CHAPTER 15: THERMODYNAMICS

15.1 THE FIRST LAW OF THERMODYNAMICS

1. What is the change in internal energy of a car if you put 12.0 gal of gasoline into its tank? The energy content of gasoline is \( 1.3 \times 10^8 \) J/gal. All other factors, such as the car’s temperature, are constant.

Solution Using the energy content of a gallon of gasoline \( 1.3 \times 10^8 \) J/gal, the energy stored in 12.0 gallons of gasoline is: \( E_{\text{gas}} = (1.3 \times 10^8 \) J/gal)(12.0 gal) = \( 1.6 \times 10^9 \) J. Therefore, the internal energy of the car increases by this energy, so that \( \Delta U = 1.6 \times 10^9 \) J.

7. (a) What is the average metabolic rate in watts of a man who metabolizes 10,500 kJ of food energy in one day? (b) What is the maximum amount of work in joules he can do without breaking down fat, assuming a maximum efficiency of 20.0%? (c) Compare his work output with the daily output of a 187-W (0.250-horsepower) motor.

Solution (a) The metabolic rate is the power, so that:

\[
P = \frac{Q}{t} = \frac{10500 \text{ kJ}}{1 \text{ day}} \times \frac{1000 \text{ J}}{1 \text{ kJ}} \times \frac{1 \text{ day}}{8.64 \times 10^4 \text{ s}} = 122 \text{ W}
\]

(b) Efficiency is defined to be the ratio of what we get to what we spend, or 

\[
Eff' = \frac{W}{E_{\text{in}}},
\]

so we can determine the work done, knowing the efficiency:

\[
W = Eff' \cdot E_{\text{in}} = 0.200(10500 \text{ kJ}) \times \frac{1000 \text{ J}}{1 \text{ kJ}} = 2.10 \times 10^6 \text{ J}
\]

(c) To compare with a 0.250 hp motor, we need to know how much work the motor...
does in one day: \( W = Pt = (0.250 \text{ hp})(1 \text{ day}) \times \frac{746 \text{ W}}{1 \text{ hp}} \times \frac{8.64 \times 10^4 \text{ s}}{1 \text{ day}} = 1.61 \times 10^7 \text{ J} \).

So, the man’s work output is: \( \frac{W_{\text{motor}}}{W_{\text{man}}} = \frac{1.61 \times 10^7 \text{ J}}{2.10 \times 10^6 \text{ J}} = 7.67 \Rightarrow 7.67 \text{ times less than the motor} \). Thus the motor produces 7.67 times the work done by the man.

15.2 THE FIRST LAW OF THERMODYNAMICS AND SOME SIMPLE PROCESSES

11. A helium-filled toy balloon has a gauge pressure of 0.200 atm and a volume of 10.0 L. How much greater is the internal energy of the helium in the balloon than it would be at zero gauge pressure?

Solution First, we must assume that the volume remains constant, so that \( V_1 = V_2 \), where state 1 is that at \( P_1 = 0.200 \text{ atm } + P_a = 0.200 \text{ atm } + 1.00 \text{ atm } = 1.20 \text{ atm } \), and state 2 is that at \( P_2 = 1.00 \text{ atm } \). Now, we can calculate the internal energy of the system in state 2 using the equation \( U = \frac{3}{2} N k T \), since helium is a monatomic gas:

\[
U_2 = \frac{3}{2} N_2 k T = \frac{3}{2} \left( \frac{P_2 V}{k T} \right) k T = \frac{3}{2} P_2 V
\]

\[
= \frac{3}{2} \left( 1 \text{ atm } \times \frac{1.013 \times 10^5 \text{ N/m}^2}{1 \text{ atm}} \right) \left( 10.0 \text{ L } \times \frac{10^{-3} \text{ m}^3}{1 \text{ L}} \right) = 1.52 \times 10^3 \text{ J}
\]

