



## The Big Idea

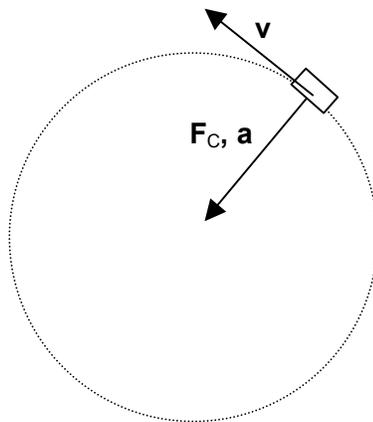
In the absence of a net force, objects move in a straight line. If they turn — that is, if their velocity changes, *even only in direction* — there must be an applied force. Forces which cause objects to turn around continuously in a circle are known as *centripetal* forces. When an object moves in a circle its velocity at any particular instant points in a direction *tangent* to the circle. The acceleration points towards the center of the circle, and so does the force acting on it. This is only natural, when you think about it — if you feel a force pushing you towards your left as you walk forward, you will walk in a circle, always turning left.

## Key Equations

$F_C = mv^2 / r$	; if an object of mass $m$ moves in a circle of radius $r$ at speed $v$ , the force acting on it must be equal to this value. The direction of this force is always pointed towards the center of the circle.
$F_G = Gm_1m_2 / r^2$	; every pair of objects in the world has a mutual attraction called gravity. This formula allows you to predict this force. Gravity is the reason the Earth circles the sun.

## Key Concepts

- An orbital period,  $T$ , is the time it takes to make one complete rotation.
- If a particle travels a distance  $2\pi r$  in an amount of time  $T$ , then its speed is distance over time or  $2\pi r/T$ .
- An object moving in a circle has an instantaneous velocity vector *tangential* to the circle of its path. The force and acceleration vectors point to the center of the circle.
- Net force and acceleration *always* have the same direction.
- Centripetal acceleration is just the acceleration provided by centripetal forces.
- Geosynchronous orbit is when a satellite completes one orbit of the Earth every 24 hours, staying above the same spot on Earth.

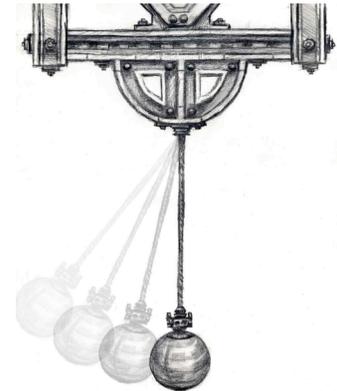


## Key Applications

- To find the maximum speed that a car can take a corner on a flat road without skidding out, set the force of friction equal to the centripetal force.
- To find the tension in the rope of a swinging pendulum, remember that it is the *sum* of the tension and gravity that produces a net upward centripetal force. A common mistake is just setting the centripetal force equal to the tension.
- To find the speed of a planet or satellite in an orbit, set the force of gravity equal to the centripetal force.

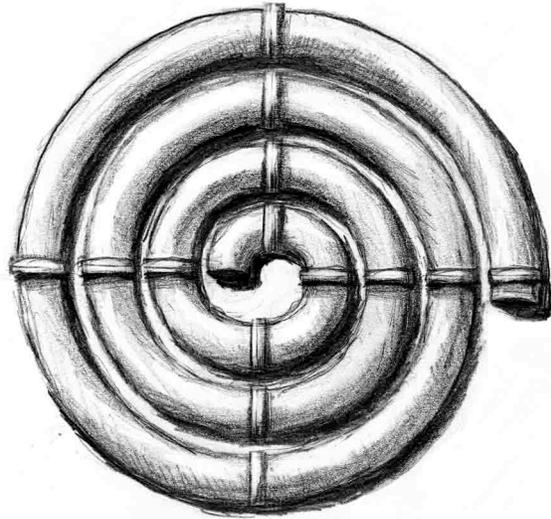
## Centripetal Forces Problem Set

- When you make a right turn at constant speed in your car what is the force that causes *you* (not the car) to change the direction of *your* velocity? Choose the best possible answer.
  - Friction between your butt and the seat
  - Inertia
  - Air resistance
  - Tension
  - All of the above
  - None of the above
- You buy new tires for your car in order to take turns a little faster (uh, not advised — always drive slowly). The new tires double your coefficient of friction with the road. With the old tires you could take a particular turn at a speed  $v_0$ . What is the maximum speed you can now take the turn without skidding out?
  - $4v_0$
  - $2v_0$
  - $v_0$
  - $\sqrt{2}v_0$
  - Not enough information given
- A pendulum consisting of a rope with a ball attached at the end is swinging back and forth. As it swings downward to the right the ball is released at its lowest point. Decide which way the ball attached at the end of the string will go at the moment it is released.
  - Straight upwards
  - Straight downwards
  - Directly right
  - Directly left
  - It will stop



