 9.50 m/s?

64. A hockey puck has a mass of 0.115 kg and is at rest. A hockey player makes a shot, exerting a constant force of 30.0 N on the puck for 0.16 s. With what speed does it head toward the goal?

65. Before a collision, a 25-kg object was moving at +12 m/s. Find the impulse that acted on the object if, after the collision, it moved at the following velocities.
   a. +8.0 m/s
   b. −8.0 m/s

66. A 0.150-kg ball, moving in the positive direction at 12 m/s, is acted on by the impulse shown in the graph in **Figure 9-16.** What is the ball's speed at 4.0 s?

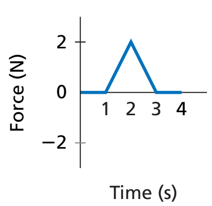


Figure 9-16

67. **Baseball** A 0.145-kg baseball is moving at 35 m/s when it is caught by a player.
   a. Find the change in momentum of the ball.
   b. If the ball is caught with the mitt held in a stationary position so that the ball stops in 0.050 s, what is the average force exerted on the ball?
   c. If, instead, the mitt is moving backward so that the ball takes 0.500 s to stop, what is the average force exerted by the mitt on the ball?

Physics Online physicspp.com/chapter_test

Applying Concepts

45. A force, F, exerted on an object over a time, Δt, causes the momentum of the object to change by the quantity, $F\Delta t$.

46. Yes, if the smaller force acts for a long enough time, it can provide a larger impulse.

47. You should move your hands in the same direction the ball is traveling to increase the time of the collision, thereby reducing the force.

48. The bullet is in the rifle a longer time, so the momentum it gains is larger.

49. After time A, the object moves with a constant, positive velocity. After time B, the object is at rest. After time C, the object moves with a constant, negative velocity.

50. When the gas pistol is fired in the opposite direction, it provides the impulse needed to move the astronaut toward the spaceship. See Solutions Manual.

51. Consider the system to be the ball, the wall, and Earth. The wall and Earth gain some momentum in the collision.

52. By shooting mass in the form of exhaust gas at high velocity in the same direction in which you are moving, its momentum would cause the ship's momentum to decrease.

53. If the two trucks had equal masses, they would have moved off at half the speed of the moving truck. Thus, the moving truck must have had a more massive load.

54. When held loosely, the recoil momentum of the rifle works against only the mass of the rifle, thereby producing a larger velocity and striking your shoulder. The recoil momentum must work against the mass of the rifle and you, resulting in a smaller velocity.

55. Momentum is conserved, so the momentum of the block and bullet after the collision equals the momentum of the bullet before the collision. The rubber bullet has a negative momentum after impact with respect to the block, so the block's momentum must be greater in this case.

Mastering Problems

9.1 Impulse and Momentum

Level 1

56. 0.013 s

57. **a.** −14 kg·m/s
   b. -3.2×10^{4} N

58. 74 kg·m/s; 1.0×10^{1} m/s

59. **a.** 2.0×10^{4} kg·m/s
b. 1.3×10^{3} N

60. -1.2×10^{3} N

61. -6.0×10^{1} N

62. 4.8 N·s

63. 3.5×10^{2} kg·m/s

64. 42 m/s

65. **a.** -1.0×10^{2} kg·m/s
b. -5.0×10^{2} kg·m/s

Level 2

66. 25 m/s

67. **a.** -5.1 kg·m/s
b. -1.0×10^{2} N
c. -1.0×10^{1} N

68. **a.** -7.1 kg·m/s
b. -1.4×10^{4} N

69. **a.** $+5.2\times10^{-23}$ N·s
b. 7.8 N

Level 3

70. 1.3×10^{3} s, or 22 min

71. 888 kg·m/s at 43.6°

72. **a.** -2.00×10^{2} kg·m/s
b. -4.0×10^{3} N
c. 4.1×10^{2} kg
d. No
e. You would not be able to protect a child on your lap in the event of a collision.

