Chapter 5 Organizer

Section/Objectives	Standards: National	Standards: State/Local	Lab and Demo Planning
Chapter Opener	See page 14T for a key to the standards.		
Section 5.1 1. **Evaluate** the sum of two or more vectors in two dimensions, graphically. 2. **Determine** the components of vectors. 3. **Solve** for the sum of two or more vectors, algebraically, by adding the components of the vectors.	UCP.2, UCP.3, A.1, A.2, B.4		**Student Lab:** **Launch Lab,** p. 119: two 5-N spring scales, 35-cm-long piece of string, 15-cm-long piece of string, 200-g object, protractor **Teacher Demonstration:** **Quick Demo,** p. 121: two or three bungee cords with arrowheads, pegboard, metric ruler
Section 5.2 4. **Define** *friction force.* 5. **Distinguish** between static and kinetic friction.	UCP.2, UCP.3, A.1, A.2, B.4		**Teacher Demonstration:** **Quick Demo,** p. 128: nonstick cookie sheet, felt-covered wood block (6-in x 3-in x 1-in)
Section 5.3 6. **Determine** the force that produces equilibrium when three forces act on an object. 7. **Analyze** the motion of an object on an inclined plane with and without friction.	UCP.2, UCP.3, UCP.4, A.1, A.2, B.4, C.5		**Student Lab:** **Additional Mini Lab,** p. 132: three 5-N spring scales, string (1 m) **Mini Lab,** p. 135: 5-N spring scale, 500-g hooked mass, protractor, smooth board or card-board, tape **Design Your Own Physics Lab,** pp. 136–137: pulley, C-clamp, masking tape, wood surface, string (1 m), 0–5-N spring scale, wood block

Differentiated Instruction

L1 Level 1 activities should be appropriate for students with learning difficulties.

L2 Level 2 activities should be within the ability range of all students.

L3 Level 3 activities are designed for above-average students.

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Assessment Resources

FAST FILE Chapters 1–5 Resources, Chapter 5
Chapter Assessment, pp. 175–180

Additional Challenge Problems, p. 5
Physics Test Prep, pp. 9–10
Pre-AP/Critical Thinking, pp. 9–10
Supplemental Problems, pp. 9–10

Technology

Interactive Chalkboard CD-ROM:
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Chapter 5

Forces in Two Dimensions

Chapter Overview

This chapter extends the discussion of Newton's laws in the previous chapter to two dimensions. The first section reviews formal vector addition in one dimension and extends it to two dimensions. The second section introduces kinetic and static friction and illustrates how to include friction in Newtonian analyses. Finally, additional two-dimensional situations are discussed, including inclined planes, as is the concept of an equilibrant.

Think About This

The climber increases the static friction between her hands, her feet, and the rock using chalk and special shoes. These allow her to apply forces in multiple directions to keep herself in equilibrium.

▶ Key Terms

component, p. 122
vector resolution, p. 122
kinetic friction, p. 126
static friction, p. 126
coefficient of kinetic friction, p. 127
coefficient of static friction, p. 127
equilibrant, p.131

Chapter 5

Forces in Two Dimensions

What You'll Learn

- You will represent vector quantities both graphically and algebraically.
- You will use Newton's laws to analyze motion when friction is involved.
- You will use Newton's laws and your knowledge of vectors to analyze motion in two dimensions.

Why It's Important

Most objects experience forces in more than one dimension. A car being towed, for example, experiences upward and forward forces from the tow truck and the downward force of gravity.

Rock Climbing How do rock climbers keep from falling? This climber has more than one support point, and there are multiple forces acting on her in multiple directions.

Think About This ▶

A rock climber approaches a portion of the rock face that forces her to hang with her back to the ground. How will she use her equipment to apply the laws of physics in her favor and overcome this obstacle?

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118
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LAUNCH Lab

Purpose to develop the concept of vector addition

Materials two 5-N spring scales, 35-cm long string, 15-cm long string, 200-g object, protractor

Teaching Strategies If students are puzzled by the results, have them try the following. Ask them each to hold a bowling ball hanging down by their side. Then tell them to slowly move it away from their bodies toward the horizontal, so that their arms are parallel to the floor. Ask students in which position the ball is easier to hold.

Expected Results Students should observe that the scales read 2 N when the strings meet at a 120° angle. The measured force increases as the angle increases.

LAUNCH Lab

Can 2 N + 2 N = 2 N?

Question

Under what conditions can two different forces equal one other force?

Procedure

1. **Measure** Use a spring scale to measure and record the weight of a 200-g object.
2. Obtain another spring scale, and attach one end of a 35-cm-long piece of string to the hooks on the bottom of each spring scale.
3. Tie one end of a 15-cm-long piece of string to the 200-g object. Loop the other end over the 35-cm-long piece of string and tie the end to the 200-g object. ***CAUTION: Avoid falling masses.***
4. Hold the spring scales parallel to each other so that the string between them forms a 120° angle. Move the string with the hanging object until both scales have the same reading. Record the readings on each scale.
5. **Collect and Organize Data** Slowly pull the string more and more horizontal while it is still supporting the 200-g object. Describe your observations.

Analysis

Does the sum of the forces measured by the two spring scales equal the weight of the hanging object? Is the sum greater than the weight? Less than the weight?

Critical Thinking Draw an equilateral triangle, with one side vertical, on a sheet of paper. If the two sides of the triangle are 2.0 N, explain the size of the third side. How is it possible that 2 N + 2 N = 2 N?

5.1 Vectors

How do rock climbers keep from falling in situations like the one shown on the preceding page? Notice that the climber has more than one support point and that there are multiple forces acting on her. She tightly grips crevices in the rock and has her feet planted on the rock face, so there are two contact forces acting on her. Gravity is pulling on her as well, so there are three total forces acting on the climber. One aspect of this situation that is different from the ones that you have studied in earlier chapters is that the forces exerted by the rock face on the climber are not horizontal or vertical forces. You know from previous chapters that you can pick your coordinate system and orient it in the way that is most useful to analyzing the situation. But what happens when the forces are not at right angles to each other? How can you set up a coordinate system and find for a net force when you are dealing with more than one dimension?

▶ Objectives

- **Evaluate** the sum of two or more vectors in two dimensions graphically.
- **Determine** the components of vectors.
- **Solve** for the sum of two or more vectors algebraically by adding the components of the vectors.

▶ Vocabulary

components
vector resolution

Analysis The sum of the weight on both scales is apparently greater than the weight of the hanging object. Because vector addition accounts for both direction and magnitude, 2 N + 2 N can equal 2 N if the two forces are added as vectors.

Critical Thinking One way to think about the net force acting on an object is to imagine replacing all of the individual forces with a single force that has the same effect. The purpose of vector resolution is to determine the magnitude and direction of that net force. When the forces on an object are in equilibrium, the force vectors cancel each other out, and there is no net effect on the object. For example, in a rugby scrum between two evenly matched teams, great forces are being exerted in many directions, but the net effect on the ball is zero. (A scrum is a mass of players from both sides that struggle for possession of a ball resembling a football.)

1 FOCUS

Bellringer Activity

Displaced Person Ask students to consider a person who walks 100 m due north and then loses all sense of direction. Without knowing the direction, the person walks another 100 m. Ask students what the magnitude of displacement could be, relative to the original starting point. They must consider the straight-line distance from the starting point. Suggest drawing arrows to represent the two 100-m vectors as they analyze the problem. The magnitude of the displacement of the person could be anywhere between 0 m and 200 m. **L2**

Tie to Prior Knowledge

Vectors, Forces, Acceleration Students learned how to add and subtract vectors in one dimension in Chapters 2 and 3. In Chapter 4, students learned what forces were and how to apply Newton's laws in one dimension. Although the emphasis in this chapter is on forces in two dimensions, knowledge of displacement, velocity, and acceleration also is necessary to analyze some of the situations presented.

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2 TEACH

Using Models

Rolling Ruler Demo To help students better visualize and understand the permissible ways to move vectors without changing them, you might use a rolling ruler to demonstrate a subset of the valid ways to move vectors. These rulers are available in stores that sell drawing and drafting equipment.

Concept Development

Designating Vectors Explain the method you will use to indicate vector quantities on the chalkboard or overhead transparency and on handouts. Usually, this is done by placing an arrow above the symbol.

Reinforcement

Magnitude To refresh students' memories, ask them what the word *magnitude* means to them. In physics, it means the size of a quantity, often represented by the length of an arrow. Discuss the need to always specify both the magnitude and direction of vector quantities. **L2 Linguistic**

Using Figure 5-2

This shows one of two correct ways to graphically add two vectors. Since adding vectors is commutative, they can be added in either order. You could put the tail of the eastward vector to the tip of the northward vector. The resultant will be the same, although the diagrams will appear different. **L2**

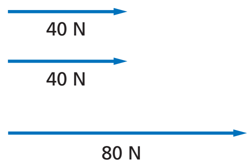


Figure 5-1 The sum of the two 40-N forces is shown by the resultant vector below them.

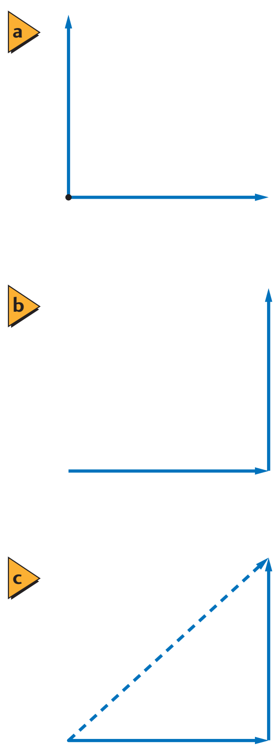
Vectors Revisited

Consider an example with force vectors. Recall the case in Chapter 4 in which you and a friend both pushed on a table together. Suppose that you each exerted 40 N of force to the right. **Figure 5-1** represents these vectors in a free-body diagram with the resultant vector, the net force, shown below it. The net force vector is 80 N, which is what you probably expected. But how was this net force vector obtained?

Vectors in Multiple Dimensions

The process for adding vectors works even when the vectors do not point along the same straight line. If you are solving one of these two-dimensional problems graphically, you will need to use a protractor, both to draw the vectors at the correct angles and also to measure the direction and magnitude of the resultant vector. You can add vectors by placing them tip-to-tail and then drawing the resultant of the vector by connecting the tail of the first vector to the tip of the second vector, as shown in **Figure 5-2**. **Figure 5-2a** shows the two forces in the free-body diagram. In **Figure 5-2b,** one of the vectors has been moved so that its tail is at the same place as the tip of the other vector. Notice that its length and direction have not changed. Because the length and direction are the only important characteristics of the vector, the vector is unchanged by this movement. This is always true: if you move a vector so that its length and direction are unchanged, the vector is unchanged. Now, as in **Figure 5-2c,** you can draw the resultant vector pointing from the tail of the first vector to the tip of the last vector and measure it to obtain its magnitude. Use a protractor to measure the direction of the resultant vector. Sometimes you will need to use trigonometry to determine the length or direction of resultant vectors. Remember that the length of the hypotenuse of a right triangle can be found by using the Pythagorean theorem. If you were adding together two vectors at right angles, vector $\boldsymbol{A}$ pointing north and vector $\boldsymbol{B}$ pointing east, you could use the Pythagorean theorem to find the magnitude of the resultant, R.

Figure 5-2 Add vectors by placing them tip-to-tail and drawing the resultant from the tail of the first vector to the tip of the last vector.



Pythagorean Theorem $R^2 = A^2 + B^2$

If vector A is at a right angle to vector B, then the sum of the squares of the magnitudes is equal to the square of the magnitude of the resultant vector.

If the two vectors to be added are at an angle other than 90°, then you can use the law of cosines or the law of sines.

Law of Cosines $R^2 = A^2 + B^2 - 2AB\cos\theta$

The square of the magnitude of the resultant vector is equal to the sum of the magnitudes of the squares of the two vectors, minus two times the product of the magnitudes of the vectors, multiplied by the cosine of the angle between them.

Law of Sines $\dfrac{R}{\sin\theta} = \dfrac{A}{\sin a} = \dfrac{B}{\sin b}$

The magnitude of the resultant, divided by the sine of the angle between two vectors, is equal to the magnitude of one of the vectors divided by the angle between that component vector and the resultant vector.

5.1 Resource MANAGER

FAST FILE Chapters 1–5 Resources
- Transparency 5–1 Master, p. 167
- Study Guide, pp. 153–158
- Section 5–1 Quiz, p. 159

Teaching Transparency 5-1
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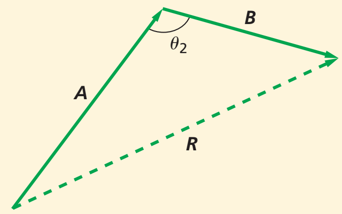
▶ EXAMPLE Problem 1

Finding the Magnitude of the Sum of Two Vectors Find the magnitude of the sum of a 15-km displacement and a 25-km displacement when the angle between them is 90° and when the angle between them is 135°.

1 Analyze and Sketch the Problem

- Sketch the two displacement vectors, $\boldsymbol{A}$ and $\boldsymbol{B}$, and the angle between them.



Known:		Unknown:
$A = 25$ km	$\theta_1 = 90°$	$R = ?$
$B = 15$ km	$\theta_2 = 135°$	

2 Solve for the Unknown

When the angle is 90°, use the Pythagorean theorem to find the magnitude of the resultant vector.

$$R^2 = A^2 + B^2$$
$$R = \sqrt{A^2 + B^2}$$
$$= \sqrt{(25\text{ km})^2 + (15\text{ km})^2} \quad \text{Substitute } A = 25\text{ km}, B = 15\text{ km}$$
$$= 29\text{ km}$$

Math Handbook
Square and Cube Roots
pages 839–840

When the angle does not equal 90°, use the law of cosines to find the magnitude of the resultant vector.

$$R^2 = A^2 + B^2 - 2AB(\cos\theta_2)$$
$$R = \sqrt{A^2 + B^2 - 2AB(\cos\theta_2)}$$
$$= \sqrt{(25\text{ km})^2 + (15\text{ km})^2 - 2(25\text{ km})(15\text{ km})(\cos 135°)} \quad \text{Substitute } A = 25\text{ km}, B = 15\text{ km}, \theta_2 = 135°$$
$$= 37\text{ km}$$

3 Evaluate the Answer

- **Are the units correct?** Each answer is a length measured in kilometers.
- **Do the signs make sense?** The sums are positive.
- **Are the magnitudes realistic?** The magnitudes are in the same range as the two combined vectors, but longer. This is because each resultant is the side opposite an obtuse angle. The second answer is larger than the first, which agrees with the graphical representation.

▶ PRACTICE Problems

Additional Problems, Appendix B

1. A car is driven 125.0 km due west, then 65.0 km due south. What is the magnitude of its displacement? Solve this problem both graphically and mathematically, and check your answers against each other.
2. Two shoppers walk from the door of the mall to their car, which is 250.0 m down a lane of cars, and then turn 90° to the right and walk an additional 60.0 m. What is the magnitude of the displacement of the shoppers' car from the mall door? Solve this problem both graphically and mathematically, and check your answers against each other.
3. A hiker walks 4.5 km in one direction, then makes a 45° turn to the right and walks another 6.4 km. What is the magnitude of her displacement?
4. An ant is crawling on the sidewalk. At one moment, it is moving south a distance of 5.0 mm. It then turns southwest and crawls 4.0 mm. What is the magnitude of the ant's displacement?

