CHAPTER 6: UNIFORM CIRCULAR MOTION AND GRAVITATION

6.1 ROTATION ANGLE AND ANGULAR VELOCITY

1. *Semi-trailer trucks have an odometer on one hub of a trailer wheel. The hub is weighted so that it does not rotate, but it contains gears to count the number of wheel revolutions—it then calculates the distance traveled. If the wheel has a 1.15 m diameter and goes through 200,000 rotations, how many kilometers should the odometer read?*

Solution Given:

$$d = 1.15\text{ m} \Rightarrow r = \frac{1.15\text{ m}}{2} = 0.575\text{ m}, \Delta\theta = 200{,}000\ \text{rot} \times \frac{2\pi\text{ rad}}{1\text{ rot}} = 1.257\times10^{6}\text{ rad}$$

Find Δs using $\Delta\theta = \frac{\Delta s}{r}$, so that

$$\Delta s = \Delta\theta \times r = \left(1.257\times10^{6}\text{ rad}\right)\left(0.575\text{ m}\right)$$
$$= 7.226\times10^{5}\text{ m} = \underline{723\text{ km}}$$

7. *A truck with 0.420 m radius tires travels at 32.0 m/s. What is the angular velocity of the rotating tires in radians per second? What is this in rev/min?*

Solution Given: $r = 0.420\text{ m}, v = 32.0\text{ m/s}$.

Use $\omega = \frac{v}{r} = \frac{32.0\ \text{m/s}}{0.420\text{ m}} = \underline{76.2\text{ rad/s}}$.

Convert to rpm by using the conversion factor:

$1\text{ rev} = 2\pi\text{ rad}$,

$$\omega = 76.2\text{ rad/s} \times \frac{1\text{ rev}}{2\pi\text{ rad}} \times \frac{60\text{ s}}{1\text{ min}}$$
$$= 728\text{ rev/s} = \underline{728\text{ rpm}}$$

6.2 CENTRIPETAL ACCELERATION

18. *Verify that the linear speed of an ultracentrifuge is about 0.50 km/s, and Earth in its orbit is about 30 km/s by calculating: (a) The linear speed of a point on an ultracentrifuge 0.100 m from its center, rotating at 50,000 rev/min. (b) The linear speed of Earth in its orbit about the Sun (use data from the text on the radius of Earth's orbit and approximate it as being circular).*

Solution (a) Use $v = r\omega$ to find the linear velocity:

$$v = r\omega = (0.100\text{ m})\left(50{,}000\text{ rev/min} \times \frac{2\pi\text{ rad}}{1\text{ rev}} \times \frac{1\text{ min}}{60\text{ s}}\right) = 524\text{ m/s} = \underline{0.524\text{ km/s}}$$

(b) Given: $\omega = 2\pi\dfrac{\text{rad}}{\text{y}} \times \dfrac{1\text{ y}}{3.16\times10^{7}\text{ s}} = 1.988\times10^{-7}\text{ rad/s}; r = 1.496\times10^{11}\text{ m}$

Use $v = r\omega$ to find the linear velocity:

$$v = r\omega = (1.496\times10^{11}\text{ m})(1.988\times10^{-7}\text{ rad/s}) = 2.975\times10^{4}\text{ m/s} = \underline{29.7\text{ km/s}}$$

6.3 CENTRIPETAL FORCE

26. *What is the ideal speed to take a 100 m radius curve banked at a 20.0° angle?*

Solution Using $\tan\theta = \frac{v^2}{rg}$ gives:

$$\tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg\tan\theta} = \sqrt{(100\text{ m})(9.8\text{ m/s}^2)\tan 20.0°} = \underline{18.9\text{ m/s}}$$

6.5 NEWTON'S UNIVERSAL LAW OF GRAVITATION

33. *(a) Calculate Earth's mass given the acceleration due to gravity at the North Pole is* 9.830 m/s^2 *and the radius of the Earth is 6371 km from pole to pole. (b) Compare this with the accepted value of* $5.979\times10^{24}\text{ kg}$.

Solution (a) Using the equation $g = \frac{GM}{r^2}$ gives:

$$g = \frac{GM}{r^2} \Rightarrow M = \frac{r^2 g}{G} = \frac{(6371\times10^3\text{ m})^2(9.830\text{ m/s}^2)}{6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2} = \underline{5.979\times10^{24}\text{ kg}}$$

(b) This is identical to the best value to three significant figures.