9.2 Conservation of Momentum

Level 1

73. **a.** Before: $m_{FB} = 95$ kg
$v_{FB} = 8.2$ m/s
$m_{DT} = 128$ kg
$v_{DT} = ?$
After: $m = 223$ kg
$v_f = 0$ m/s
See Solutions Manual.
b. 7.8×10^{2} kg·m/s
c. -7.8×10^{2} kg·m/s
d. $+7.8\times10^{2}$ kg·m/s
e. -7.8×10^{2} kg·m/s
f. -6.1 m/s

74. **a.** Before: $m_C = 5.0$ g
$m_D = 10.0$ g
$v_{Ci} = 20.0$ cm/s
$v_{Di} = 10.0$ cm/s

68. Hockey A hockey puck has a mass of 0.115 kg and strikes the pole of the net at 37 m/s. It bounces off in the opposite direction at 25 m/s, as shown in **Figure 9-17.**
a. What is the impulse on the puck?
b. If the collision takes 5.0×10^{-4} s, what is the average force on the puck?

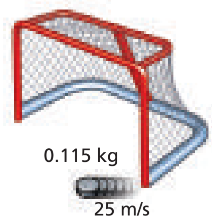


Figure 9-17

69. A nitrogen molecule with a mass of 4.7×10^{-26} kg, moving at 550 m/s, strikes the wall of a container and bounces back at the same speed.
a. What is the impulse the molecule delivers to the wall?
b. If there are 1.5×10^{23} collisions each second, what is the average force on the wall?

70. Rockets Small rockets are used to make tiny adjustments in the speeds of satellites. One such rocket has a thrust of 35 N. If it is fired to change the velocity of a 72,000-kg spacecraft by 63 cm/s, how long should it be fired?

71. An animal rescue plane flying due east at 36.0 m/s drops a bale of hay from an altitude of 60.0 m, as shown in **Figure 9-18.** If the bale of hay weighs 175 N, what is the momentum of the bale the moment before it strikes the ground? Give both magnitude and direction.

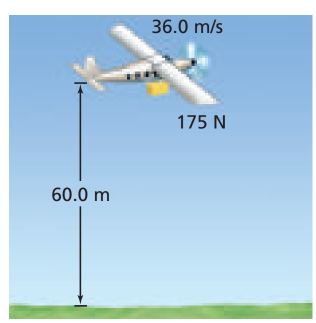


Figure 9-18

72. Accident A car moving at 10.0 m/s crashes into a barrier and stops in 0.050 s. There is a 20.0-kg child in the car. Assume that the child's velocity is changed by the same amount as that of the car, and in the same time period.
a. What is the impulse needed to stop the child?
b. What is the average force on the child?
c. What is the approximate mass of an object whose weight equals the force in part b?
d. Could you lift such a weight with your arm?
e. Why is it advisable to use a proper restraining seat rather than hold a child on your lap?

9.2 Conservation of Momentum

73. Football A 95-kg fullback, running at 8.2 m/s, collides in midair with a 128-kg defensive tackle moving in the opposite direction. Both players end up with zero speed.
a. Identify the "before" and "after" situations and draw a diagram of both.
b. What was the fullback's momentum before the collision?
c. What was the change in the fullback's momentum?
d. What was the change in the defensive tackle's momentum?
e. What was the defensive tackle's original momentum?
f. How fast was the defensive tackle moving originally?

74. Marble C, with mass 5.0 g, moves at a speed of 20.0 cm/s. It collides with a second marble, D, with mass 10.0 g, moving at 10.0 cm/s in the same direction. After the collision, marble C continues with a speed of 8.0 cm/s in the same direction.
a. Sketch the situation and identify the system. Identify the "before" and "after" situations and set up a coordinate system.
b. Calculate the marbles' momenta before the collision.
c. Calculate the momentum of marble C after the collision.
d. Calculate the momentum of marble D after the collision.
e. What is the speed of marble D after the collision?

75. Two lab carts are pushed together with a spring mechanism compressed between them. Upon release, the 5.0-kg cart repels one way with a velocity of 0.12 m/s, while the 2.0-kg cart goes in the opposite direction. What is the velocity of the 2.0-kg cart?

 For more problems, go to Additional Problems, Appendix B.