▶ IN-CLASS Example

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Question Find the magnitude of the sum of two forces, one 20.0 N and the other 7.0 N, when the angle between them is 30.0°.

Answer Use the law of cosines $R^2 = A^2 + B^2 - 2AB\cos\theta =$ $(20.0\text{ N})^2 + (7.0\text{ N})^2 - 2(20.0\text{ N})(7.0\text{ N})\cos 30.0° = 14.4\text{ N} = 14\text{ N}$

▶ PRACTICE Problems

1. See Solutions Manual. $R = 141$ km
2. See Solutions Manual. $R = 257$ m
3. 1.0×10^1 km
4. 8.3 mm

QUICK DEMO

Vector Addition

Estimated Time 10 minutes

Materials two or three bungee cords with arrow heads, pegboard, metric ruler

Procedure Use the pegboard and bungee cords to demonstrate vector addition. Create a number of vector addition problems. Use the bungee cords to represent different vectors. Vectors can be started from the same origin or can be stacked graphically to show addition. Use the ruler to measure the length of each vector as well as the resultant.

HELPING STRUGGLING STUDENTS Activity

Vector Addition This text describes vector addition using the tip-to-tail method. If this does not seem to work for some students, you may want to have them try the parallelogram method of vector addition. In this method, the student draws a copy of the first vector, with the tail at the tip of the second vector, parallel to the first vector and the same length. Then the student draws a copy of the second vector, with the tail at the tip of the first vectors. The two original vectors and two copies should make a parallelogram. The student finds the resultant vector by drawing a vector whose tail is at the tails of the original vector, and whose tip is at the tips of the copies. **L1 Visual-Spatial**

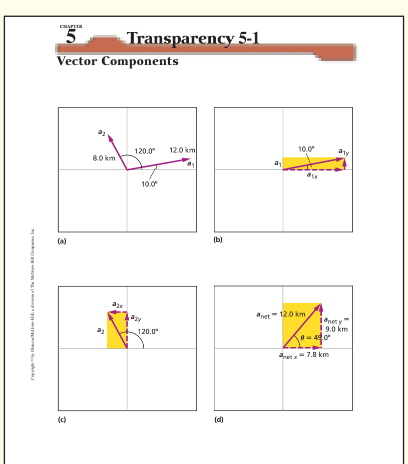
Identifying Misconceptions

Pythagorean Theorem Students will often want to apply the Pythagorean theorem whenever they add two vectors. In fact, the Pythagorean theorem is only applicable when adding two vectors at right angles to each other.

ACTIVITY

Vector Addition It may be easier for students to understand vector addition and subtraction when it applies more directly to their own motion. Choose a location such as a gym, field, or open classroom, where students can determine their displacements graphically. Have students move from one position to another, and then measure the vectors to their new positions. Have students create a scale to calculate and measure their displacements; they should also check the two quantities against each other. **L1 Kinesthetic**

Page 167, FAST FILE Chapters 1–5 Resources



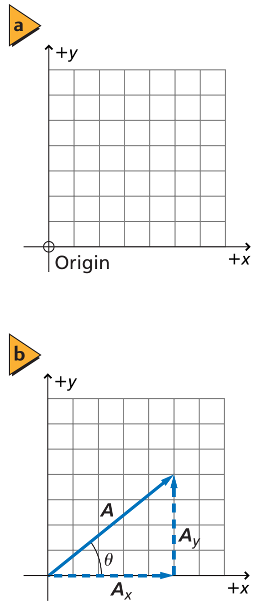
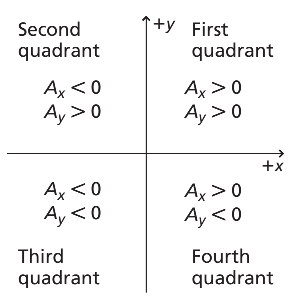


Figure 5-3 A coordinate system has an origin and two perpendicular axes **(a)**. The direction of a vector, ***A***, is measured counterclockwise from the *x*-axis **(b).**

Figure 5-4 The sign of a component depends upon which of the four quadrants the component is in.



Components of Vectors

Choosing a coordinate system, such as the one in **Figure 5-3a,** is similar to laying a grid drawn on a sheet of transparent plastic on top of a vector problem. You have to choose where to put the center of the grid (the origin) and establish the directions in which the axes point. Notice that in the coordinate system shown in Figure 5-3a, the *x*-axis is drawn through the origin with an arrow pointing in the positive direction. The positive *y*-axis is located 90° counterclockwise from the positive *x*-axis and crosses the *x*-axis at the origin.

How do you choose the direction of the *x*-axis? There is never a single correct answer, but some choices make the problem easier to solve than others. When the motion you are describing is confined to the surface of Earth, it is often convenient to have the *x*-axis point east and the *y*-axis point north. When the motion involves an object moving through the air, the positive *x*-axis is often chosen to be horizontal and the positive *y*-axis vertical (upward). If the motion is on a hill, it's convenient to place the positive *x*-axis in the direction of the motion and the *y*-axis perpendicular to the *x*-axis.

Component vectors Defining a coordinate system allows you to describe a vector in a different way. Vector ***A*** shown in **Figure 5-3b,** for example, could be described as going 5 units in the positive *x*-direction and 4 units in the positive *y*-direction. You can represent this information in the form of two vectors like the ones labeled $\boldsymbol{A}_x$ and $\boldsymbol{A}_y$ in the diagram. Notice that $\boldsymbol{A}_x$ is parallel to the *x*-axis, and $\boldsymbol{A}_y$ is parallel to the *y*-axis. Further, you can see that if you add $\boldsymbol{A}_x$ and $\boldsymbol{A}_y$, the resultant is the original vector, ***A***. A vector can be broken into its **components,** which are a vector parallel to the *x*-axis and another parallel to the *y*-axis. This can always be done and the following vector equation is always true.

$$\boldsymbol{A} = \boldsymbol{A}_x + \boldsymbol{A}_y$$

This process of breaking a vector into its components is sometimes called **vector resolution.** Notice that the original vector is the hypotenuse of a right triangle. This means that the magnitude of the original vector will always be larger than the magnitudes of either component vector.

Another reason for choosing a coordinate system is that the direction of any vector can be specified relative to those coordinates. The direction of a vector is defined as the angle that the vector makes with the *x*-axis, measured counterclockwise. In Figure 5-3b, the angle, θ, tells the direction of the vector, ***A***. All algebraic calculations involve only the positive components of vectors, not the vectors themselves. In addition to measuring the lengths of the component vectors graphically, you can find the components by using trigonometry. The components are calculated using the equations below, where the angle, θ, is measured counterclockwise from the positive *x*-axis.

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{A_x}{A};\ \text{therefore, } A_x = A\cos\theta$$

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{A_y}{A};\ \text{therefore, } A_y = A\sin\theta$$

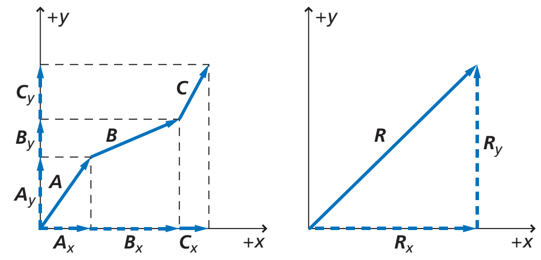
When the angle that a vector makes with the *x*-axis is larger than 90°, the sign of one or more components is negative, as shown in **Figure 5-4.**

CHALLENGE Activity

Vector Operations Some students may ask if it is possible to multiply and divide vectors. Refer them to a university-level physics text, which will demonstrate the multiplication of vectors to form a scalar (or dot) product, and a different type of multiplication to form a vector (or cross) product. Have them attempt to solve one of the physics problem sets in the textbook. They can compare their answers and solutions with each other. **L3 Logical-Mathematical**

Algebraic Addition of Vectors

You might be wondering why you need to resolve vectors into their components. The answer is that doing this often makes adding vectors together much easier mathematically. Two or more vectors (**A**, **B**, **C**, etc.) may be added by first resolving each vector into its *x*- and *y*-components. The *x*-components are added to form the *x*-component of the resultant: $R_x = A_x + B_x + C_x$. Similarly, the *y*-components are added to form the *y*-component of the resultant: $R_y = A_y + B_y + C_y$. This process is illustrated graphically in **Figure 5-5.** Because R_x and R_y are at a right angle (90°), the magnitude of the resultant vector can be calculated using the Pythagorean theorem, $R^2 = R_x^2 + R_y^2$. To find the angle or direction of the resultant, recall that the tangent of the angle that the vector makes with the *x*-axis is given by the following.



■ **Figure 5-5** $\boldsymbol{R_x}$ is the sum of the *x*-components of **A**, **B**, and **C**. $\boldsymbol{R_y}$ is the sum of the *y*-components. The vector sum of $\boldsymbol{R_x}$ and $\boldsymbol{R_y}$ is the vector sum of **A**, **B**, and **C**.

Angle of the Resultant Vector $\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

The angle of the resultant vector is equal to the inverse tangent of the quotient of the *y*-component divided by the *x*-component of the resultant vector.

You can find the angle by using the $\tan^{-1}$ key on your calculator. Note that when $\tan\theta > 0$, most calculators give the angle between 0° and 90°, and when $\tan\theta < 0$, the angle is reported to be between 0° and −90°.

▶ PROBLEM-SOLVING Strategies

Vector Addition

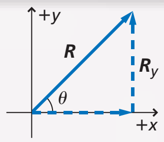
Use the following technique to solve problems for which you need to add or subtract vectors.

1. Choose a coordinate system.
2. Resolve the vectors into their *x*-components using $A_x = A\cos\theta$, and their *y*-components using $A_y = A\sin\theta$, where θ is the angle measured counterclockwise from the positive *x*-axis.
3. Add or subtract the component vectors in the *x*-direction.
4. Add or subtract the component vectors in the *y*-direction.
5. Use the Pythagorean theorem, $R = \sqrt{R_x^2 + R_y^2}$, to find the magnitude of the resultant vector.
6. Use $\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$ to find the angle of the resultant vector.

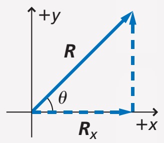
▶ Connecting Math to Physics

Math Review

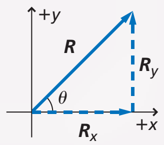
$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{R_y}{R}$



$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{R_x}{R}$



$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{R_y}{R_x}$



Critical Thinking

Permissible Operations Ask students which arithmetic operations are permitted between a vector and a scalar. It is not possible to add or subtract a combination of vectors and scalars, it is possible to multiply a vector by a scalar. Multiplication changes the length of the vector but not its direction, except that when the scalar is negative, it reverses the direction of the vector. As an example, the method for vector subtraction described in this text essentially multiplies the second vector by −1. **L2**

Reinforcement

Vector Addition Vectors are difficult to understand. Students may resort to ordinary addition when presented with word problems involving vectors. If this occurs, ask them to imagine they are walking along the edge of a 50-m-long park, then they turn 90° and walk another 50 m. Ask how far they would be from the starting point. 71 m If they answer 100 m, ask why they would go over and then up and not cut across going from the beginning point to the end point on a diagonal. **L1**

DIFFERENTIATED INSTRUCTION Activity

Visually Impaired The qualitative aspects of vector addition can be communicated to students who have limited vision by using cardboard or plastic arrows of varying lengths. Fasteners can be used to place two vector arrows tip to tail, and a third arrow can be used to represent the resultant. Alternatively, pencils can be taped to a flat surface so students can feel the geometry. When moving arrows or pencils you can translate them (move up, down, right, or left), but do not rotate them. Rotation does change the value of the vector, whereas translating does not change its value. **L1** **Kinesthetic**

▶ IN-CLASS Example

Question Add the following two vectors via the component method: A is 4.0 m south and B is 7.3 m northwest.

Answer Treat east as $+x$ and north as $+y$.

$A_x = (4.0\text{ m})\cos 270° = 0$
$B_x = (7.3\text{ m})\cos 135° = -5.16\text{ m}$

$A_y = (4.0\text{ m})\sin 270° = -4.0\text{ m}$
$B_y = (7.3\text{ m})\sin 135° = 5.16\text{ m}$

$R_x = A_x + B_x = 0 + (-5.16\text{ m})$
$= -5.16\text{ m}$
$R_y = A_y + B_y$
$= (-4.0\text{ m}) + (5.16\text{ m}) = 1.16\text{ m}$

$R^2 = R_x^{\,2} + R_y^{\,2}$
$= (-5.16\text{ m})^2 + (-1.16\text{ m})^2$
$R = 5.3\text{ m}$

Direction: $\theta = \tan^{-1} R_y/R_x = \tan^{-1}$ $(1.16\text{ m})/(-5.16\text{ m}) = 167°$, or 12.7° north of west

Critical Thinking

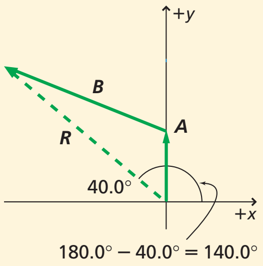
Adding Three Vectors Remind the students that they have explored addition of two vectors. Ask them how this might solve a problem involving three vectors. Add two vectors, then add the third vector to the resultant. Point out that they could find the magnitude of the sum of the first two vectors using the Pythagorean theorem or the law of cosines, but they could not find its angle. L2

▶ EXAMPLE Problem 2

Finding Your Way Home A GPS receiver indicates that your home is 15.0 km and 40.0° north of west, but the only path through the woods leads directly north. If you follow the path 5.0 km before it opens into a field, how far, and in what direction, would you have to walk to reach your home?

1 Analyze and Sketch the Problem

- Draw the resultant vector, $\boldsymbol{R}$, from your original location to your home.
- Draw $\boldsymbol{A}$, the known vector, and draw $\boldsymbol{B}$, the unknown vector.



Known:
$\boldsymbol{A}$ = 5.0 km, due north
$\boldsymbol{R}$ = 15.0 km, 40.0° north of west
θ = 140.0°

Unknown:
$\boldsymbol{B}$ = ?

2 Solve for the Unknown

Find the components of $\boldsymbol{R}$.

$R_x = R\cos\theta$
$= (15.0\text{ km})\cos 140.0°$ — Substitute $\boldsymbol{R}$ = 15.0 km, θ = 140.0°
$= -11.5\text{ km}$

$R_y = R\sin\theta$
$= (15.0\text{ km})\sin 140.0°$ — Substitute $\boldsymbol{R}$ = 15.0 km, θ = 140.0°
$= 9.64\text{ km}$

Math Handbook
Inverses of Sine, Cosine, and Tangent
page 856

Because $\boldsymbol{A}$ is due north, A_x = 0.0 km and A_y = 5.0 km.

Use the components of $\boldsymbol{R}$ and $\boldsymbol{A}$ to find the components of $\boldsymbol{B}$.

$B_x = R_x - A_x$
$= -11.5\text{ km} - 0.0\text{ km}$ — Substitute R_x = −11.5 km, A_x = 0.0 km
$= -11.5\text{ km}$ — The negative sign means that this component points west.

$B_y = R_y - A_y$
$= 9.64\text{ km} - 5.0\text{ km}$ — Substitute R_y = 9.64 km, A_y = 5.0 km
$= 4.6\text{ km}$ — This component points north.

Use the components of vector $\boldsymbol{B}$ to find the magnitude of vector $\boldsymbol{B}$.

$B = \sqrt{B_x^{\,2} + B_y^{\,2}}$
$= \sqrt{(-11.5\text{ km})^2 + (4.6\text{ km})^2}$ — Substitute B_y = −4.6 km, B_x = −11.5 km
$= 12\text{ km}$

Use the tangent to find the direction of vector $\boldsymbol{B}$.