4. A ball is spiraling outward in the tube shown to the right. Which way will the ball go after it leaves the tube?

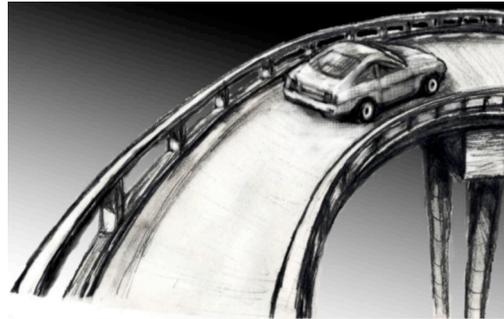
- Towards the top of the page
- Towards the bottom of the page
- Continue spiraling outward in the clockwise direction
- Continue in a circle with the radius equal to that of the spiral as it leaves the tube
- None of the above



5. An object of mass 10 kg is in a circular orbit of radius 10 m at a velocity of 10 m/s.
- Calculate the centripetal force (in N) required to maintain this orbit.
  - What is the acceleration of this object?
6. Suppose you are spinning a child around in a circle by her arms . The radius of her orbit around you is 1 meter. Her speed is 1 m/s. Her mass is 25 kg.
- What is the tension in your arms?
  - In her arms?
7. A racecar is traveling at a speed of 80.0 m/s on a circular racetrack of radius 450 m.
- What is its centripetal acceleration in  $\text{m/s}^2$ ?
  - What is the centripetal force on the racecar if its mass is 500 kg?
  - What provides the necessary centripetal force in this case?
8. The radius of the Earth is 6380 km. Calculate the velocity of a person standing at the equator due to the Earth's 24 hour rotation. Calculate the centripetal acceleration of this person and express it as a fraction of the acceleration  $g$  due to gravity. Is there any danger of "flying off"?
9. Neutron stars are the corpses of stars left over after supernova explosions. They are the size of a small city, but can spin several times per second. (Try to imagine this in your head.) Consider a neutron star of radius 10 km that spins with a period of 0.8 seconds. Imagine a person is standing at the equator of this neutron star.
- Calculate the centripetal acceleration of this person and express it as a multiple of the acceleration  $g$  due to gravity (on Earth).
  - Now, find the minimum acceleration due to gravity that the neutron star must have in order to keep the person from flying off.
10. Calculate the force of gravity between the Sun and the Earth. (The relevant data are included in Appendix B.)

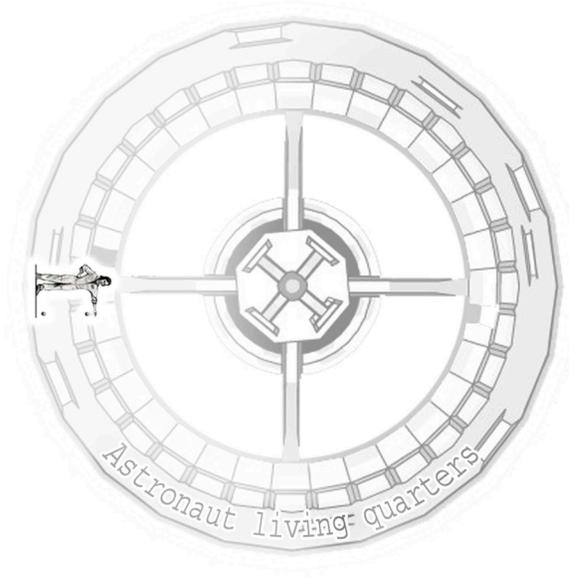
11. Calculate the force of gravity between two human beings, assuming that each has a mass of 80 kg and that they are standing 1 m apart. Is this a large force?
12. Prove  $g$  is *approximately*  $10 \text{ m/s}^2$  on Earth by following these steps:
- Calculate the force of gravity between a falling object (for example an apple) and that of Earth. Use the symbol  $m_o$  to represent the mass of the falling object.
  - Now divide that force by the object's mass to find the acceleration  $g$  of the object.
13. Our Milky Way galaxy is orbited by a few hundred “globular” clusters of stars, some of the most ancient objects in the universe. Globular cluster M13 is orbiting at a distance of 26,000 light-years (one light-year is  $9.46 \times 10^{15} \text{ m}$ ) and has an orbital period of 220 million years. The mass of the cluster is  $10^6$  times the mass of the Sun.
- What is the amount of centripetal force required to keep this cluster in orbit?
  - What is the source of this force?
  - Based on this information, what is the mass of our galaxy? If you assume that the galaxy contains nothing but Solar-mass stars (each with an approximate mass of  $2 \times 10^{30} \text{ kg}$ ), how many stars are in our galaxy?
14. Calculate the centripetal acceleration of the Earth around the Sun.