After: $m_C = 5.0$ g
$m_D = 10.0$ g
$v_{Cf} = 8.0$ cm/s
$v_{Df} = ?$
See Solutions Manual.
b. $p_{Ci} = 1.0\times10^{-3}$ kg·m/s
$p_{Di} = 1.0\times10^{-3}$ kg·m/s
c. 4.0×10^{-4} kg·m/s
d. 1.6×10^{-3} kg·m/s
e. 16 cm/s

75. -0.30 m/s

76. -4.94 m/s, or 4.94 m/s backwards

Level 2

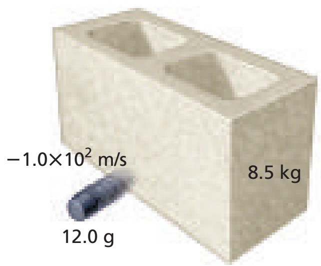
77. 0.35 m/s

78. 1.26 m/s in the same direction as she was riding

79. cue ball: 2.8 m/s
eight ball: 2.8 m/s

76. A 50.0-g projectile is launched with a horizontal velocity of 647 m/s from a 4.65-kg launcher moving in the same direction at 2.00 m/s. What is the launcher's velocity after the launch?

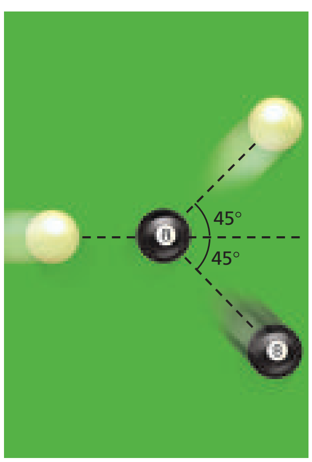
77. A 12.0-g rubber bullet travels at a velocity of 150 m/s, hits a stationary 8.5-kg concrete block resting on a frictionless surface, and ricochets in the opposite direction with a velocity of -1.0×10^2 m/s, as shown in **Figure 9-19.** How fast will the concrete block be moving?



■ **Figure 9-19**

78. **Skateboarding** Kofi, with mass 42.00 kg, is riding a skateboard with a mass of 2.00 kg and traveling at 1.20 m/s. Kofi jumps off and the skateboard stops dead in its tracks. In what direction and with what velocity did he jump?

79. **Billiards** A cue ball, with mass 0.16 kg, rolling at 4.0 m/s, hits a stationary eight ball of similar mass. If the cue ball travels 45° above its original path and the eight ball travels 45° below the horizontal, as shown in **Figure 9-20,** what is the velocity of each ball after the collision?



■ **Figure 9-20**

80. A 2575-kg van runs into the back of an 825-kg compact car at rest. They move off together at 8.5 m/s. Assuming that the friction with the road is negligible, calculate the initial speed of the van.

81. **In-line Skating** Diego and Keshia are on in-line skates and stand face-to-face, then push each other away with their hands. Diego has a mass of 90.0 kg and Keshia has a mass of 60.0 kg.
  a. Sketch the event, identifying the "before" and "after" situations, and set up a coordinate axis.
  b. Find the ratio of the skaters' velocities just after their hands lose contact.
  c. Which skater has the greater speed?
  d. Which skater pushed harder?

82. A 0.200-kg plastic ball moves with a velocity of 0.30 m/s. It collides with a second plastic ball of mass 0.100 kg, which is moving along the same line at a speed of 0.10 m/s. After the collision, both balls continue moving in the same, original direction. The speed of the 0.100-kg ball is 0.26 m/s. What is the new velocity of the 0.200-kg ball?

Mixed Review

83. A constant force of 6.00 N acts on a 3.00-kg object for 10.0 s. What are the changes in the object's momentum and velocity?

84. The velocity of a 625-kg car is changed from 10.0 m/s to 44.0 m/s in 68.0 s by an external, constant force.
  a. What is the resulting change in momentum of the car?
  b. What is the magnitude of the force?

85. **Dragster** An 845-kg dragster accelerates on a race track from rest to 100.0 km/h in 0.90 s.
  a. What is the change in momentum of the dragster?
  b. What is the average force exerted on the dragster?
  c. What exerts that force?

86. **Ice Hockey** A 0.115-kg hockey puck, moving at 35.0 m/s, strikes a 0.365-kg jacket that is thrown onto the ice by a fan of a certain hockey team. The puck and jacket slide off together. Find their velocity.