$\theta = \tan^{-1}\dfrac{B_y}{B_x}$
$= \tan^{-1}\dfrac{4.6\text{ km}}{-11.5\text{ km}}$ — Substitute B_y = 4.6 km, B_x = −11.5 km
$= -22°$ or $158°$ — Tangent of an angle is negative in quadrants II and IV, so two answers are possible.

Locate the tail of vector $\boldsymbol{B}$ at the origin of a coordinate system and draw the components B_x and B_y. The direction is in the third quadrant, at 158°, or 22° north of west. Thus, $\boldsymbol{B}$ = 12 km at 22° north of west.

3 Evaluate the Answer

- **Are the units correct?** Kilometers and degrees are correct.
- **Do the signs make sense?** They agree with the diagram.
- **Is the magnitude realistic?** The length of $\boldsymbol{B}$ should be longer than R_x because the angle between A and B is greater than 90°.

HELPING STRUGGLING STUDENTS Activity

Vector Addition The vector sum of the legs of an angle is not equivalent to their algebraic sum. For this activity, you need a large space, some masking tape, a meterstick, and a calculator. Clear a space large enough to mark out a relatively large right triangle on the floor with masking tape. Have two students practice taking the same size steps. Then have them both stand at the vertex of one of the acute angles. Starting at the same time, have one student move to the other acute vertex along the hypotenuse, while the other student moves along the two legs. Make sure that both students take the same sized steps at the same pace. The student who moves along the hypotenuse will get there first. L1 Kinesthetic

PRACTICE Problems

Additional Problems, Appendix B

Solve problems 5–10 algebraically. You may also choose to solve some of them graphically to check your answers.

5. Sudhir walks 0.40 km in a direction 60.0° west of north, then goes 0.50 km due west. What is his displacement?

6. Afua and Chrissy are going to sleep overnight in their tree house and are using some ropes to pull up a box containing their pillows and blankets, which have a total mass of 3.20 kg. The girls stand on different branches, as shown in **Figure 5-6,** and pull at the angles and with the forces indicated. Find the *x*- and *y*-components of the net force on the box. *Hint: Draw a free-body diagram so that you do not leave out a force.*

7. You first walk 8.0 km north from home, then walk east until your displacement from home is 10.0 km. How far east did you walk?

8. A child's swing is held up by two ropes tied to a tree branch that hangs 13.0° from the vertical. If the tension in each rope is 2.28 N, what is the combined force (magnitude and direction) of the two ropes on the swing?

9. Could a vector ever be shorter than one of its components? Equal in length to one of its components? Explain.

10. In a coordinate system in which the *x*-axis is east, for what range of angles is the *x*-component positive? For what range is it negative?

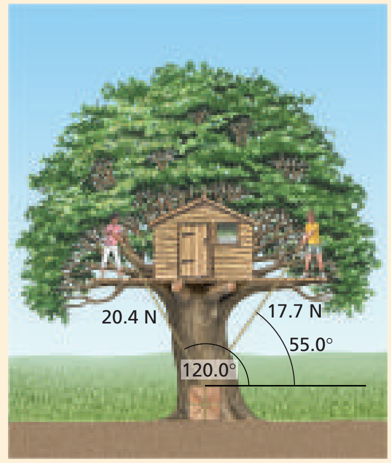


Figure 5-6 (Not to scale)

You will use these techniques to resolve vectors into their components throughout your study of physics. You will get more practice at it, particularly in the rest of this chapter and the next. Resolving vectors into components allows you to analyze complex systems of vectors without using graphical methods.

5.1 Section Review

11. Distance v. Displacement Is the distance that you walk equal to the magnitude of your displacement? Give an example that supports your conclusion.

12. Vector Difference Subtract vector ***K*** from vector ***L***, shown in **Figure 5-7.**

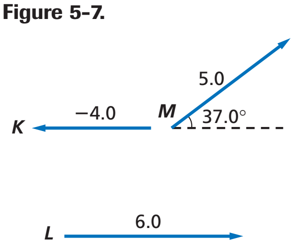


Figure 5-7

13. Components Find the components of vector ***M***, shown in Figure 5-7.

14. Vector Sum Find the sum of the three vectors shown in Figure 5-7.

15. Commutative Operations The order in which vectors are added does not matter. Mathematicians say that vector addition is commutative. Which ordinary arithmetic operations are commutative? Which are not?

16. Critical Thinking A box is moved through one displacement and then through a second displacement. The magnitudes of the two displacements are unequal. Could the displacements have directions such that the resultant displacement is zero? Suppose the box was moved through three displacements of unequal magnitude. Could the resultant displacement be zero? Support your conclusion with a diagram.

Physics Online physicspp.com/self_check_quiz

PRACTICE Problems

5. 0.87 km at 77° west of north

6. The net force is 0.8 N in the upward direction

7. 6.0 km

8. 4.44 N upward

9. It could never be shorter than one of its components, but if it lies along either the *x*- or *y*-axis, then one of its components equals its length.

10. The *x*-component is positive for angles less than 90° and for angles greater than 270°. It's negative for angles greater than 90°, but less than 270°.

5.1 Section Review

11. Not necessarily; for example, I could walk around the block (one kilometer per side). My displacement would be zero, but the distance that I walk would be 4 km.

12. 10.0 to the right

13. $\boldsymbol{M}_x$ = 4.0 to the right
$\boldsymbol{M}_y$ = 3.0 upward

14. ***R*** = 6.7 at 27°

15. Addition and multiplication are commutative. Subtraction and division are not.

16. No, but if there are three displacements, the sum can be zero if the three vectors form a triangle when they are placed tip-to-tail or if the sum of two displacements equals the third. See Solutions Manual.

3 ASSESS

Check for Understanding

Vector Addition Activity Ask students to make up a problem involving vector addition in one dimension. Have them explain how to solve it. Then have them make up another problem involving vector addition in two dimensions, where the vectors are at right angles, and ask them to outline the solution. Finally, have them create a problem involving two-dimensional vector addition without right angles and explain its solution. **L2 Linguistic**

Reteach

Vector Addition Activity Review the methods of adding vectors, both graphically and mathematically. Emphasize when the Pythagorean theorem does and does not apply. Draw a number of sets of vectors on the board. Have a student use a meterstick to measure the vectors and determine the resultant vector. Then have the students determine the resultant for each set using trigonometry. **L2 Visual-Spatial**

Section 5.2

1 FOCUS

Bellringer Activity

Surface Friction Push a low-friction object, such as ice, across a table. Then push a higher friction object, such as a book, across the same surface. Ask students to describe the difference in the behaviors of the two objects and to suggest a possible reason for this difference. Students probably will say that friction was a factor. Get students to articulate what they mean by *friction,* so that you will be able to help guide them to the scientific definition later.
L1 Visual-Spatial

Tie to Prior Knowledge

Resistance In previous chapters, the notion of friction has been called *resistance.* Make this connection clear to the students once you have formally defined friction. Also, students are well acquainted with the effects of friction from events in their daily lives.

2 TEACH

Using Figure 5-8

Sketch Figures 5-8b and 5-8c on the board. Draw force vectors showing the applied force and the friction resistance in each case. Explain that in Figure 5-8b the static friction will match the applied force to prevent motion. Only when the applied force exceeds the resistance force will the object move, as in Figure 5-8c.

5.2 Friction

Objectives
- **Define** the friction force.
- **Distinguish** between static and kinetic friction.

Vocabulary
kinetic friction
static friction
coefficient of kinetic friction
coefficient of static friction

Push your hand across your desktop and feel the force called friction opposing the motion. Push your book across the desk. When you stop pushing, the book will continue moving for a little while, then it will slow down and stop. The frictional force acting on the book gave it an acceleration in the direction opposite to the one in which it was moving. So far, you have neglected friction in solving problems, but friction is all around you. You need it to both start and stop a bicycle and a car. If you have ever walked on ice, you understand the importance of friction.

Static and Kinetic Friction

There are two types of friction. Both always oppose motion. When you pushed your book across the desk, it experienced a type of friction that acts on moving objects. This force is known as **kinetic friction,** and it is exerted on one surface by another when the two surfaces rub against each other because one or both of them are moving.

To understand the other kind of friction, imagine trying to push a heavy couch across the floor. You give it a push, but it does not move. Because it does not move, Newton's laws tell you that there must be a second horizontal force acting on the couch, one that opposes your force and is equal in size. This force is **static friction,** which is the force exerted on one surface by another when there is no motion between the two surfaces. You might push harder and harder, as shown in **Figures 5-8a** and **5-8b,** but if the couch still does not move, the force of friction must be getting larger. This is because the static friction force acts in response to other forces. Finally, when you push hard enough, as shown in **Figure 5-8c,** the couch will begin to move. Evidently, there is a limit to how large the static friction force can be. Once your force is greater than this maximum static friction, the couch begins moving and kinetic friction begins to act on it instead of static friction.

A model for friction forces On what does a frictional force depend? The materials that the surfaces are made of play a role. For example, there is more friction between skis and concrete than there is between skis and snow. It may seem reasonable to think that the force of friction also might depend on either the surface area in contact or the speed of the motion, but experiments have shown that this is not true. The normal force between the two objects does matter, however. The harder one object is pushed against the other, the greater the force of friction that results.

Figure 5-8 There is a limit to the ability of the static friction force to match the applied force.

5.2 Resource MANAGER

FAST FILE Chapters 1–5 Resources
Transparency 5–2 Master, p. 169
Transparency 5–3 Master, p. 171
Study Guide, pp. 153–158
Section 5–2 Quiz, p. 160
Teaching Transparency 5-2
Teaching Transparency 5-3
Connecting Math to Physics

Technology
TeacherWorks™ CD-ROM
Interactive Chalkboard CD-ROM
Exam*View*® Pro Testmaker CD-ROM
physicspp.com
physicspp.com/vocabulary_puzzlemaker

If you pull a block along a surface at a constant velocity, according to Newton's laws, the frictional force must be equal and opposite to the force with which you pull. You can pull a block of known mass along a table at a constant velocity and use a spring scale, as shown in **Figure 5-9,** to measure the force that you exert. You can then stack additional blocks on the block to increase the normal force and repeat the measurement.

Plotting the data will yield a graph like the one in **Figure 5-10.** There is a direct proportion between the kinetic friction force and the normal force. The different lines correspond to dragging the block along different surfaces. Note that the line corresponding to the sandpaper surface has a steeper slope than the line for the highly polished table. You would expect it to be much harder to pull the block along sandpaper than along a polished table, so the slope must be related to the magnitude of the resulting frictional force. The slope of this line, designated μ_k, is called the **coefficient of kinetic friction** between the two surfaces and relates the frictional force to the normal force, as shown below.

Kinetic Friction Force $F_{f,\,kinetic} = \mu_k F_N$

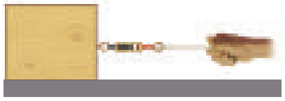
The kinetic friction force is equal to the product of the coefficient of the kinetic friction and the normal force.

The maximum static friction force is related to the normal force in a similar way as the kinetic friction force. Remember that the static friction force acts in response to a force trying to cause a stationary object to start moving. If there is no such force acting on an object, the static friction force is zero. If there is a force trying to cause motion, the static friction force will increase up to a maximum value before it is overcome and motion starts.

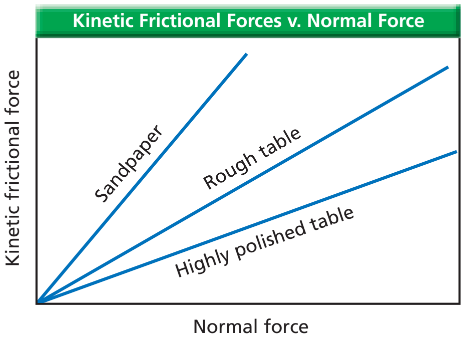
Static Friction Force $F_{f,\,static} \leq \mu_s F_N$

The static friction force is less than or equal to the product of the coefficient of the static friction and the normal force.

In the equation for the maximum static friction force, μ_s is the **coefficient of static friction** between the two surfaces, and $\mu_s F_N$ is the maximum static friction force that must be overcome before motion can begin. In Figure 5-8c, the static friction force is balanced the instant before the couch begins to move.



■ **Figure 5-9** The spring scale pulls the block with a constant force.



■ **Figure 5-10** There is a linear relationship between the frictional force and the normal force.

Critical Thinking

High or Low Friction Ask students if it is better to have low friction between surfaces or high friction between surfaces. It depends on the situation. Ask students to give examples of each. Low-friction situations include pistons in engines and skis on snow. Examples of situations where high friction is desired include erasers on paper or a flag twirler's glove gripping the flagpole.
L2 **Logical-Mathematical**

Using an Analogy

Investigating Friction Activity A simplified analogy for the friction force between two objects is the interaction of two pieces of nylon fastening tape with minute hooks. On a microscopic level, the two surfaces in contact do partially mesh into each other. Ask students to research how this tape works and draw some diagrams to help explain it. **L2**

Identifying Misconceptions

Constant Speed Some students use friction as a cover for hanging onto the misconception that motion at a constant speed only happens when a force is being continuously applied. To prevent this misconception, do not let students develop the habit of answering that friction is responsible for any decrease in speed, such as a thrown object slowing on its way up.

PHYSICS PROJECT Activity

Surface Friction Ask students why some material surfaces scrape past each other, whereas others slip almost effortlessly. As studies of friction at the atomic level are showing, surface texture often has little to do with it. Some objects become less slippery when wet, and a rough object can sometimes slide more easily than a smooth one. Have interested students do library research on this topic. One possible reference is "Friction on the Atomic Scale," *Scientific American*, October 1996, page 74. **L2** **Linguistic**

▶ IN-CLASS Example

Question A small child is dragging a heavy, rubber-soled shoe by its laces across a sidewalk at a constant speed of 0.35 m/s. If the shoe has a mass of 1.56 kg, what is the horizontal component of the force exerted by the child?

Answer $F_{net} = ma = 0$

$F_{net} = F_x - F_f$

$ma = 0$

Substituting yields, $F_x = F_f$

$F_x = \mu F_n = \mu mg$

$= (0.65)(1.56\ kg)(9.8\ m/s^2)$

$= 9.9\ N$

Discussion

Question What are some situations in which it is advantageous to have a high coefficient of friction between two surfaces?

Answer Some examples are running shoes on a track, sandpaper on wood, food graters on cheese, car brakes on wheels, and car tires on roads. L2

▶ PRACTICE Problems

17. 0.69

18. 0.0991

19. 74 N

20. 84 N

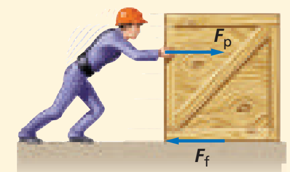
21. 0.6 N

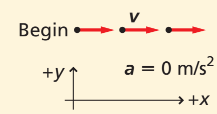
▶ EXAMPLE Problem 3

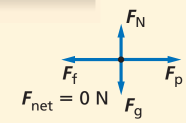
Balanced Friction Forces You push a 25.0-kg wooden box across a wooden floor at a constant speed of 1.0 m/s. How much force do you exert on the box?

1 Analyze and Sketch the Problem

- Identify the forces and establish a coordinate system.
- Draw a motion diagram indicating constant v and $a = 0$.
- Draw the free-body diagram.