Next, we can use the ideal gas law, in combination with the equation \( U = \frac{3}{2} N k T \) to get an expression for \( U_1 \):

\[
\frac{U_1}{U_2} = \frac{\frac{3}{2} N_1 k T}{\frac{3}{2} N_2 k T} = \frac{N_1}{N_2} = \frac{P_1 V / k T}{P_2 V / k T} = \frac{P_1}{P_2}, \text{ so that}
\]

\[
U_1 = \left( \frac{P_1}{P_2} \right) U_2 = \left( \frac{1.20 \text{ atm}}{1.00 \text{ atm}} \right) \left( 1.52 \times 10^3 \text{ J} \right) = 1.82 \times 10^3 \text{ J}
\]
and so the internal energy inside the balloon is:

\[ U_1 - U_2 = 1.82 \times 10^3 \text{ J} - 1.52 \times 10^3 \text{ J} = 300 \text{ J}, \text{ greater than it would be at zero gauge pressure.} \]

\[ \text{15.3 INTRODUCTION TO THE SECOND LAW OF THERMODYNAMICS: HEAT ENGINES AND THEIR EFFICIENCY} \]

21. \( \text{With } 2.56 \times 10^6 \text{ J of heat transfer into this engine, a given cyclical heat engine can do only } 1.50 \times 10^5 \text{ J of work. (a) What is the engine’s efficiency? (b) How much heat transfer to the environment takes place?} \]

Solution  (a) The efficiency is the work out divided by the heat in:

\[
Eff = \frac{W}{Q_h} = \frac{1.50 \times 10^5 \text{ J}}{2.56 \times 10^6 \text{ J}} = 0.0586, \text{ or } 5.86\%
\]

(b) The work output is the difference between the heat input and the wasted heat, so from the first law of thermodynamics:

\[
W = Q_h - Q_c \Rightarrow Q_c = Q_h - W = 2.56 \times 10^6 \text{ J} - 1.50 \times 10^5 \text{ J} = 2.41 \times 10^6 \text{ J}
\]

\[ \text{15.5 APPLICATIONS OF THERMODYNAMICS: HEAT PUMPS AND REFRIGERATORS} \]

42. \( \text{(a) What is the best coefficient of performance for a refrigerator that cools an environment at } -30.0^\circ \text{C and has heat transfer to another environment at } 45.0^\circ \text{C? (b) How much work in joules must be done for a heat transfer of 4186 kJ from the cold environment? (c) What is the cost of doing this if the work costs 10.0 cents per } 3.60 \times 10^6 \text{ J (a kilowatt-hour)? (d) How many kJ of heat transfer occurs into the warm environment? (e) Discuss what type of refrigerator might operate between these temperatures.} \]
Solution

(a) Using the equation \( \text{COP}_{\text{ref}} = \frac{Q_c}{W} = \text{COP}_{\text{hp}} - 1 \), and for the best coefficient of performance, that means make the Carnot substitution, remembering to use the absolute temperatures:

\[
\text{COP}_{\text{ref}} = \frac{Q_c}{W} = \text{COP}_{\text{hp}} - 1 = \frac{1}{\text{Eff}_c} - 1 = \frac{T_h}{T_h - T_c} - 1
\]

\[
= \frac{T_h - (T_h - T_c)}{T_h - T_c} = \frac{T_c}{T_h - T_c} = \frac{243 \text{ K}}{318 \text{ K} - 243 \text{ K}} = 3.24
\]

(b) Using \( \text{COP}_{\text{ref}} = \frac{Q_c}{W} \), again, and solve for the work done given \( Q_c = 1000 \text{ kcal} \):

\[
\text{COP}_{\text{ref}} = \frac{Q_c}{W} \Rightarrow W = \frac{Q_c}{\text{COP}_{\text{ref}}} = \frac{1000 \text{ kcal}}{3.24} = 308.6 \text{ kcal} = 309 \text{ kcal}
\]

(c) The cost is found by converting the units of energy into units of cents:

\[
\text{cost} = (308.6 \text{ kcal}) \left( \frac{4186 \text{ J}}{1 \text{ kcal}} \right) \left( \frac{10.0 \text{¢}}{3.60 \times 10^6 \text{ J}} \right) = 3.59 \text{¢}
\]