87. A 50.0-kg woman, riding on a 10.0-kg cart, is moving east at 5.0 m/s. The woman jumps off the front of the cart and lands on the ground at 7.0 m/s eastward, relative to the ground.
  a. Sketch the "before" and "after" situations and assign a coordinate axis to them.
  b. Find the cart's velocity after the woman jumps off.

80. 11 m/s

Level 3

81. a. Before: $m_K = 60.0$ kg
$m_D = 90.0$ kg
$v_i = 0.0$ m/s
After: $m_K = 60.0$ kg
$m_D = 90.0$ kg
$v_{Kf} = ?$
$v_{Df} = ?$
See Solutions Manual.
b. -1.50
c. Keshia, who has the smaller mass, has the greater speed.
d. The forces were equal and opposite.

82. 0.22 m/s in the original direction

Mixed Review

Level 1

83. 20.0 m/s

84. a. 2.12×10^4 kg·m/s
b. 312 N

85. a. 2.35×10^4 kg·m/s
b. 2.6×10^4 N
c. The force is exerted by the track through friction.

Level 2

86. 8.39 m/s

87. a. Before: $m_w = 50.0$ kg
$m_c = 10.0$ kg
$v_i = 5.0$ m/s
After: $m_w = 50.0$ kg
$m_c = 10.0$ kg
$v_{wf} = 7.0$ m/s
$v_{cf} = ?$
See Solutions Manual.
b. -5.0 m/s, or 5.0 m/s west

88. a. She spins around the center of mass of her body, first in the tuck position and then also as she straightens out.
b. giant swing (greatest), straight, tuck (least)
c. tuck (greatest), straight, giant swing (least)

Level 3

89. a. 1.5×10^2 kg·m/s down
b. -1.5×10^2 N·s up
c. 3.0×10^3 N
d. 5.98×10^2 N; The force is about 5 times the weight.

Thinking Critically

90. a. Before: $m_A = 92$ kg
$m_B = 75$ kg
$m_C = 75$ kg
$v_{Ai} = 5.0$ m/s
$v_{Bi} = -2.0$ m/s
$v_{Ci} = -4.0$ m/s
After: $m_A = 92$ kg
$m_B = 75$ kg
$m_C = 75$ kg
$v_f = ?$
See Solutions Manual.
b. 0.041 m/s
c. Yes. The velocity is positive, so the football crosses the goal line for a touchdown.

91. The student and the stool would spin slowly in the direction opposite to that of the wheel. Without friction there are no external torques. Thus, the angular momentum of the system is not changed. The angular momentum of the student and stool must be equal and opposite to the angular momentum of the spinning wheel.

92. Dotted lines show that the changes of momentum for each ball are equal and opposite: $\Delta(m_A v_A) = \Delta(m_B v_B)$. Because the masses have a 3:2 ratio, a 2:3 ratio of velocity changes will compensate.

Writing in Physics

93. The change in a car's momentum does not depend on how it is brought to a stop. Thus, the impulse also does not change. To reduce the force, the time over which a car is stopped must be increased. Using barriers that can extend the time it takes to stop a car will reduce the force. Flexible, plastic containers filled with sand are often used.

94. There are two ways an airbag reduces injury. First, an airbag extends the time over which the impulse acts, thereby reducing the force. Second, an airbag spreads the force over a larger area, thereby reducing the pressure. Thus, the injuries due to forces from small objects are reduced. The dangers of airbags mostly center on the fact that airbags must be inflated very rapidly. The surface of an airbag can approach the passenger at speeds of up to 322 km/h (200 mph). Injuries can occur when the moving bag collides with the person. Systems are being developed that will adjust the rate at which gases fill the airbags to match the size of the person.

Cumulative Review

95. −6.0 N

96. 4.3×10^7 m

97. 1.44 kg·m^2

88. Gymnastics **Figure 9-21** shows a gymnast performing a routine. First, she does giant swings on the high bar, holding her body straight and pivoting around her hands. Then, she lets go of the high bar and grabs her knees with her hands in the tuck position. Finally, she straightens up and lands on her feet.

a. In the second and final parts of the gymnast's routine, around what axis does she spin?
b. Rank in order, from greatest to least, her moments of inertia for the three positions.
c. Rank in order, from greatest to least, her angular velocities in the three positions.