Known:	Unknown:
$m = 25.0$ kg	$F_p = ?$
$v = 1.0$ m/s	
$a = 0.0$ m/s^2	
$\mu_k = 0.20$ (Table 5-1)	

2 Solve for the Unknown

The normal force is in the y-direction, and there is no acceleration.

$F_N = F_g$

$= mg$

$= (25.0\ kg)(9.80 m/s^2)$ **Substitute $m = 25.0$ kg, $g = 9.80$ m/s^2**

$= 245$ N

Math Handbook
Operations with Significant Digits
pages 835–836

The pushing force is in the x-direction; v is constant, thus there is no acceleration.

$F_p = \mu_k mg$

$= (0.20)(25.0\ kg)(9.80\ m/s^2)$ **Substitute $\mu_k = 0.20$, $m = 25.0$ kg, $g = 9.80$ m/s^2**

$= 49$ N

3 Evaluate the Answer

- **Are the units correct?** Performing dimensional analysis on the units verifies that force is measured in kg·m/s^2 or N.
- **Does the sign make sense?** The positive sign agrees with the sketch.
- **Is the magnitude realistic?** The force is reasonable for moving a 25.0-kg box.

▶ PRACTICE Problems

Additional Problems, Appendix B

17. A girl exerts a 36-N horizontal force as she pulls a 52-N sled across a cement sidewalk at constant speed. What is the coefficient of kinetic friction between the sidewalk and the metal sled runners? Ignore air resistance.

18. You need to move a 105-kg sofa to a different location in the room. It takes a force of 102 N to start it moving. What is the coefficient of static friction between the sofa and the carpet?

19. Mr. Ames is dragging a box full of books from his office to his car. The box and books together have a combined weight of 134 N. If the coefficient of static friction between the pavement and the box is 0.55, how hard must Mr. Ames push the box in order to start it moving?

20. Suppose that the sled in problem 17 is resting on packed snow. The coefficient of kinetic friction is now only 0.12. If a person weighing 650 N sits on the sled, what force is needed to pull the sled across the snow at constant speed?

21. Suppose that a particular machine in a factory has two steel pieces that must rub against each other at a constant speed. Before either piece of steel has been treated to reduce friction, the force necessary to get them to perform properly is 5.8 N. After the pieces have been treated with oil, what will be the required force?

QUICK DEMO

Friction

Estimated Time 10 minutes

Materials nonstick cookie sheet, felt-covered wood block (6 in × 3 in × 1 in)

Procedure Ask students to predict which side of the block will require the greatest force to be pushed across the cookie sheet at a constant speed. Ask some students to slide the largest side of the felt-covered block along the nonstick cookie sheet. Repeat with the smallest side of the block. Ask students to summarize their results for the class. The force of friction is the same in both cases, regardless of the area of the surfaces in contact.

Table 5-1		
Typical Coefficients of Friction		
Surface	μ_s	μ_k
Rubber on dry concrete	0.80	0.65
Rubber on wet concrete	0.60	0.40
Wood on wood	0.50	0.20
Steel on steel (dry)	0.78	0.58
Steel on steel (with oil)	0.15	0.06

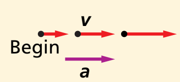
Note that the equations for the kinetic and maximum static friction forces involve only the magnitudes of the forces. The forces themselves, $\boldsymbol{F}_f$ and $\boldsymbol{F}_N$, are at right angles to each other. **Table 5-1** shows coefficients of friction between various surfaces. Although all the listed coefficients are less than 1.0, this does not mean that they must always be less than 1.0. For example, coefficients as large as 5.0 are experienced in drag racing.

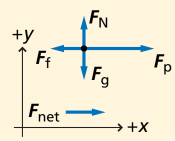
EXAMPLE Problem 4

Unbalanced Friction Forces If the force that you exert on the 25.0-kg box in Example Problem 3 is doubled, what is the resulting acceleration of the box?

1 Analyze and Sketch the Problem

- Draw a motion diagram showing v and a.
- Draw the free-body diagram with a doubled $\boldsymbol{F}_p$.





Known:

$m = 25.0$ kg $\mu_k = 0.20$

$v = 1.0$ m/s $F_p = 2(49\text{ N}) = 98$ N

Unknown:

$a = ?$

2 Solve for the Unknown

The normal force is in the y-direction, and there is no acceleration.

$$F_N = F_g \qquad \text{Substitute } F_g = mg$$
$$= mg$$

In the x-direction there is an acceleration. So the forces must be unequal.

$$F_{net} = F_p - F_f$$
$$ma = F_p - F_f \qquad \text{Substitute } F_{net} = ma$$
$$a = \frac{F_p - F_f}{m}$$

Find F_f and substitute it into the expression for a.

$$F_f = \mu_k F_N$$
$$= \mu_k mg \qquad \text{Substitute } F_N = mg$$
$$a = \frac{F_p - \mu_k mg}{m} \qquad \text{Substitute } F_f = \mu_k mg$$
$$= \frac{98\text{ N} - (0.20)(25.0\text{ kg})(9.80\text{ m/s}^2)}{25.0\text{ kg}} \qquad \text{Substitute } F_p = 98\text{ N},\ m = 25.0\text{ kg},\ \mu_k = 0.20,\ g = 9.80\text{ m/s}^2$$
$$= 2.0\text{ m/s}^2$$

Math Handbook
Isolating a Variable
page 845

3 Evaluate the Answer

- **Are the units correct?** a is measured in m/s^2.
- **Does the sign make sense?** In this coordinate system, the sign should be positive.
- **Is the magnitude realistic?** If the force were cut in half, a would be zero.

Concept Development

Friction and Normal Force Demo Have a student push an empty box or crate at constant speed across a level floor. Then put a heavy load in the container. Have the student push the box across the floor again. Have the student report the change in the force required. Ask why the force required is greater in the second case. The required force increases because an increased normal force produces increased friction. **L2 Kinesthetic**

IN-CLASS Example

INTERACTIVE CHALKBOARD with Image Bank PowerPoint® Presentations

Question If the child in the previous In-Class Example pulls with only half the force as before, what will happen?

Answer If the new force is half of the original force, then a is 0 and the shoe will not move.

Question If the child pulls with an extra 2.0 N in the horizontal direction, what will be the acceleration of the shoe?

Answer $F_{net} = ma$

$F_x - F_f = ma$

$$a = \frac{F_x - F_f}{m}$$
$$= \frac{11.9\text{ N} - 9.9\text{ N}}{1.56\text{ kg}}$$
$$= 1.3\text{ m/s}^2$$

Teacher F.Y.I. — REAL-LIFE PHYSICS

Hydroplaning As students can see in Table 5-1, the coefficients of friction between rubber tires and the concrete of a road are much less when the road is wet than when it is dry. If water pools on the surface of a road, there can be enough of a layer of water between the road and the tires that there is almost no contact between the tires and the road, so that the tires are essentially riding on a layer of water. Since the coefficient of friction between rubber and water is nearly zero, it can be almost impossible to stop in a short distance.

Reinforcement

Static v. Kinetic Friction Demo Use a demonstration-sized spring scale so the class can read the scale as you pull a book across a table. Show that it takes much more force to get the book started than it does to keep it moving. Ask students to explain their observations in terms of static and kinetic frictional forces. This activity could also be done in small groups with smaller spring scales. **L2 Visual-Spatial**

PRACTICE Problems

22. 0.128

23. 0.15

24. 5.5 m

25. 0.50 s

26. 66 m, so he hits the branch before he can stop.

APPLYING PHYSICS

As surface friction varies, so does the ability of certain materials to adhere to the surface. To prevent graffiti in public places, the walls often are coated with a material whose surface makes it impossible to apply any pen, permanent marker, or pencil.

3 ASSESS

Check for Understanding

Friction Coefficients To see whether students understand when to use each of the coefficients of friction, give students a problem that requires the use of only one of the coefficients to solve the problem, but provide them with both coefficients. After they have solved the problem, ask them which coefficient they used, and why. **L2 Logical-Mathematical**

Reteach

Demonstrate Static Friction Emphasize that the static friction force can vary in size and that the coefficient of static friction gives the maximum possible static friction force. Pull on a heavy object with a demonstration-sized spring scale with various forces and have students describe the variation in forces. **L2**

APPLYING PHYSICS

Causes of Friction All surfaces, even those that appear to be smooth, are rough at a microscopic level. If you look at a photograph of a graphite crystal magnified by a scanning tunneling microscope, the atomic level surface irregularities of the crystal are revealed. When two surfaces touch, the high points on each are in contact and temporarily bond. This is the origin of both static and kinetic friction. The details of this process are still unknown and are the subject of research in both physics and engineering.

PRACTICE Problems

Additional Problems, Appendix B

22. A 1.4-kg block slides across a rough surface such that it slows down with an acceleration of 1.25 m/s^2. What is the coefficient of kinetic friction between the block and the surface?

23. You help your mom move a 41-kg bookcase to a different place in the living room. If you push with a force of 65 N and the bookcase accelerates at 0.12 m/s^2, what is the coefficient of kinetic friction between the bookcase and the carpet?

24. A shuffleboard disk is accelerated to a speed of 5.8 m/s and released. If the coefficient of kinetic friction between the disk and the concrete court is 0.31, how far does the disk go before it comes to a stop? The courts are 15.8 m long.

25. Consider the force pushing the box in Example Problem 4. How long would it take for the velocity of the box to double to 2.0 m/s?

26. Ke Min is driving along on a rainy night at 23 m/s when he sees a tree branch lying across the road and slams on the brakes when the branch is 60.0 m in front of him. If the coefficient of kinetic friction between the car's locked tires and the road is 0.41, will the car stop before hitting the branch? The car has a mass of 2400 kg.

Here are a few important things to remember when dealing with frictional situations. First, friction always acts in a direction opposite to the motion (or in the case of static friction, intended motion). Second, the magnitude of the force of friction depends on the magnitude of the normal force between the two rubbing surfaces; it does not necessarily depend on the weight of either object. Finally, multiplying the coefficient of static friction and the normal force gives you the maximum static friction force. Keep these things in mind as you review this section.

5.2 Section Review

27. Friction In this section, you learned about static and kinetic friction. How are these two types of friction similar? What are the differences between static and kinetic friction?

28. Friction At a wedding reception, you notice a small boy who looks like his mass is about 25 kg running part way across the dance floor, then sliding on his knees until he stops. If the kinetic coefficient of friction between the boy's pants and the floor is 0.15, what is the frictional force acting on him as he slides?

29. Velocity Derek is playing cards with his friends, and it is his turn to deal. A card has a mass of 2.3 g, and it slides 0.35 m along the table before it stops. If the coefficient of kinetic friction between the card and the table is 0.24, what was the initial speed of the card as it left Derek's hand?

30. Force The coefficient of static friction between a 40.0-kg picnic table and the ground below it is 0.43. What is the greatest horizontal force that could be exerted on the table while it remains stationary?

31. Acceleration Ryan is moving to a new apartment and puts a dresser in the back of his pickup truck. When the truck accelerates forward, what force accelerates the dresser? Under what circumstances could the dresser slide? In which direction?

32. Critical Thinking You push a 13-kg table in the cafeteria with a horizontal force of 20 N, but it does not move. You then push it with a horizontal force of 25 N, and it accelerates at 0.26 m/s^2. What, if anything, can you conclude about the coefficients of static and kinetic friction?

Physics Online physicspp.com/self_check_quiz

5.2 Section Review

27. Similar: both act in a direction opposite to the motion (or intended motion) and they result from two surfaces rubbing against each other. Static friction applies when there is no relative motion between the two surfaces. Kinetic friction is the type of friction when there is relative motion. The coefficient of static friction between two surfaces is greater than the coefficient of kinetic friction between the same two surfaces.

28. 37 N

29. 1.3 m/s

30. 170 N

31. Friction between the dresser and the truck accelerates the dresser forward. The dresser will slide backward if the force accelerating it is greater than $\mu_s mg$.

32. $\mu_k = 0.17$; $0.16 \leq \mu_s < 0.20$

5.3 Force and Motion in Two Dimensions

You have already worked with several situations dealing with forces in two dimensions. For example, when friction acts between two surfaces, you must take into account both the frictional force that is parallel to the surface and the normal force that is perpendicular to it. So far, you have considered only the motion along a level surface. Now you will use your skill in adding vectors to analyze situations in which the forces acting on an object are at angles other than 90°.

▸ **Objectives**
- **Determine** the force that produces equilibrium when three forces act on an object.
- **Analyze** the motion of an object on an inclined plane with and without friction.

▸ **Vocabulary**

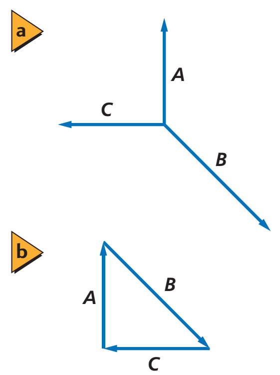
equilibrant

Equilibrium Revisited

Recall from Chapter 4 that when the net force on an object is zero, the object is in equilibrium. According to Newton's laws, the object will not accelerate because there is no net force acting on it; an object in equilibrium is motionless or moves with constant velocity. You have already analyzed several equilibrium situations in which two forces acted on an object. It is important to realize that equilibrium can occur no matter how many forces act on an object. As long as the resultant is zero, the net force is zero and the object is in equilibrium.

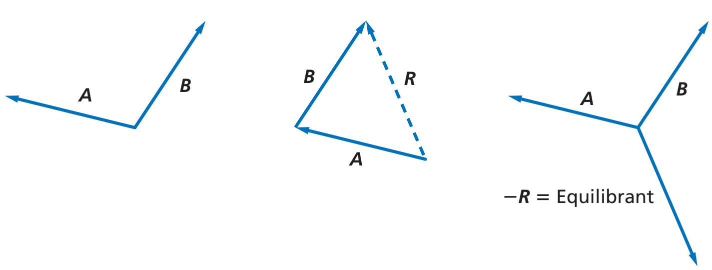
Figure 5-11a shows three forces exerted on a point object. What is the net force acting on the object? Remember that vectors may be moved if you do not change their direction (angle) or length. **Figure 5-11b** shows the addition of the three forces, ***A*, *B*,** and ***C*.** Note that the three vectors form a closed triangle. There is no net force; thus, the sum is zero and the object is in equilibrium.

Suppose that two forces are exerted on an object and the sum is not zero. How could you find a third force that, when added to the other two, would add up to zero, and therefore cause the object to be in equilibrium? To find this force, first find the sum of the two forces already being exerted on the object. This single force that produces the same effect as the two individual forces added together is called the resultant force. The force that you need to find is one with the same magnitude as the resultant force, but in the opposite direction. A force that puts an object in equilibrium is called the **equilibrant. Figure 5-12** illustrates the procedure for finding this force for two vectors. Note that this general procedure works for any number of vectors.



■ **Figure 5-11** An object is in equilibrium when all the forces on it add up to zero.

■ **Figure 5-12** The equilibrant is the same magnitude as the resultant, but opposite in direction.



Section 5.3

1 FOCUS

Bellringer Activity

Tug-of-War Explain to students that if you pull one end of a rope, and your opponent holds the other end still, the other side must exert just as much force as you do. When your opponent starts moving, the forces on the two ends of the rope still are equal. Sketch a free-body diagram to show students that when the net force acting on the rope is zero, neither side wins. It takes an imbalance of forces for one side to win. **L2 Visual-Spatial**

Tie to Prior Knowledge

Equilibrium Students should be familiar with the concept of equilibrium from Chapter 4. However, so far they have applied it only in one dimension.