39. *Astrology, that unlikely and vague pseudoscience, makes much of the position of the planets at the moment of one's birth. The only known force a planet exerts on Earth is gravitational. (a) Calculate the gravitational force exerted on a 4.20 kg baby by a 100 kg father 0.200 m away at birth (he is assisting, so he is close to the child). (b) Calculate the force on the baby due to Jupiter if it is at its closest distance to Earth, some* $6.29\times10^{11}\text{ m}$ *away. How does the force of Jupiter on the baby compare to the force of the father on the baby? Other objects in the room and the hospital building also exert similar gravitational forces. (Of course, there could be an unknown force acting, but scientists first need to be convinced that there is even an effect, much less that an unknown force causes it.)*

Solution (a) Use $F = \frac{GMm}{r^2}$ to calculate the force:

$$F_{\text{f}} = \frac{GMm}{r^2} = \frac{\left(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(100\ \text{kg}\right)\left(4.20\ \text{kg}\right)}{\left(0.200\ \text{m}\right)^2} = \underline{7.01\times10^{-7}\ \text{N}}$$

(b) The mass of Jupiter is:

$$m_{\text{J}} = 1.90\times10^{27}\ \text{kg}$$

$$F_{\text{J}} = \frac{\left(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.90\times10^{27}\ \text{kg}\right)\left(4.20\ \text{kg}\right)}{\left(6.29\times10^{11}\ \text{m}\right)^2} = \underline{1.35\times10^{-6}\ \text{N}}$$

$$\frac{F_{\text{f}}}{F_{\text{J}}} = \frac{7.01\times10^{-7}\ \text{N}}{1.35\times10^{-6}\ \text{N}} = \underline{0.521}$$

6.6 SATELLITES AND KEPLER'S LAWS: AN ARGUMENT FOR SIMPLICITY

45. *Find the mass of Jupiter based on data for the orbit of one of its moons, and compare your result with its actual mass.*

Solution Using $\frac{r^3}{T^2} = \frac{G}{4\pi^2}M$, we can solve the mass of Jupiter:

$$M_{\text{J}} = \frac{4\pi^2}{G}\times\frac{r^3}{T^2}$$

$$= \frac{4\pi^2}{6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2}\times\frac{\left(4.22\times10^{8}\ \text{m}\right)^3}{\left[\left(0.00485\ \text{y}\right)\left(3.16\times10^{7}\ \text{s/y}\right)\right]^2} = \underline{1.89\times10^{27}\ \text{kg}}$$

This result matches the value for Jupiter's mass given by NASA.

48. ***Integrated Concepts*** *Space debris left from old satellites and their launchers is becoming a hazard to other satellites. (a) Calculate the speed of a satellite in an orbit 900 km above Earth's surface. (b) Suppose a loose rivet is in an orbit of the same*

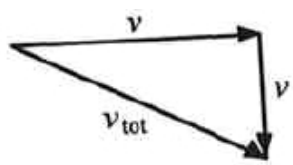
radius that intersects the satellite's orbit at an angle of 90° *relative to Earth. What is the velocity of the rivet relative to the satellite just before striking it? (c) Given the rivet is 3.00 mm in size, how long will its collision with the satellite last? (d) If its mass is 0.500 g, what is the average force it exerts on the satellite? (e) How much energy in joules is generated by the collision? (The satellite's velocity does not change appreciably, because its mass is much greater than the rivet's.)*

Solution (a) Use $F_c = ma_c$, then substitute using $a = \frac{v^2}{r}$ and $F = \frac{GmM}{r^2}$.

$$\frac{GmM}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM_E}{r_S}} = \sqrt{\frac{\left(6.673\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(5.979\times10^{24}\ \mathrm{kg}\right)}{900\times10^3\ \mathrm{m}}} = \underline{2.11\times10^4\ \mathrm{m/s}}$$

(b)



In the satellite's frame of reference, the rivet has two perpendicular velocity components equal to v from part (a):

$$v_{\mathrm{tot}} = \sqrt{v^2 + v^2} = \sqrt{2v^2} = \sqrt{2}\left(2.105\times10^4\ \mathrm{m/s}\right) = \underline{2.98\times10^4\ \mathrm{m/s}}$$

(c) Using kinematics: $d = v_{\mathrm{tot}}t \Rightarrow t = \frac{d}{v_{\mathrm{tot}}} = \frac{3.00\times10^{-3}\ \mathrm{m}}{2.98\times10^4\ \mathrm{m/s}} = \underline{1.01\times10^{-7}\ \mathrm{s}}$

(d) $\overline{F} = \frac{\Delta p}{\Delta t} = \frac{mv_{\mathrm{tot}}}{t} = \frac{\left(0.500\times10^{-3}\ \mathrm{kg}\right)\left(2.98\times10^4\ \mathrm{m/s}\right)}{1.01\times10^{-7}\ \mathrm{s}} = \underline{1.48\times10^8\ \mathrm{N}}$

(e) The energy is generated from the rivet. In the satellite's frame of reference, $v_{\mathrm{i}} = v_{\mathrm{tot}}$, and $v_{\mathrm{f}} = 0$. So, the change in the kinetic energy of the rivet is:

$$\Delta\,\mathrm{KE} = \frac{1}{2}mv_{\mathrm{tot}}^{\ 2} - \frac{1}{2}mv_{\mathrm{i}}^{\ 2} = \frac{1}{2}\left(0.500\times10^{-3}\ \mathrm{kg}\right)\left(2.98\times10^4\ \mathrm{m/s}\right)^2 - 0\ \mathrm{J} = \underline{2.22\times10^5\ \mathrm{J}}$$