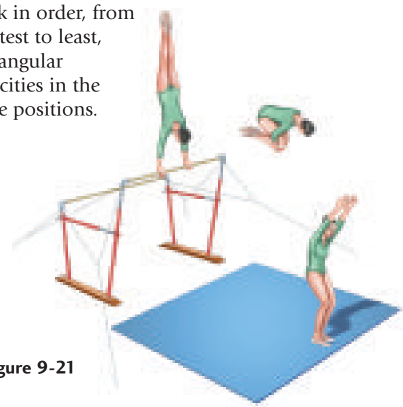


Figure 9-21

89. A 60.0-kg male dancer leaps 0.32 m high.
a. With what momentum does he reach the ground?
b. What impulse is needed to stop the dancer?
c. As the dancer lands, his knees bend, lengthening the stopping time to 0.050 s. Find the average force exerted on the dancer's body.
d. Compare the stopping force with his weight.

Thinking Critically

90. Apply Concepts A 92-kg fullback, running at 5.0 m/s, attempts to dive directly across the goal line for a touchdown. Just as he reaches the line, he is met head-on in midair by two 75-kg linebackers, both moving in the direction opposite the fullback. One is moving at 2.0 m/s and the other at 4.0 m/s. They all become entangled as one mass.
a. Sketch the event, identifying the "before" and "after" situations.
b. What is the velocity of the football players after the collision?
c. Does the fullback score a touchdown?

91. Analyze and Conclude A student, holding a bicycle wheel with its axis vertical, sits on a stool that can rotate without friction. She uses her hand to get the wheel spinning. Would you expect the student and stool to turn? If so, in which direction? Explain.

92. Analyze and Conclude Two balls during a collision are shown in **Figure 9-22,** which is drawn to scale. The balls enter from the left, collide, and then bounce away. The heavier ball, at the bottom of the diagram, has a mass of 0.600 kg, and the other has a mass of 0.400 kg. Using a vector diagram, determine whether momentum is conserved in this collision. Explain any difference in the momentum of the system before and after the collision.

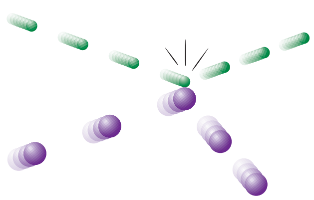


Figure 9-22

Writing in Physics

93. How can highway barriers be designed to be more effective in saving people's lives? Research this issue and describe how impulse and change in momentum can be used to analyze barrier designs.

94. While air bags save many lives, they also have caused injuries and even death. Research the arguments and responses of automobile makers to this statement. Determine whether the problems involve impulse and momentum or other issues.

Cumulative Review

95. A 0.72-kg ball is swung vertically from a 0.60-m string in uniform circular motion at a speed of 3.3 m/s. What is the tension in the cord at the top of the ball's motion? (Chapter 6)

96. You wish to launch a satellite that will remain above the same spot on Earth's surface. This means the satellite must have a period of exactly one day. Calculate the radius of the circular orbit this satellite must have. *Hint: The Moon also circles Earth and both the Moon and the satellite will obey Kepler's third law. The Moon is 3.9×10^8 m from Earth and its period is 27.33 days.* (Chapter 7)

97. A rope is wrapped around a drum that is 0.600 m in diameter. A machine pulls with a constant 40.0 N force for a total of 2.00 s. In that time, 5.00 m of rope is unwound. Find α, ω at 2.00 s, and I. (Chapter 8)

 For more problems, go to Additional Problems, Appendix B.

Standardized Test Practice

Multiple Choice

1. When a star that is much larger than the Sun nears the end of its lifetime, it begins to collapse, but continues to rotate. Which of the following describes the conditions of the collapsing star's moment of inertia (I), angular momentum (L), and angular velocity (ω)?

Ⓐ I increases, L stays constant, ω decreases
Ⓑ I decreases, L stays constant, ω increases
Ⓒ I increases, L increases, ω increases
Ⓓ I increases, L increases, ω stays constant

2. A 40.0-kg ice-skater glides with a speed of 2.0 m/s toward a 10.0-kg sled at rest on the ice. The ice-skater reaches the sled and holds on to it. The ice-skater and the sled then continue sliding in the same direction in which the ice-skater was originally skating. What is the speed of the ice-skater and the sled after they collide?