2 TEACH

Using Figure 5-11

The figure shows that three forces are added to produce a resultant of zero. The same result may be obtained by canceling force *A* and the vertical component of force *B*, and then canceling force *C* and the horizontal component of force *B*. **L2**

Concept Development

Equilibrium Demo Attach an object to two spring scales using strings of equal length. Hold up the object using the spring scales. Demonstrate to the students that equilibrium can be achieved in many different ways by arranging the spring scales so that the angle between the strings varies and the forces exerted by each string vary.

5.3 Resource MANAGER

FAST FILE Chapters 1–5 Resources
- Transparency 5–4 Master, p. 173
- Study Guide, pp. 153–158
- Reinforcement, pp. 163–164
- Enrichment, pp. 165–166
- Section 5–3 Quiz, p. 161
- Mini Lab Worksheet, p. 147
- Physics Lab Worksheet, pp. 149–152

Teaching Transparency 5-4
Connecting Math to Physics

Technology
- TeacherWorks™ CD-ROM
- Interactive Chalkboard CD-ROM
- Exam*View*® Pro Testmaker CD-ROM
- physicspp.com
- physicspp.com/vocabulary_puzzlemaker

Identifying Misconceptions

Equilibrant In trying to find the equilibrant of two forces, students will often calculate just the resultant of the two vectors and call it the equilibrant. To help students avoid this mistake, ask them in what direction the equilibrant acts, reminding them that the equilibrant is a vector of the same magnitude but opposite direction of the resultant. **L2**

CHALLENGE PROBLEM

$F_{1x} = 58.3$ N
$F_{1y} = 17.8$ N

$F_{2x} = 16.7$ N
$F_{2y} = 34.2$ N

$F_{3x} = -7.52$ N
$F_{3y} = 53.5$ N

$F_{4x} = -74.3$ N
$F_{4y} = 56.0$ N

$F_{5x} = -60.7$ N
$F_{5y} = -23.3$ N

$F_{6x} = -26.4$ N
$F_{6y} = -98.5$ N

$F_{7x} = 0.0$ N
$F_{7y} = -26.0$ N

$F_{8x} = 28.8$ N
$F_{8y} = -71.4$ N

$F_{9x} = 27.8$ N
$F_{9y} = -42.8$ N

$F_{10x} = 81.7$ N
$F_{10y} = -7.15$ N

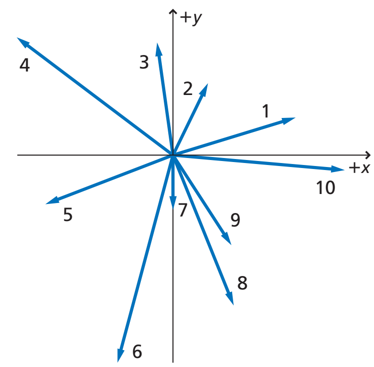
$F_x = 44.38$ N
$F_y = -107.65$ N

$$F_R = \sqrt{(F_x)^2 + (F_y)^2}$$
$$= \sqrt{(44.38\ \text{N})^2 + (-107.65\ \text{N})^2}$$
$$= 116\ \text{N}$$
$$\theta_R = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$
$$= \tan^{-1}\left(\frac{-107.65\ \text{N}}{44.38\ \text{N}}\right)$$
$$= -67.6°$$
$$F_{\text{equilibrant}} = 116\ \text{N at } 112.4°$$
$$= 116\ \text{N at } 22.4°\ \text{W of N}$$

CHALLENGE PROBLEM

Find the equilibrant for the following forces.

F_1 = 61.0 N at 17.0° north of east
F_2 = 38.0 N at 64.0° north of east
F_3 = 54.0 N at 8.0° west of north
F_4 = 93.0 N at 53.0° west of north
F_5 = 65.0 N at 21.0° south of west
F_6 = 102.0 N at 15.0° west of south
F_7 = 26.0 N south
F_8 = 77.0 N at 22.0° east of south
F_9 = 51.0 N at 33.0° east of south
F_{10} = 82.0 N at 5.0° south of east

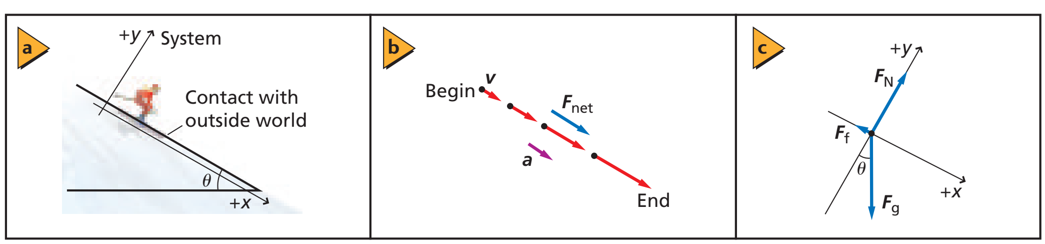


Motion Along an Inclined Plane

You have applied Newton's laws to a variety of equilibrium situations, but only to motions that were either horizontal or vertical. How would you apply them in a situation like the one in **Figure 5-13a,** in which a skier glides down a slope?

Start by identifying the forces acting on the object, the skier, as shown in **Figure 5-13b** and sketching a free-body diagram. The gravitational force on the skier is in the downward direction toward the center of Earth. There is a normal force perpendicular to the hill, and the frictional forces opposing the skier's motion are parallel to the hill. The resulting free-body diagram is shown in **Figure 5-13c.** You can see that, other than the force of friction, only one force acts horizontally or vertically, and you know from experience that the acceleration of the skier will be along the slope. How do you find the net force that causes the skier to accelerate?

Figure 5-13 A skier slides down a slope **(a).** Identify the forces that are acting upon the skier **(b)** and draw a free-body diagram describing those forces **(c).** It is important to draw the direction of the normal and the friction forces correctly in order to properly analyze these types of situations.



Additional MINI LAB

Equilibrium

Purpose to create equilibrium among three forces and analyze the force vectors

Materials three 5-N spring scales, string (1 m)

Procedure Fold string in half around the hook of spring scale A. Attach spring scales B and C to the ends of the string. Ask students to predict the reading on scale A if scales B and C are moved apart while the readings on scales B and C remain the same. Move scales B and C apart until the string makes an angle of 45°. Record all readings. Scale A should be less than the sum of the readings on scales B and C.

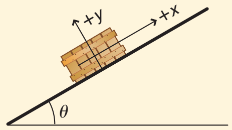
Assessment Discuss why the reading on scale A was less than the sum of the readings on scales B and C. Ask what the reading on scale A would be if they separated scales B and C until the string was straight (zero) and how that informs their discussion.

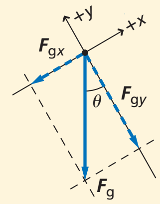
▶ EXAMPLE Problem 5

Components of Weight for an Object on an Incline A crate weighing 562 N is resting on a plane inclined 30.0° above the horizontal. Find the components of the weight forces that are parallel and perpendicular to the plane.

1 Analyze and Sketch the Problem

- Include a coordinate system with the positive *x*-axis pointing uphill.
- Draw the free-body diagram showing $\mathbf{F}_g$, the components $\mathbf{F}_{gx}$ and $\mathbf{F}_{gy}$, and the angle θ.





Known:	**Unknown:**
$F_g = 562$ N	$F_{gx} = ?$
$\theta = 30.0°$	$F_{gy} = ?$

2 Solve for the Unknown

$\mathbf{F}_{gx}$ and $\mathbf{F}_{gy}$ are negative because they point in directions opposite to the positive axes.

$$F_{gx} = -F_g(\sin\theta)$$
$$= -(562\text{ N})(\sin 30.0°) \quad \text{Substitute } F_g = 562,\ \theta = 30.0°$$
$$= -281\text{ N}$$
$$F_{gy} = -F_g(\cos\theta)$$
$$= -(562\text{ N})(\cos 30.0°) \quad \text{Substitute } F_g = 562,\ \theta = 30.0°$$
$$= -487\text{ N}$$

Math Handbook
Trigonometric Ratios
page 855

3 Evaluate the Answer

- **Are the units correct?** Force is measured in newtons.
- **Do the signs make sense?** The components point in directions opposite to the positive axes.
- **Are the magnitudes realistic?** The values are less than F_g.

▶ PRACTICE Problems

Additional Problems, Appendix B

33. An ant climbs at a steady speed up the side of its anthill, which is inclined 30.0° from the vertical. Sketch a free-body diagram for the ant.

34. Scott and Becca are moving a folding table out of the sunlight. A cup of lemonade, with a mass of 0.44 kg, is on the table. Scott lifts his end of the table before Becca does, and as a result, the table makes an angle of 15.0° with the horizontal. Find the components of the cup's weight that are parallel and perpendicular to the plane of the table.

35. Kohana, who has a mass of 50.0 kg, is at the dentist's office having her teeth cleaned, as shown in **Figure 5-14.** If the component of her weight perpendicular to the plane of the seat of the chair is 449 N, at what angle is the chair tilted?

36. Fernando, who has a mass of 43.0 kg, slides down the banister at his grandparents' house. If the banister makes an angle of 35.0° with the horizontal, what is the normal force between Fernando and the banister?

37. A suitcase is on an inclined plane. At what angle, relative to the vertical, will the component of the suitcase's weight parallel to the plane be equal to half the perpendicular component of its weight?

Figure 5-14

▶ IN-CLASS Example

Question Jeff, who weighs 640.0 N, sits on the slope of a hill that descends at an angle of 35.0° from horizontal. What are the components of his weight parallel and perpendicular to the surface of the hill?

Answer Choosing axes such that +*y* is downward and normal to the slope and +*x* is down the slope, yields:

$F_{gx} = (640.0\text{ N})(\sin 35.0°) = 524\text{ N}$

$F_{gy} = (640.0\text{ N})(\cos 35.0°) = 367\text{ N}$

Critical Thinking

Force Components Ask students to look at pictures or videos of weightlifters and determine how the material in this section pertains to successful weightlifting technique. Drawing force diagrams may help them articulate their explanation. Only the force exerted perpendicular to the bar by the lifter can actually elevate the bar. Weightlifters will use a grip that enables them to maximize this perpendicular force. **L2 Visual-Spatial**

▶ PRACTICE Problems

33. See Solutions Manual.

34. $F_{g,\text{ parallel}} = 1.1$ N
$F_{g,\text{ perpendicular}} = 4.2$ N

35. 23.6°

36. 345 N

37. 63.4° relative to the vertical

Teacher F.Y.I. — CONTENT BACKGROUND

Igloo Construction and Equilibrants The Inuit people use equilibrant forces to build igloos. An igloo is a dome-shaped structure built of blocks of ice. As gravity pulls the blocks closer together, each block pushes against the blocks on either side, above, and below it. The top center block is the most important one. Until the center block is in place, the top row of blocks (in particular) will move because none of them are in equilibrium. However, once the top block is inserted, all the blocks in the igloo stay in place because the sum of the forces results in a net force of zero.

IN-CLASS Example

Question A sled slides from rest at the top of a snow-covered hill. If the hill is inclined at 30.0° and the coefficient of kinetic friction between the sled's runners and the snow is 0.18, what is the sled's acceleration?

Answer $F_{gx} - F_{fx} = ma_x$

$mg(\sin\theta) - \mu mg(\cos\theta) = ma_x$

$a_x = g(\sin\theta) - \mu g(\cos\theta)$

$= (9.80\text{ m/s}^2)(\sin 30.0°) - 0.18(9.80\text{ m/s}^2)(\cos 30.0°)$

$= 3.4\text{ m/s}^2$

Discussion

Question If an object slides down an inclined plane, does its acceleration depend on its mass? The coefficient of kinetic friction between the two surfaces? The angle of inclination?

Answer It depends on the angle and the coefficient of friction but not on the object's mass. L2

PRACTICE Problems

38. 4.90 m/s^2; 19.6 m/s

39. 3.8 m/s^2

40. 5.2 m/s^2

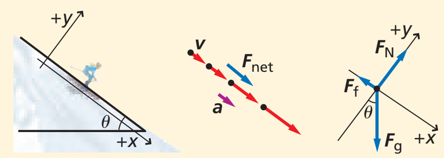
41. 0.75

EXAMPLE Problem 6

Skiing Downhill A 62-kg person on skis is going down a hill sloped at 37°. The coefficient of kinetic friction between the skis and the snow is 0.15. How fast is the skier going 5.0 s after starting from rest?

1 Analyze and Sketch the Problem

- Establish a coordinate system.
- Draw a free-body diagram showing the skier's velocity and direction of acceleration.
- Draw a motion diagram showing increasing v, and both a and F_{net} in the $+x$ direction, like the one shown in Figure 5-13.



Known:	Unknown:
$m = 62$ kg	$a = ?$
$\theta = 37°$	$v_f = ?$
$\mu_k = 0.15$	
$v_i = 0.0$ m/s	
$t = 5.0$ s	

2 Solve for the Unknown

y-direction:

$F_{net,\,y} = ma_y = 0.0\text{ N}$ — There is no acceleration in the y-direction, so $a_y = 0.0\text{ m/s}^2$.

Solve for F_N.

$F_N - F_{gy} = F_{net,\,y}$ — F_{gy} is negative. It is in the negative direction as defined by the coordinate system.

$F_N = F_{gy}$ — Substitute $F_{net,\,y} = 0.0$ N and rearrange

$= mg(\cos\theta)$ — Substitute $F_{gy} = mg\cos\theta$

Math Handbook
Isolating a Variable
page 845

x-direction:

Solve for a.

$F_{net,\,x} = F_{gx} - F_f$ — F_f is negative because it is in the negative direction as defined by the coordinate system.

$ma_x = mg(\sin\theta) - \mu_k F_N$ — Substitute $F_{net,\,x} = ma$, $F_{gx} = mg\sin\theta$, $F_f = \mu_k F_N$

$= mg(\sin\theta) - \mu_k mg(\cos\theta)$ — Substitute $a = a_x$ because all the acceleration is in the x-direction; substitute $F_N = mg\cos\theta$

$a = g(\sin\theta - \mu_k\cos\theta)$

$= (9.80\text{ m/s}^2)(\sin 37° - (0.15)\cos 37°)$ — Substitute $g = 9.80\text{ m/s}^2$, $\theta = 37°$, $\mu_k = 0.15$

$= 4.7\text{ m/s}^2$

Because v_i, a, and t are all known, use the following.

$v_f = v_i + at$

$= 0.0 + (4.7\text{ m/s}^2)(5.0\text{ s})$ — Substitute $v_i = 0.0$ m/s, $a = 4.7\text{ m/s}^2$, $t = 5.0$ s

$= 24$ m/s

3 Evaluate the Answer

- **Are the units correct?** Performing dimensional analysis on the units verifies that v_f is in m/s and a is in m/s^2.
- **Do the signs make sense?** Because v_f and a are both in the $+x$ direction, the signs do make sense.
- **Are the magnitudes realistic?** The velocity is fast, over 80 km/h (50 mph), but 37° is a steep incline, and the friction between the skis and the snow is not large.

Teacher F.Y.I. — REAL-LIFE PHYSICS

Equilibrium and Static Balance One of the most important applications of force vectors is made by architects and engineers in achieving equilibrium, known as static balance, in the structures they design. Whether the structure is a bridge, a building, or a highway, equilibrium is vital. Buildings with peaked roofs must be constructed so that the outward force where the roof joins the wall is counterbalanced. This can be done through an external buttress or an internal truss. In either instance, a horizontal inward force is applied to counterbalance the outward component of the vector of the roof.