(d) We want to determine \( Q_h \), so using \( W = Q_h - Q_c \) gives:

\[
W = Q_h - Q_c \Rightarrow
\]

\[
Q_h = W + Q_c = 309 \text{ kcal} + 1000 \text{ kcal} = 1309 \text{ kcal} = 1309 \text{ kcal} \times \left( \frac{4.186 \text{ kJ}}{1 \text{ kcal}} \right) = 5479 \text{ kJ}
\]

(e) The inside of this refrigerator (actually freezer) is at \(-22^\circ\text{F} \approx -30.0^\circ\text{C}\), so this probably is a commercial meat packing freezer. The exhaust is generally vented to the outside, so as to not heat the building too much.

15.6 Entropy and the Second Law of Thermodynamics: Disorder and the Unavailability of Energy

47. (a) On a winter day, a certain house loses \( 5.00 \times 10^8 \text{ J} \) of heat to the outside (about 500,000 Btu). What is the total change in entropy due to this heat transfer alone, assuming an average indoor temperature of \( 21.0^\circ\text{C} \) and an average outdoor...
temperature of 5.00°C? (b) This large change in entropy implies a large amount of energy has become unavailable to do work. Where do we find more energy when such energy is lost to us?

Solution
(a) Use \( \Delta S = \frac{Q}{T} \) to calculate the change in entropy, remembering to use temperatures in Kelvin:

\[
\Delta S = \frac{-Q}{T} + \frac{Q}{T} = Q\left(\frac{1}{T_c} - \frac{1}{T_h}\right) = \left(5.00 \times 10^5 \text{ J}\right)\left(\frac{1}{278 \text{ K}} - \frac{1}{294 \text{ K}}\right) = 9.78 \times 10^4 \text{ J/K}
\]

(b) In order to gain more energy, we must generate it from things within the house, like a heat pump, human bodies, and other appliances. As you know, we use a lot of energy to keep our houses warm in the winter, because of the loss of heat to the outside.

53. What is the decrease in entropy of 25.0 g of water that condenses on a bathroom mirror at a temperature of 35.0°C, assuming no change in temperature and given the latent heat of vaporization to be 2450 kJ/kg?

Solution
When water condenses, it should seem reasonable that its entropy decreases, since the water gets ordered, so

\[
\Delta S = \frac{Q}{T} = \frac{-mL_v}{T} = \frac{-\left(25.0 \times 10^{-3} \text{ kg}\right)\left(2450 \times 10^3 \text{ J/kg}\right)}{308 \text{ K}} = -199 \text{ J/K}
\]

The entropy of the water decreases by 199 J/K when it condenses.

15.7 STATISTICAL INTERPRETATION OF ENTROPY AND THE SECOND LAW OF THERMODYNAMICS: THE UNDERLYING EXPLANATION

59. (a) If tossing 100 coins, how many ways (microstates) are there to get the three most likely macrostates of 49 heads and 51 tails, 50 heads and 50 tails, and 51 heads and
49 tails? (b) What percent of the total possibilities is this? (Consult Table 15.4.)

Solution  
(a) From Table 15.4, we can tabulate the number of ways of getting the three most likely microstates:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>No. of ways</th>
</tr>
</thead>
<tbody>
<tr>
<td>49</td>
<td>51</td>
<td>$9.9 \times 10^{28}$</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>$1.0 \times 10^{29}$</td>
</tr>
<tr>
<td>51</td>
<td>49</td>
<td>$9.9 \times 10^{28}$</td>
</tr>
</tbody>
</table>

Total $= 2.98 \times 10^{29} = 3.0 \times 10^{29}$

(b) The total number of ways is $1.27 \times 10^{30}$, so the percent represented by the three most likely microstates is:

$$
\% = \frac{\text{total # of ways to get 3 macrostates}}{\text{total # of ways}} = \frac{3.0 \times 10^{29}}{1.27 \times 10^{30}} = 0.236 = 24\%
$$