Ⓐ 0.4 m/s　Ⓒ 1.6 m/s
Ⓑ 0.8 m/s　Ⓓ 3.2 m/s

3. A bicyclist applies the brakes and slows the motion of the wheels. The angular momentum of each wheel then decreases from 7.0 kg·m^2/s to 3.5 kg·m^2/s over a period of 5.0 s. What is the angular impulse on each wheel?

Ⓐ −0.7 kg·m^2/s
Ⓑ −1.4 kg·m^2/s
Ⓒ −2.1 kg·m^2/s
Ⓓ −3.5 kg·m^2/s

4. A 45.0-kg ice-skater stands at rest on the ice. A friend tosses the skater a 5.0-kg ball. The skater and the ball then move backwards across the ice with a speed of 0.50 m/s. What was the speed of the ball at the moment just before the skater caught it?

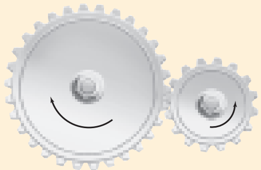
Ⓐ 2.5 m/s　Ⓒ 4.0 m/s
Ⓑ 3.0 m/s　Ⓓ 5.0 m/s

5. What is the difference in momentum between a 50.0-kg runner moving at a speed of 3.00 m/s and a 3.00×10^3-kg truck moving at a speed of only 1.00 m/s?

Ⓐ 1275 kg·m/s　Ⓒ 2850 kg·m/s
Ⓑ 2550 kg·m/s　Ⓓ 2950 kg·m/s

6. When the large gear in the diagram rotates, it turns the small gear in the opposite direction at the same linear speed. The larger gear has twice the radius and four times the mass of the smaller gear. What is the angular momentum of the larger gear as a function of the angular momentum of the smaller gear? *Hint: The moment of inertia for a disk is $\frac{1}{2}mr^2$, where m is mass and r is the radius of the disk.*

Ⓐ $-2L_{small}$　Ⓒ $-8L_{small}$
Ⓑ $-4L_{small}$　Ⓓ $-16L_{small}$



7. A force of 16 N exerted against a rock with an impulse of 0.8 kg·m/s causes the rock to fly off the ground with a speed of 4.0 m/s. What is the mass of the rock?

Ⓐ 0.2 kg
Ⓑ 0.8 kg
Ⓒ 1.6 kg
Ⓓ 4.0 kg

Extended Answer

8. A 12.0-kg rock falls to the ground. What is the impulse on the rock if its velocity at the moment it strikes the ground is 20.0 m/s?

✓ Test-Taking TIP

If It Looks Too Good To Be True

Beware of answer choices in multiple-choice questions that seem ready-made and obvious. Remember that only one answer choice for each question is correct. The rest are made up by test-makers to distract you. This means that they might look very appealing. Check each answer choice carefully before making your final selection.

Standardized Test Practice

Rubric

The following rubric is a sample scoring device for extended response questions.

Extended Response

Points	Description
4	The student demonstrates a thorough understanding of the physics involved. The response may contain minor flaws that do not detract from the demonstration of a thorough understanding.
3	The student demonstrates an understanding of the physics involved. The response is essentially correct and demonstrates an essential but less than thorough understanding of the physics.
2	The student demonstrates only a partial understanding of the physics involved. Although the student may have used the correct approach to a solution or may have provided a correct solution, the work lacks an essential understanding of the underlying physical concepts.
1	The student demonstrates a very limited understanding of the physics involved. The response is incomplete and exhibits many flaws.
0	The student provides a completely incorrect solution or no response at all.

Multiple Choice

1. B　**2.** C　**3.** D
4. D　**5.** C　**6.** C
7. A

Extended Answer

8. $F\Delta t = m\Delta v$
$= (12.0\text{ kg})(20.0\text{ m/s} - 0.00\text{ m/s})$
$= 2.40\times10^2$ kg·m/s
$= 2.40\times10^2$ N·s

The rock's impulse on the ground is 2.40×10^2 N·s. Thus, the ground's impact on the rock is -2.40×10^2 N·s.