PRACTICE Problems Additional Problems, Appendix B

38. Consider the crate on the incline in Example Problem 5. Calculate the magnitude of the acceleration. After 4.00 s, how fast will the crate be moving?

39. If the skier in Example Problem 6 were on a 31° downhill slope, what would be the magnitude of the acceleration?

40. Stacie, who has a mass of 45 kg, starts down a slide that is inclined at an angle of 45° with the horizontal. If the coefficient of kinetic friction between Stacie's shorts and the slide is 0.25, what is her acceleration?

41. After the skier on the 37° hill in Example Problem 6 had been moving for 5.0 s, the friction of the snow suddenly increased and made the net force on the skier zero. What is the new coefficient of friction?

The most important decision in problems involving motion along a slope is what coordinate system to use. Because an object's acceleration is usually parallel to the slope, one axis, usually the *x*-axis, should be in that direction. The *y*-axis is perpendicular to the *x*-axis and perpendicular to the surface of the slope. With this coordinate system, you now have two forces, the normal and frictional forces, in the directions of the coordinate axes; however, the weight is not. This means that when an object is placed on an inclined plane, the magnitude of the normal force between the object and the plane will usually not be equal to the object's weight.

You will need to apply Newton's laws once in the *x*-direction and once in the *y*-direction. Because the weight does not point in either of these directions, you will need to break this vector into its *x*- and *y*-components before you can sum your forces in these two directions. Example Problem 5 and Example Problem 6 both showed this procedure.

MINI LAB

What's Your Angle?

Prop a board up so that it forms an inclined plane at a 45° angle. Hang a 500-g object from the spring scale.

1. Measure and record the weight of the object. Set the object on the bottom of the board and slowly pull it up the inclined plane at a constant speed.

2. Observe and record the reading on the spring scale.

Analyze and Conclude

3. Calculate the component of weight for the 500-g object that is parallel to the inclined plane.

4. Compare the spring-scale reading along the inclined plane with the component of weight parallel to the inclined plane.

5.3 Section Review

42. Forces One way to get a car unstuck is to tie one end of a strong rope to the car and the other end to a tree, then push the rope at its midpoint at right angles to the rope. Draw a free-body diagram and explain why even a small force on the rope can exert a large force on the car.

43. Mass A large scoreboard is suspended from the ceiling of a sports arena by 10 strong cables. Six of the cables make an angle of 8.0° with the vertical while the other four make an angle of 10.0°. If the tension in each cable is 1300.0 N, what is the scoreboard's mass?

44. Acceleration A 63-kg water skier is pulled up a 14.0° incline by a rope parallel to the incline with a tension of 512 N. The coefficient of kinetic friction is 0.27. What are the magnitude and direction of the skier's acceleration?

45. Equilibrium You are hanging a painting using two lengths of wire. The wires will break if the force is too great. Should you hang the painting as shown in **Figures 5-15a** or **5-15b?** Explain.

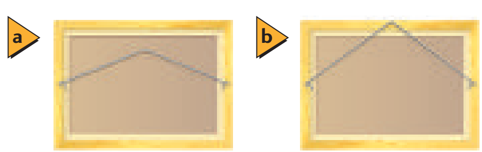


Figure 5-15

46. Critical Thinking Can the coefficient of friction ever have a value such that a skier would be able to slide uphill at a constant velocity? Explain why or why not. Assume there are no other forces acting on the skier.

MINI LAB

What's Your Angle?

See page 147 of **FAST FILE Chapters 1–5 Resources** *for the accompanying Mini Lab Worksheet.*

Purpose to explore vector components and vector addition

Materials 5 N spring scale, 500-g hooked mass, protractor, smooth board or cardboard, tape

Expected Results The scale will read 4.9 N when the object is hung from it. As long as the inclined plane is relatively friction free, as you pull the object up the ramp, the scale will read 3.5 N to 3.6 N.

Analyze and Conclude

3. $F_x = mg\sin\theta = (0.5\text{ kg})(9.80\text{ m/s}^2)(0.707) = 3.5\text{ N}$

4. Answers may vary, although the inclined plane reading should be nearly the same as the component from question 3.

3 ASSESS

Check for Understanding

Equilibrants Ask how two forces of 6.0 N and 8.0 N could be applied to an object to give a resultant force of 10.0 N. They could be applied at an angle of 90° to each other. Sketch the two forces. Then ask how a third force could be added to produce equilibrium. The third force should have a magnitude of 10.0 N and point in the direction opposite the 10.0 N resultant. Remind students that this third force is the equilibrant. **L2 Logical-Mathematical**

Reteach

Vector Resolution One of the most common vector resolutions is the one for the weight of an object on an inclined plane. Repeat this resolution step by step, stressing why each component goes where it does, and that neither component can be greater than the total weight of the object. Drawing a large triangle helps.

5.3 Section Review

42. See Solutions Manual. The vectors shown in the free-body diagram indicate that even a small force perpendicular to the rope can increase the tension in the rope enough to overcome the friction force. $T = \frac{F}{2\sin\theta}$, so for small values of θ, the tension, T, will increase greatly. (θ is between rope's original position and its displaced position.)

43. 1.31×10^3 kg

44. 3.2 m/s^2, the direction is up the ramp

45. $F_T = F_g\,(2\sin\theta)$, so F_T gets smaller as θ gets larger, and θ is larger in Figure 5–15b.

46. No, because both the frictional force opposing the motion of the skier and the component of Earth's gravity parallel to the slope point downhill, not uphill.

PHYSICS LAB

Time Allotment

one laboratory period

Process Skills measure in SI, use numbers, collect and organize data, estimate

Safety Precautions Make sure that the C-clamps are securely attached to the table. Students should be cautious when handling the wood blocks, as there may be splinters.

Alternative Materials A sheet of sandpaper taped to a moveable surface, such as cardboard, may be substituted for wood. Almost any object that has flat sides may be substituted for the blocks.

Teaching Strategies

- **Caution students to exert only as much force as necessary to pull the block at a constant speed. It is important that students do not accelerate the block once it starts moving.**

Sample Data

Object material = wood
Surface material = wood
Data Table 1

F_N(N)	Static Friction Force, F_s(N)			
	Trial 1	Trial 2	Trial 3	Average
2.1	1.5	1.7	1.4	1.5

Data Table 2

F_N(N)	Kinetic Friction Force, F_f(N)			
	Trial 1	Trial 2	Trial 3	Average
2.10	0.85	0.65	0.75	0.75

Data Table 3

F_N(N)	F_s(N)	F_f(N)	μ_s	μ_k
2.10	1.53	0.75	0.73	0.36

Data Table 4, Angle, θ, when sliding begins on an incline

θ	tan θ
21°	0.38

PHYSICS LAB

Alternate CBL instructions can be found on the Web site.
physicspp.com

The Coefficient of Friction

Static and kinetic friction are forces that are a result of two surfaces in contact with each other. Static friction is the force that must be overcome to cause an object to begin moving, while kinetic friction occurs between two objects in motion relative to each other. The kinetic friction force, $F_{f,\,kinetic}$, is defined by $F_{f,\,kinetic} = \mu_k F_N$, where μ_k is the coefficient of kinetic friction and F_N is the normal force acting on the object. The maximum static frictional force, $F_{f,\,max\,static}$, is defined by $F_{f,\,static} = \mu_s F_N$ where μ_s is the coefficient of static friction and F_N is the normal force on the object. The maximum static frictional force that must be overcome before movement is able to begin is $\mu_s F_N$. If you apply a constant force to pull an object along a horizontal surface at a constant speed, then the frictional force opposing the motion is equal and opposite to the applied force, F_p. Therefore, $F_p = F_f$. The normal force is equal and opposite to the object's weight when the object is on a horizontal surface and the applied force is horizontal.

QUESTION

How can the coefficient of static and kinetic friction be determined for an object on a horizontal surface?

Objectives

- **Measure** the normal and frictional forces acting on an object starting in motion and already in motion.
- **Use numbers** to calculate μ_s and μ_k.
- **Compare and contrast** values of μ_s and μ_k.
- **Analyze** the kinetic friction results.
- **Estimate** the angle where sliding will begin for an object on an inclined plane.

Safety Precautions

Materials

pulley
C-clamp
masking tape
wood surface
string (1 m)
spring scale, 0-5 N
wood block

Procedure

1. Check your spring scale to make sure that it reads zero when it is held vertically. If necessary, follow your teacher's instructions to zero it.
2. Attach the pulley to the edge of the table with a C-clamp.
3. Attach the string to the spring scale hook and the wood block.
4. Measure the weight of the block of wood, or other small object, and record the value as the normal force, F_N, in Data Tables 1, 2, and 3.
5. Unhook the string from the spring scale and run it through the pulley. Then reattach it to the spring scale.
6. Move the wood block as far away from the pulley as the string permits, while having it remain on the wood surface.
7. With the spring scale oriented vertically so that a right angle is formed between the wood block, the pulley, and the spring scale, slowly pull up on the spring scale. Observe the force that is necessary to cause the wood block to begin sliding. Record this value for the static frictional force in Data Table 1.

Horizons Companies

Material Table	
Object material	
Surface material	

Data Table 1

F_N (N)	Static Friction Force, F_s (N)			
	Trial 1	Trial 2	Trial 3	Average

Data Table 2

F_N (N)	Kinetic Friction Force, F_f (N)			
	Trial 1	Trial 2	Trial 3	Average

Data Table 3

F_N(N)	F_s (N)	F_f (N)	μ_s	μ_k

Data Table 4 (Angle, θ, when sliding begins on an incline)

θ*	tan θ*

8. Repeat steps 6 and 7 for two additional trials.
9. Repeat steps 6 and 7. However, once the block begins sliding, pull just hard enough to keep it moving at a constant speed across the other horizontal surface. Record this force as the kinetic frictional force in Data Table 2.
10. Repeat step 9 for two additional trials.
11. Place the block on the end of the surface. Slowly raise one end of the surface to make an incline. Gently tap the block to cause it to move and overcome static friction. If the block stops, replace it at the top of the incline and repeat the procedure. Continue increasing the angle, θ, between the horizontal and the inclined surface, and tapping the block until it slides at a constant speed down the incline. Record the angle, θ, in Data Table 4.

Analyze

1. Average the data for the static frictional force, $F_{s,\,max}$, from the three trials and record the result in the last column of Data Table 1 and in Data Table 3.
2. Average the data for the kinetic frictional force, F_f, from the three trials and record the result in the last column of Data Table 2 and in Data Table 3.
3. Use the data in Data Table 3 to calculate the coefficient of static friction, μ_s, and record the value in Data Table 3.
4. Use the data in Data Table 3 to calculate the coefficient of kinetic friction, μ_k, and record the value in Data Table 3.
5. Calculate tan θ for your value in Data Table 4.

Conclude and Apply

1. **Compare and Contrast** Examine your values for μ_s and μ_k. Explain whether your results are reasonable or not.
2. **Use Models** Draw a free-body diagram showing the forces acting on the block if it is placed on an incline of angle θ. Make certain that you include the force due to friction in your diagram.
3. From your diagram, assuming that the angle, θ, is where sliding begins, what does tan θ represent?
4. Compare your value for tan θ (experimental), μ_s, and μ_k.

Going Further

Repeat the experiment with additional surfaces that have different characteristics.

Real-World Physics

If you were downhill skiing and wished to determine the coefficient of kinetic friction between your skis and the slope, how could you do this? Be specific about how you could find a solution to this problem.

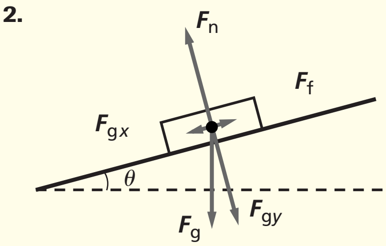
Physics Online

To find out more about friction, visit the Web site: **physicspp.com**

Analyze

1. answers in Tables 1 and 3
2. answers in Tables 2 and 3
3. Answer is in Table 3.
4. Answer is in Table 3.
5. Answer is in Table 4.

Conclude and Apply

1. Students should draw on their experiences moving objects and conclude that the force necessary to start an object moving usually is greater than the force needed to keep it moving, so the value for μ_s is larger than for μ_k. It is reasonable that $\mu_s > \mu_k$.
2. 

3. tan θ represents the coefficient of kinetic friction, because, tan $\theta = F_f/F_{gy} = F_f/F_N = \mu_k$
4. The value for tan θ and μ_k should be similar. The value for μ_s should be larger than tan θ and μ_k.

Going Further

Answers will vary depending on materials selected. Values for μ in the classroom, however, are most likely between 0 and 1.

Real-World Physics

Because tan $\theta = \mu_k$, as long as the skier descends at a constant speed, only the angle of the hill needs to be observed.

ALTERNATIVE INQUIRY LAB

To Make this Lab an Inquiry Lab: Have different lab groups choose from a variety of surfaces and materials to compare their results to the sample values for μ from Chapter 5. Students should determine how to measure the various forces by pulling the block across the surface. Students can use the collected data to determine the coefficient of friction.

Background

A discussion of psychology and physiology is appropriate here because they determine how we perceive the acceleration and motion of our own bodies. It is important to recognize that the topics in this feature and the points raised are exceedingly complex and not thoroughly understood. Research in the physiology of the senses and the perception of body attitude is ongoing, and there are many conflicting theories.

Teaching Strategies

- Involve students in a discussion of how we perceive motion, force, and acceleration when applied to our own bodies. This is not a simple task because it requires the objective examination of sensations that are inherent in us from birth.
- Using a picture or diagram of a roller coaster, ask students to describe the forces at work at various points along the ride.

Activity

Skeletal Feedback System Have students research current work on understanding the vestibular and skeletal feedback systems, and learn how they enable us to keep our balance. The skeletal feedback system provides our sense of proprioception (the unconscious perception of movement and spatial orientation arising from stimuli within the body itself). Ask students to share their findings in class.

Technology and Society

Roller Coasters

Why are roller coasters fun? A roller-coaster ride would be no fun at all if not for the forces acting on the coaster car and the rider. What forces do riders experience as they ride a roller coaster? The force of gravity acts on the rider and the coaster car in the downward direction. The seat of the car exerts a force on the rider in the opposite direction. When the coaster car makes a turn, the rider experiences a force in the opposite direction. Also, there are forces present due to the friction between the rider and the seat, the side of the car, and the safety bar.

The Force Factor

Designers of roller coasters take into account the magnitude of the forces exerted on the rider. They design the coaster in such a way that the forces thrill the rider without causing injury or too much discomfort.

Designers measure the amount of force exerted on the rider by calculating the force factor. The force factor is equal to the force exerted by the seat on the rider divided by the weight of the rider. Suppose the rider weighs about 68 kg. When the roller coaster is at the bottom of a hill, the rider may experience a force factor of 2. That means that at the bottom of the hill, the rider will feel as though he or she weighs twice as much, or in this case 136 kg. Conversely, at the top of a hill the force factor may be 0.5 and the rider will feel as though he or she weighs half his or her normal weight. Thus, designers create excitement by designing portions that change the rider's apparent weight.

The thrill of a roller-coaster ride is produced by the forces acting on the rider and the rider's reaction to visual cues.

The Thrill Factors Roller-coaster designers manipulate the way in which the body perceives the external world to create that "thrilling" sensation. For example, the roller coaster moves up the first hill very slowly, tricking the rider into thinking that the hill is higher than it is.

The organs of the inner ear sense the position of the head both when it is still and when it is moving. These organs help maintain balance by providing information to the brain. The brain then sends nerve impulses to the skeletal muscles to contract or relax to maintain balance. The constant change in position during a roller-coaster ride causes the organs of the inner ear to send conflicting messages to the brain. As a result, the skeletal muscles contract and relax throughout the ride.