15. You are speeding around a turn of radius 30.0 m at a constant speed of 15.0 m/s.



- What is the minimum coefficient of friction  $\mu$  between your car tires and the road necessary for you to retain control?
  - Even if the road is terribly icy, you will still move in a circle because you are slamming into the walls. What centripetal forces must the walls exert on you if you do not lose speed? Assume  $m = 650 \text{ kg}$ .
16. Calculate the gravitational force that your pencil or pen pulls on you. Use the center of your chest as the center of mass (and thus the mark for the distance measurement) and estimate all masses and distances.
- If there were no other forces present, what would your acceleration be towards your pencil? Is this a large or small acceleration?
  - Why, in fact, doesn't your pencil accelerate towards you?
17. A digital TV satellite is placed in geosynchronous orbit around Earth, so it is always in the same spot in the sky.
- Using the fact that the satellite will have the same period of revolution as Earth, calculate the radius of its orbit.
  - What is the ratio of the radius of this orbit to the radius of the Earth?
  - Draw a sketch, to scale, of the Earth and the orbit of this digital TV satellite.
  - If the mass of the satellite were to double, would the radius of the satellite's orbit be larger, smaller, or the same? Why?

18. A top secret spy satellite is designed to orbit the Earth twice each day (*i.e.*, twice as fast as the Earth's rotation). What is the height of this orbit above the Earth's surface?
19. Two stars with masses  $3.00 \times 10^{31}$  kg and  $7.00 \times 10^{30}$  kg are orbiting each other under the influence of each other's gravity. We want to send a satellite in between them to study their behavior. However, the satellite needs to be at a point where the gravitational forces from the two stars are equal. The distance between the two stars is  $2.0 \times 10^{10}$  m. Find the distance from the more massive star to where the satellite should be placed. (*Hint*: distance from the satellite to one of the stars is the variable.)
20. Calculate the mass of the Earth using *only*: (i) Newton's Universal Law of Gravity; (ii) the Moon-Earth distance (Appendix B); and (iii) the fact that it takes the Moon 27 days to orbit the Earth.
21. A student comes up to you and says, "I can visualize the force of tension, the force of friction, and the other forces, but I can't visualize *centripetal* force." As you know, a centripetal force must be *provided* by tension, friction, or some other "familiar" force. Write a two or three sentence explanation, in your own words, to help the confused student.
22. A space station was established far from the gravitational field of Earth. Extended stays in zero gravity are not healthy for human beings. Thus, for the comfort of the astronauts, the station is rotated so that the astronauts *feel* there is an internal gravity. The rotation speed is such that the *apparent* acceleration of gravity is  $9.8 \text{ m/s}^2$ . The direction of rotation is counter-clockwise.



- If the radius of the station is 80 m, what is its rotational speed,  $v$ ?
- Draw vectors representing the astronaut's velocity and acceleration.
- Draw a free body diagram for the astronaut.
- Is the astronaut exerting a force on the space station? If so, calculate its magnitude. Her mass  $m = 65$  kg.
- The astronaut drops a ball, which *appears* to accelerate to the 'floor', (see picture) at  $9.8 \text{ m/s}^2$ .
  - Draw the velocity and acceleration vectors for the ball while it is in the air.
  - What force(s) are acting on the ball while it is in the air?
  - Draw the acceleration and velocity vectors after the ball hits the floor and comes to rest.
  - What force(s) act on the ball after it hits the ground?