You know that you are moving at high speeds because your eyes see the surroundings move past at high speed. So, designers make use of the surrounding landscape along with twists, turns, tunnels, and loops to give the rider plenty of visual cues. These visual cues, along with the messages from the inner ear, can result in disorientation and in some cases, nausea. To enthusiasts the disorientation is part of the thrill.

In order to attract visitors, amusement parks are constantly working on designing new rides that take the rider to new thrill levels. As roller-coaster technology improves, your most thrilling roller-coaster ride may be over the next hill.

Going Further

1. **Compare and Contrast** Compare and contrast your experience as a rider in the front of a roller coaster versus the back of it. Explain your answer in terms of the forces acting on you.
2. **Critical Thinking** While older roller coasters rely on chain systems to pull the coaster up the first hill, newer ones depend on hydraulic systems to do the same job. Research each of these two systems. What do you think are the advantages and disadvantages of using each system?

Going Further

1. Each rider will feel basically the same effects at the same points along the ride, those toward the back will experience forces that are cumulatively enhanced by the acceleration of the cars ahead of them. Therefore, at the top of a given hill, the last car accelerates faster than the first one, and the riders in the back will feel a more pronounced weightlessness.
2. The advantages of a hydraulic launch are: the mechanism takes up less space, force can be applied more precisely, and top speed can be achieved more quickly. However, hydraulic launch also requires a more elaborate braking system.

Chapter 5 Study Guide

5.1 Vectors

Vocabulary

- components (p. 122)
- vector resolution (p. 122)

Key Concepts

- When two vectors are at right angles, you can use the Pythagorean theorem to determine the magnitude of the resultant vector.

$$R^2 = A^2 + B^2$$

- The law of cosines and law of sines can be used to find the magnitude of the resultant of any two vectors.

$$R^2 = A^2 + B^2 - 2AB\cos\theta$$
$$\frac{R}{\sin\theta} = \frac{A}{\sin a} = \frac{B}{\sin b}$$

- The components of a vector are projections of the component vectors.

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{A_x}{A}; \text{ therefore, } A_x = A\cos\theta$$
$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{A_y}{A}; \text{ therefore, } A_y = A\sin\theta$$
$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

- Vectors can be summed by separately adding the x- and y-components.

5.2 Friction

Vocabulary

- kinetic friction (p. 126)
- static friction (p. 126)
- coefficient of kinetic friction (p. 127)
- coefficient of static friction (p. 127)

Key Concepts

- A frictional force acts when two surfaces touch.
- The frictional force is proportional to the force pushing the surfaces together.
- The kinetic friction force is equal to the coefficient of kinetic friction times the normal force.

$$F_{\text{f, kinetic}} = \mu_k F_N$$

- The static friction force is less than or equal to the coefficient of static friction times the normal force.

$$F_{\text{f, static}} \leq \mu_k F_N$$

5.3 Force and Motion in Two Dimensions

Vocabulary

- equilibrant (p. 131)

Key Concepts

- The force that must be exerted on an object to cause it to be in equilibrium is called the equilibrant.
- The equilibrant is found by finding the net force on an object, then applying a force with the same magnitude but opposite direction.
- An object on an inclined plane has a component of the force of gravity in a direction parallel to the plane; the component can accelerate the object down the plane.

Key Concepts

Summary statements can be used by students to review the major concepts of the chapter.

Physics Online

Visit physicspp.com
/self_check_quiz
/vocabulary_puzzlemaker
/chapter_test
/standardized_test

VOCABULARY PuzzleMaker

For additional help with vocabulary, have students access the Vocabulary PuzzleMaker online.

physicspp.com/vocabulary_puzzlemaker

Concept Mapping

47. See Solutions Manual.

Mastering Concepts

48. Make scale drawings of arrows representing the vector quantities. Place the arrows for the quantities to be added tip-to-tail. Draw an arrow from the tail of the first to the tip of the last. Measure the length of that arrow and find its direction.

49. Allowed: moving the vector without changing length or direction.

50. The resultant is the vector sum of two or more vectors. It represents the quantity that results from adding the vectors.

51. It is not affected

52. Reverse the direction of the second vector and then add them.

53. ***A*** is the symbol for the vector quantity. *A* is the signed magnitude (length) of the vector.

54. *a* and *b* represent the lengths of two vectors that are at right angles to one another. *c* represents the length of the sum of the two vectors.

55. The angle is measured counterclockwise from the *x*-axis.

56. The frictional force is greater than the normal force. You can pull the object along the surface, measuring the force needed to move it at constant speed. Also measure the weight of the object.

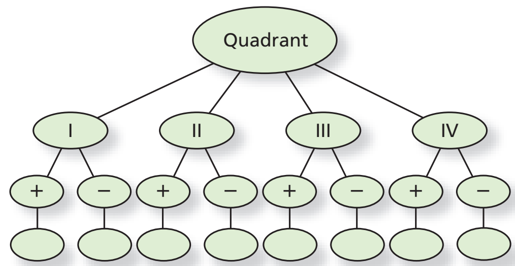
57. It would make no difference. Friction does not depend upon surface area.

58. One axis is vertical, with the positive direction either up or down.

Chapter 5 Assessment

Concept Mapping

47. Complete the concept map below with the terms *sine*, *cosine*, or *tangent* to indicate whether each function is positive or negative in each quadrant. Some circles could remain blank, and others can have more than one term.



Mastering Concepts

48. How would you add two vectors graphically? (5.1)

49. Which of the following actions is permissible when you graphically add one vector to another: moving the vector, rotating the vector, or changing the vector's length? (5.1)

50. In your own words, write a clear definition of the resultant of two or more vectors. Do not explain how to find it; explain what it represents. (5.1)

51. How is the resultant displacement affected when two displacement vectors are added in a different order? (5.1)

52. Explain the method that you would use to subtract two vectors graphically. (5.1)

53. Explain the difference between **A** and *A*. (5.1)

54. The Pythagorean theorem usually is written $c^2 = a^2 + b^2$. If this relationship is used in vector addition, what do *a*, *b*, and *c* represent? (5.1)

55. When using a coordinate system, how is the angle or direction of a vector determined with respect to the axes of the coordinate system? (5.1)

56. What is the meaning of a coefficient of friction that is greater than 1.0? How would you measure it? (5.2)

57. **Cars** Using the model of friction described in this textbook, would the friction between a tire and the road be increased by a wide rather than a narrow tire? Explain. (5.2)

58. Describe a coordinate system that would be suitable for dealing with a problem in which a ball is thrown up into the air. (5.3)

59. If a coordinate system is set up such that the positive *x*-axis points in a direction 30° above the horizontal, what should be the angle between the *x*-axis and the *y*-axis? What should be the direction of the positive *y*-axis? (5.3)

60. Explain how you would set up a coordinate system for motion on a hill. (5.3)

61. If your textbook is in equilibrium, what can you say about the forces acting on it? (5.3)

62. Can an object that is in equilibrium be moving? Explain. (5.3)

63. What is the sum of three vectors that, when placed tip to tail, form a triangle? If these vectors represent forces on an object, what does this imply about the object? (5.3)

64. You are asked to analyze the motion of a book placed on a sloping table. (5.3)
- **a.** Describe the best coordinate system for analyzing the motion.
- **b.** How are the components of the weight of the book related to the angle of the table?

65. For a book on a sloping table, describe what happens to the component of the weight force parallel to the table and the force of friction on the book as you increase the angle that the table makes with the horizontal. (5.3)
- **a.** Which components of force(s) increase when the angle increases?
- **b.** Which components of force(s) decrease?

Applying Concepts

66. A vector that is 1 cm long represents a displacement of 5 km. How many kilometers are represented by a 3-cm vector drawn to the same scale?

67. **Mowing the Lawn** If you are pushing a lawn mower across the grass, as shown in **Figure 5-16,** can you increase the horizontal component of the force that you exert on the mower without increasing the magnitude of the force? Explain.

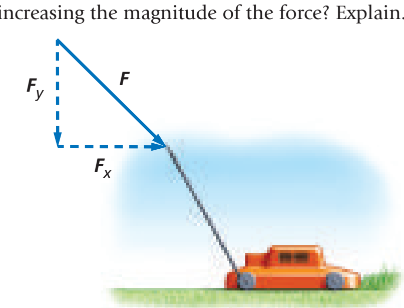


Figure 5-16

 For more problems, go to Additional Problems, Appendix B.

59. The two axes must be at right angles. The positive *y*-axis points 30 degrees away from the vertical so that it is at right angles to the *x*-axis.

60. For motion on a hill, the vertical (*y*) axis is usually set up perpendicular, or normal, to the surface of the hill.

61. The net force acting on the book is zero.

62. Yes, Newton's first law permits motion as long as the object's velocity is constant. It cannot accelerate.

68. A vector drawn 15 mm long represents a velocity of 30 m/s. How long should you draw a vector to represent a velocity of 20 m/s?

69. What is the largest possible displacement resulting from two displacements with magnitudes 3 m and 4 m? What is the smallest possible resultant? Draw sketches to demonstrate your answers.

70. How does the resultant displacement change as the angle between two vectors increases from 0° to 180°?

71. A and B are two sides of a right triangle, where $\tan\theta = A/B$.
  a. Which side of the triangle is longer if $\tan\theta$ is greater than 1.0?
  b. Which side is longer if $\tan\theta$ is less than 1.0?
  c. What does it mean if $\tan\theta$ is equal to 1.0?

72. **Traveling by Car** A car has a velocity of 50 km/h in a direction 60° north of east. A coordinate system with the positive x-axis pointing east and a positive y-axis pointing north is chosen. Which component of the velocity vector is larger, x or y?

73. Under what conditions can the Pythagorean theorem, rather than the law of cosines, be used to find the magnitude of a resultant vector?

74. A problem involves a car moving up a hill, so a coordinate system is chosen with the positive x-axis parallel to the surface of the hill. The problem also involves a stone that is dropped onto the car. Sketch the problem and show the components of the velocity vector of the stone.

75. **Pulling a Cart** According to legend, a horse learned Newton's laws. When the horse was told to pull a cart, it refused, saying that if it pulled the cart forward, according to Newton's third law, there would be an equal force backwards; thus, there would be balanced forces, and, according to Newton's second law, the cart would not accelerate. How would you reason with this horse?

76. **Tennis** When stretching a tennis net between two posts, it is relatively easy to pull one end of the net hard enough to remove most of the slack, but you need a winch to take the last bit of slack out of the net to make the top almost completely horizontal. Why is this true?

77. The weight of a book on an inclined plane can be resolved into two vector components, one along the plane, and the other perpendicular to it.
  a. At what angle are the components equal?
  b. At what angle is the parallel component equal to zero?
  c. At what angle is the parallel component equal to the weight?

78. **TV Towers** The transmitting tower of a TV station is held upright by guy wires that extend from the top of the tower to the ground. The force along the guy wires can be resolved into two perpendicular components. Which one is larger?

Mastering Problems

5.1 Vectors

79. **Cars** A car moves 65 km due east, then 45 km due west. What is its total displacement?

80. Find the horizontal and vertical components of the following vectors, as shown in **Figure 5-17.**
  a. E
  b. F
  c. A

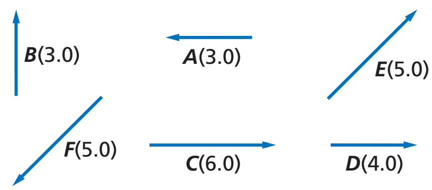


Figure 5-17

81. Graphically find the sum of the following pairs of vectors, whose lengths and directions are shown in Figure 5-17.
  a. D and A
  b. C and D
  c. C and A
  d. E and F

82. Graphically add the following sets of vectors, as shown in Figure 5-17.
  a. A, C, and D
  b. A, B, and E
  c. B, D, and F

83. You walk 30 m south and 30 m east. Find the magnitude and direction of the resultant displacement both graphically and algebraically.

84. **Hiking** A hiker's trip consists of three segments. Path A is 8.0 km long heading 60.0° north of east. Path B is 7.0 km long in a direction due east. Path C is 4.0 km long heading 315° counterclockwise from east.
  a. Graphically add the hiker's displacements in the order A, B, C.
  b. Graphically add the hiker's displacements in the order C, B, A.
  c. What can you conclude about the resulting displacements?

Chapter 5 Assessment

63. The vector sum of forces forming a closed triangle is zero. If these are the only forces acting on the object, the net force on the object is zero and the object is in equilibrium.

64. a. Set up the y-axis perpendicular, or normal, to the surface of the table and the x-axis pointing uphill and parallel to the surface.
  b. One component is parallel to the inclined surface and the other is perpendicular to it.

65. a. The component of the book's weight force along the table increases.
  b. The component of the weight force normal to the table decreases and the friction force decreases.

Applying Concepts

66. 15 km

67. Yes, lower the handle to make the angle between the handle and the horizontal smaller.

68. 10 mm

69. The largest is 7 m, the smallest 1 m. See Solutions Manual.

70. The resultant increases.

71. a. A
  b. B
  c. A and B are equal

72. The northward component (y) is longer.

73. The Pythagorean theorem can be used only if the two vectors to be added are at right angles to one another.

74. See Solutions Manual.

75. The equal and opposite forces referred to in Newton's third law are acting on different objects. The horse would pull on the cart, and the cart would pull on the horse. The cart would have an unbalanced net force on it (neglecting friction) and would thus accelerate.

76. When stretching the net between the two posts, there is no perpendicular component upward to balance the weight of the net. All the force exerted on the net is horizontal. Stretching the net to remove the last bit of slack requires great force in order to reduce the flexibility of the net and to increase the internal forces that hold it together.

77. **a.** 45°
b. 0°
c. 90°

78. The component perpendicular to the ground is larger if the angle between the guy wire and horizontal is greater than 45°.

Mastering Problems

5.1 Vectors

Level 1

79. 2.0×10^{1} km, east

80. **a.** $E_x = 3.5$, $E_y = 3.5$
b. $F_x = -3.5$, $F_y = -3.5$
c. $A_x = -3.0$, $A_y = 0.0$

81. **a.** See Solutions Manual.
b. See Solutions Manual.
c. See Solutions Manual.
d. See Solutions Manual.

Level 2

82. **a.** See Solutions Manual.
b. See Solutions Manual.
c. See Solutions Manual.

83. 4×10^{1} m, 45° east of south; See Solutions Manual.

84. **a.** See Solutions Manual.
b. See Solutions Manual.
c. You can add vectors in any order. The result is always the same.

85. 640.3 N at 51.34°

86. 79 N at 54°

Level 3

87. 509.9 km, 78.69° south of west

88. **a.** 6.5 m/s
b. 32° from vertical

89. 5 km, 53° south of east

5.2 Friction

Level 1

90. 0.255

91. 1.2 m/s^2

85. What is the net force acting on the ring in **Figure 5-18?**

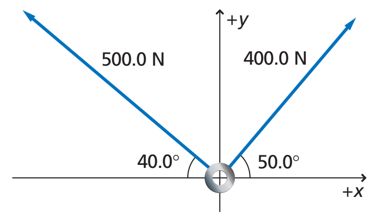


Figure 5-18

86. What is the net force acting on the ring in **Figure 5-19?**

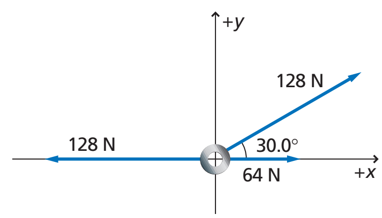


Figure 5-19

87. A Ship at Sea A ship at sea is due into a port 500.0 km due south in two days. However, a severe storm comes in and blows it 100.0 km due east from its original position. How far is the ship from its destination? In what direction must it travel to reach its destination?

88. Space Exploration A descent vehicle landing on Mars has a vertical velocity toward the surface of Mars of 5.5 m/s. At the same time, it has a horizontal velocity of 3.5 m/s.
a. At what speed does the vehicle move along its descent path?
b. At what angle with the vertical is this path?

89. Navigation Alfredo leaves camp and, using a compass, walks 4 km E, then 6 km S, 3 km E, 5 km N, 10 km W, 8 km N, and, finally, 3 km S. At the end of three days, he is lost. By drawing a diagram, compute how far Alfredo is from camp and which direction he should take to get back to camp.

5.2 Friction

90. If you use a horizontal force of 30.0 N to slide a 12.0-kg wooden crate across a floor at a constant velocity, what is the coefficient of kinetic friction between the crate and the floor?

91. A 225-kg crate is pushed horizontally with a force of 710 N. If the coefficient of friction is 0.20, calculate the acceleration of the crate.

92. A force of 40.0 N accelerates a 5.0-kg block at 6.0 m/s^2 along a horizontal surface.
a. How large is the frictional force?
b. What is the coefficient of friction?

93. Moving Appliances Your family just had a new refrigerator delivered. The delivery man has left and you realize that the refrigerator is not quite in the right position, so you plan to move it several centimeters. If the refrigerator has a mass of 180 kg, the coefficient of kinetic friction between the bottom of the refrigerator and the floor is 0.13, and the static coefficient of friction between these same surfaces is 0.21, how hard do you have to push horizontally to get the refrigerator to start moving?

94. Stopping at a Red Light You are driving a 2500.0-kg car at a constant speed of 14.0 m/s along a wet, but straight, level road. As you approach an intersection, the traffic light turns red. You slam on the brakes. The car's wheels lock, the tires begin skidding, and the car slides to a halt in a distance of 25.0 m. What is the coefficient of kinetic friction between your tires and the wet road?

5.3 Force and Motion in Two Dimensions

95. An object in equilibrium has three forces exerted on it. A 33.0-N force acts at 90.0° from the *x*-axis and a 44.0-N force acts at 60.0° from the *x*-axis. What are the magnitude and direction of the third force?

96. Five forces act on an object: (1) 60.0 N at 90.0°, (2) 40.0 N at 0.0°, (3) 80.0 N at 270.0°, (4) 40.0 N at 180.0°, and (5) 50.0 N at 60.0°. What are the magnitude and direction of a sixth force that would produce equilibrium?

97. Advertising Joe wishes to hang a sign weighing 7.50×10^{2} N so that cable *A*, attached to the store, makes a 30.0° angle, as shown in **Figure 5-20.** Cable *B* is horizontal and attached to an adjoining building. What is the tension in cable *B*?

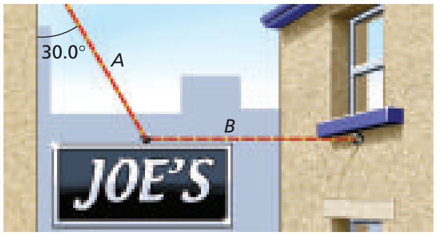


Figure 5-20

 For more problems, go to Additional Problems, Appendix B.

Level 2

92. **a.** 1.0×10^{1} N
b. 0.20

93. 370 N

Level 3

94. 0.400

5.3 Force and Motion in Two Dimensions

Level 1

95. 74.4 N, 253°

Level 2

96. 34.2 N, 223°

97. 435 N, right

98. A street lamp weighs 150 N. It is supported by two wires that form an angle of 120.0° with each other. The tensions in the wires are equal.

a. What is the tension in each wire supporting the street lamp?

b. If the angle between the wires supporting the street lamp is reduced to 90.0°, what is the tension in each wire?

99. A 215-N box is placed on an inclined plane that makes a 35.0° angle with the horizontal. Find the component of the weight force parallel to the plane's surface.

100. Emergency Room You are shadowing a nurse in the emergency room of a local hospital. An orderly wheels in a patient who has been in a very serious accident and has had severe bleeding. The nurse quickly explains to you that in a case like this, the patient's bed will be tilted with the head downward to make sure the brain gets enough blood. She tells you that, for most patients, the largest angle that the bed can be tilted without the patient beginning to slide off is 32.0° from the horizontal.

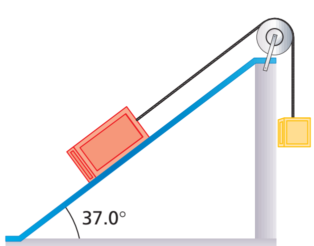
a. On what factor or factors does this angle of tilting depend?

b. Find the coefficient of static friction between a typical patient and the bed's sheets.

101. Two blocks are connected by a string over a frictionless, massless pulley such that one is resting on an inclined plane and the other is hanging over the top edge of the plane, as shown in **Figure 5-21.** The hanging block has a mass of 16.0 kg, and the one on the plane has a mass of 8.0 kg. The coefficient of kinetic friction between the block and the inclined plane is 0.23. The blocks are released from rest.

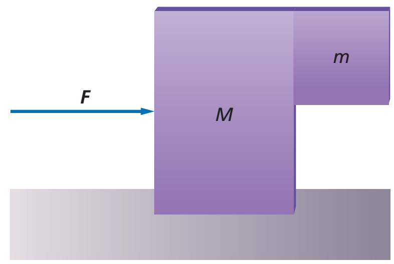
a. What is the acceleration of the blocks?

b. What is the tension in the string connecting the blocks?



■ **Figure 5-21**

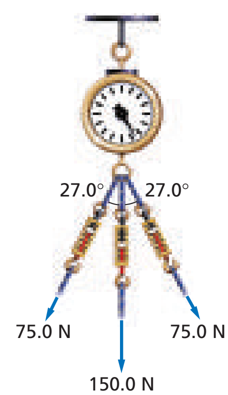
102. In **Figure 5-22,** a block of mass M is pushed with a force, F, such that the smaller block of mass m does not slide down the front of it. There is no friction between the larger block and the surface below it, but the coefficient of static friction between the two blocks is μ_s. Find an expression for F in terms of M, m, μ_s, and g.



■ **Figure 5-22**

Mixed Review

103. The scale in **Figure 5-23** is being pulled on by three ropes. What net force does the scale read?



■ **Figure 5-23**

104. Sledding A sled with a mass of 50.0 kg is pulled along flat, snow-covered ground. The static friction coefficient is 0.30, and the kinetic friction coefficient is 0.10.

a. What does the sled weigh?

b. What force will be needed to start the sled moving?

c. What force is needed to keep the sled moving at a constant velocity?

d. Once moving, what total force must be applied to the sled to accelerate it at 3.0 m/s^2?

Chapter 5 Assessment

98. a. 1.5×10^2 N

b. 1.1×10^2 N

99. 123 N

Level 3

100. a. The coefficient of static friction between the patient and the bed's sheets.

b. 0.625

101. a. 4.0 m/s^2

b. 93 N

102. $F=\left(\frac{g}{\mu_s}\right)(m+M)$

Mixed Review

Level 1

103. 283.6 N

104. a. 4.90×10^2 N

b. 1.5×10^2 N

c. 49 N, kinetic friction

d. 2.0×10^2 N

Level 2

105. **a.** 166 N
b. 3.6 km

Level 3

106. 24 m

Thinking Critically

107. See Solutions Manual; 49 m

108. **a.** Kako is correct.
b. They will reach the bottom at the same time.

Writing in Physics

109. Answers will vary. Answers may include lubricants and the reduction of the normal force to reduce the force of friction.

110. Answers will vary

Cumulative Review

111. **a.** 90.0 g
b. 1.68 km
c. 128.6 kg
d. 47.9 s

112. 10 km/h

113. 13 m/s^2

105. Mythology Sisyphus was a character in Greek mythology who was doomed in Hades to push a boulder to the top of a steep mountain. When he reached the top, the boulder would slide back down the mountain and he would have to start all over again. Assume that Sisyphus slides the boulder up the mountain without being able to roll it, even though in most versions of the myth, he rolled it.

a. If the coefficient of kinetic friction between the boulder and the mountainside is 0.40, the mass of the boulder is 20.0 kg, and the slope of the mountain is a constant 30.0°, what is the force that Sisyphus must exert on the boulder to move it up the mountain at a constant velocity?

b. If Sisyphus pushes the boulder at a velocity of 0.25 m/s and it takes him 8.0 h to reach the top of the mountain, what is the mythical mountain's vertical height?

106. Landscaping A tree is being transported on a flatbed trailer by a landscaper, as shown in **Figure 5-24.** If the base of the tree slides on the trailer, the tree will fall over and be damaged. If the coefficient of static friction between the tree and the trailer is 0.50, what is the minimum stopping distance of the truck, traveling at 55 km/h, if it is to accelerate uniformly and not have the tree slide forward and fall on the trailer?

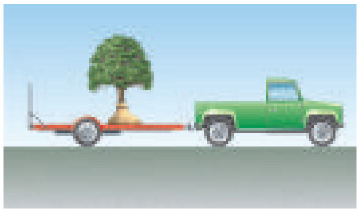


Figure 5-24

Thinking Critically

107. Use Models Using the Example Problems in this chapter as models, write an example problem to solve the following problem. Include the following sections: Analyze and Sketch the Problem, Solve for the Unknown (with a complete strategy), and Evaluate the Answer. A driver of a 975-kg car traveling 25 m/s puts on the brakes. What is the shortest distance it will take for the car to stop? Assume that the road is concrete, the force of friction of the road on the tires is constant, and the tires do not slip.

108. Analyze and Conclude Margaret Mary, Doug, and Kako are at a local amusement park and see an attraction called the Giant Slide, which is simply a very long and high inclined plane. Visitors at the amusement park climb a long flight of steps to the top of the 27° inclined plane and are given canvas sacks. They sit on the sacks and slide down the 70-m-long plane. At the time when the three friends walk past the slide, a 135-kg man and a 20-kg boy are each at the top preparing to slide down. "I wonder how much less time it will take the man to slide down than it will take the boy," says Margaret Mary. "I think the boy will take less time," says Doug. "You're both wrong," says Kako. "They will reach the bottom at the same time."

a. Perform the appropriate analysis to determine who is correct.

b. If the man and the boy do not take the same amount of time to reach the bottom of the slide, calculate how many seconds of difference there will be between the two times.

Writing in Physics

109. Investigate some of the techniques used in industry to reduce the friction between various parts of machines. Describe two or three of these techniques and explain the physics of how they work.

110. Olympics In recent years, many Olympic athletes, such as sprinters, swimmers, skiers, and speed skaters, have used modified equipment to reduce the effects of friction and air or water drag. Research a piece of equipment used by one of these types of athletes and the way it has changed over the years. Explain how physics has impacted these changes.

Cumulative Review

111. Add or subtract as indicated and state the answer with the correct number of significant digits. (Chapter 1)

a. 85.26 g + 4.7 g
b. 1.07 km + 0.608 km
c. 186.4 kg − 57.83 kg
d. 60.08 s − 12.2 s

112. You ride your bike for 1.5 h at an average velocity of 10 km/h, then for 30 min at 15 km/h. What is your average velocity? (Chapter 3)

113. A 45-N force is exerted in the upward direction on a 2.0-kg briefcase. What is the acceleration of the briefcase? (Chapter 4)

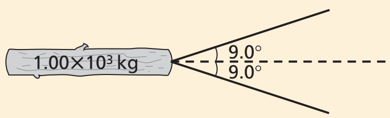
 For more problems, go to Additional Problems, Appendix B.

Standardized Test Practice

Multiple Choice

1. Two tractors pull against a 1.00×10^3-kg log. If the angle of the tractors' chains in relation to each other is 18.0°, and each tractor pulls with a force of 8×10^2 N, what forces will they be able to exert?

   (A) 250 N
   (B) 1.52×10^3 N
   (C) 1.58×10^3 N
   (D) 1.60×10^3 N



2. An airplane pilot tries to fly directly east with a velocity of 800.0 km/h. If a wind comes from the southwest at 80.0 km/h, what is the relative velocity of the airplane to the surface of Earth?

   (A) 804 km/h, 5.7° N of E
   (B) 858 km/h, 3.8° N of E
   (C) 859 km/h, 4.0° N of E
   (D) 880 km/h 45° N of E

3. For a winter fair, some students decide to build 30.0-kg wooden pull-carts on sled skids. If two 90.0-kg passengers get in, how much force will the puller have to exert to move a pull-cart? The coefficient of maximum static friction between the cart and the snow is 0.15.

   (A) 1.8×10^2 N
   (B) 3.1×10^2 N
   (C) 2.1×10^3 N
   (D) 1.4×10^4 N

4. It takes a minimum force of 280 N to move a 50.0-kg crate. What is the coefficient of maximum static friction between the crate and the floor?

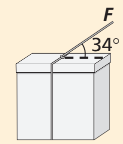
   (A) 0.18
   (B) 0.57
   (C) 1.8
   (D) 5.6

5. What is the *y*-component of a 95.3-N force that is exerted at 57.1° to the horizontal?

   (A) 51.8 N
   (B) 80.0 N
   (C) 114 N
   (D) 175 N

6. A string exerts a force of 18 N on a box at an angle of 34° from the horizontal. What is the horizontal component of the force on the box?

   (A) 10 N
   (B) 15 N
   (C) 21.7 N
   (D) 32 N



7. Sukey is riding her bicycle on a path when she comes around a corner and sees that a fallen tree is blocking the way 42 m ahead. If the coefficient of friction between her bicycle's tires and the gravel path is 0.36, and she is traveling at 50.0 km/h, how much stopping distance will she require? Sukey and her bicycle, together, have a mass of 95 kg.

   (A) 3.00 m
   (B) 4.00 m
   (C) 8.12 m
   (D) 27.3 m

Extended Answer

8. A man starts from a position 310 m north of his car and walks for 2.7 min in a westward direction at a constant velocity of 10 km/h. How far is he from his car when he stops?

9. Jeeves is tired of his 41.2-kg son sliding down the banister, so he decides to apply an extremely sticky paste that increases the coefficient of static friction to 0.72 to the top of the banister. What will be the magnitude of the static friction force on the boy if the banister is at an angle of 52.4° from the horizontal?

✓ Test-Taking TIP

Calculators Are Only Machines

If your test allows you to use a calculator, use it wisely. Figure out which numbers are relevant, and determine the best way to solve the problem before you start punching keys.

Standardized Test Practice

Rubric

The following rubric is a sample scoring device for extended response questions.

Extended Response

Points	Description
4	The student demonstrates a thorough understanding of the physics involved. The response may contain minor flaws that do not detract from the demonstration of a thorough understanding.
3	The student demonstrates an understanding of the physics involved. The response is essentially correct and demonstrates an essential but less than thorough understanding of the physics.
2	The student demonstrates only a partial understanding of the physics involved. Although the student may have used the correct approach to a solution or may have provided a correct solution, the work lacks an essential understanding of the underlying physical concepts.
1	The student demonstrates a very limited understanding of the physics involved. The response is incomplete and exhibits many flaws.
0	The student provides a completely incorrect solution or no response at all.

Multiple Choice

1. C **2.** C **3.** B
4. B **5.** B **6.** B
7. D

Extended Answer

8. 5.5×10^2 m
9. 1.8×10^2 N