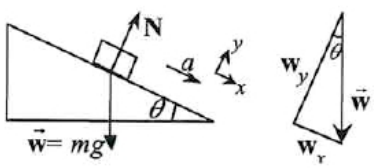
CHAPTER 5: FURTHER APPLICATION OF NEWTON'S LAWS: FRICTION, DRAG, AND ELASTICITY

5.1 FRICTION

8. *Show that the acceleration of any object down a frictionless incline that makes an angle θ with the horizontal is $a = g\sin\theta$. (Note that this acceleration is independent of mass.)*

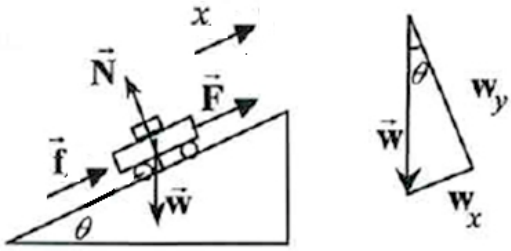
Solution The component of w down the incline leads to the acceleration:
$w_x = \text{net } F_x = ma = mg\sin\theta$ so that $\underline{a = g\sin\theta}$



The component of w perpendicular to the incline equals the normal force.
$w_y = \text{net } F_y = 0 = N - mg\sin\theta$

14. *Calculate the maximum acceleration of a car that is heading up a $4°$ slope (one that makes an angle of $4°$ with the horizontal) under the following road conditions. Assume that only half the weight of the car is supported by the two drive wheels and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the acceleration. (Ignore rolling.) (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that $\mu_s = 0.100$, the same as for shoes on ice.*

Solution



Take the positive *x*-direction as up the slope. For max acceleration,

$$\text{net } F_x = ma = f - w_x = \frac{1}{2}\mu_s mg\cos\theta - mg\sin\theta$$

So the maximum acceleration is: $a = g\left(\frac{1}{2}\mu_s\cos\theta - \sin\theta\right)$

(a) $\mu_s = 1.00,\ a = (9.80\text{ m/s}^2)\left[\frac{1}{2}(1.00)\cos 4° - \sin 4°\right] = \underline{4.20\text{ m/s}^2}$

(b) $\mu_s = 0.700,\ a = (9.80\text{ m/s}^2)\left[\frac{1}{2}(0.700)\cos 4° - \sin 4°\right] = \underline{2.74\text{ m/s}^2}$

(c) $\mu_s = 0.100,\ a = (9.80\text{ m/s}^2)\left[\frac{1}{2}(0.100)\cos 4° - \sin 4°\right] = \underline{-0.195\text{ m/s}^2}$

The negative sign indicates downwards acceleration, so the car cannot make it up the grade.

5.3 ELASTICITY: STRESS AND STRAIN

29. *During a circus act, one performer swings upside down hanging from a trapeze holding another, also upside-down, performer by the legs. If the upward force on the lower performer is three times her weight, how much do the bones (the femurs) in her upper legs stretch? You may assume each is equivalent to a uniform rod 35.0 cm long and 1.80 cm in radius. Her mass is 60.0 kg.*

Solution Use the equation $\Delta L = \frac{1}{Y}\frac{F}{A}L_0$, where $Y = 1.6\times10^{10}\text{ N/m}^2$ (from Table 5.3), $L_0 = 0.350\text{ m},\ A = \pi r^2 = \pi(0.0180\text{ m})^2 = 1.018\times10^{-3}\text{ m}^2$, and

$F_{\text{tot}} = 3w = 3(60.0\text{ kg})(9.80\text{ m/s}^2) = 1764\text{ N}$, so that the force on each leg is $F_{\text{leg}} = F_{\text{tot}}/2 = 882\text{ N}$. Substituting in the value gives:

$$\Delta L = \frac{1}{1.6\times10^{10}\text{ N/m}^2}\frac{(882\text{ N})}{(1.018\times10^{-3}\text{ m}^2)}(0.350\text{ m}) = 1.90\times10^{-5}\text{ m}.$$

So each leg is stretched by $\underline{1.90\times10^{-3}\text{ cm}}$.

35. *As an oil well is drilled, each new section of drill pipe supports its own weight and that of the pipe and drill bit beneath it. Calculate the stretch in a new 6.00 m length of steel pipe that supports 3.00 km of pipe having a mass of 20.0 kg/m and a 100-kg drill bit. The pipe is equivalent in strength to a solid cylinder 5.00 cm in diameter.*

Solution Use the equation $\Delta L = \frac{1}{Y}\frac{F}{A}L_0$, where $L_0 = 6.00\text{ m}, Y = 1.6\times10^{10}\text{ N/m}^2$. To calculate the mass supported by the pipe, we need to add the mass of the new pipe to the mass of the 3.00 km piece of pipe and the mass of the drill bit:

$$m = m_{\text{p}} + m_{3\text{ km}} + m_{\text{bit}}$$
$$= (6.00\text{ m})(20.0\text{ kg/m}) + (3.00\times10^3\text{ m})(20.0\text{ kg/m}) + 100\text{ kg} = 6.022\times10^4\text{ kg}$$

So that the force on the pipe is:
$$F = w = mg = (6.022\times10^4\text{ kg})(9.80\text{ m/s}^2) = 5.902\times10^5\text{ N}$$

Finally the cross sectional area is given by: $A = \pi r^2 = \pi\left(\frac{0.0500\text{ m}}{2}\right)^2 = 1.963\times10^{-3}\text{ m}^2$

Substituting in the values gives:
$$\Delta L = \frac{1}{2.10\times10^{11}\text{ N/m}^2}\frac{(5.902\times10^5\text{ N})}{(1.963\times10^{-3}\text{ m}^2)}(6.00\text{ m}) = 8.59\times10^{-3}\text{ m} = \underline{8.59\text{ mm}}$$

41. *A farmer making grape juice fills a glass bottle to the brim and caps it tightly. The juice expands more than the glass when it warms up, in such a way that the volume increases by 0.2% (that is, $\Delta V/V_0 = 2\times10^{-3}$) relative to the space available. Calculate the force exerted by the juice per square centimeter if its bulk modulus is $1.8\times10^9\text{ N/m}^2$, assuming the bottle does not break. In view of your answer, do you think the bottle will survive?*

Solution Using the equation $\Delta V = \frac{1}{B}\frac{F}{A}V_0$ gives:

$$\frac{F}{A} = B\frac{\Delta V}{V_0} = \left(1.8\times10^{9}\ \text{N/m}^2\right)\left(2\times10^{-3}\right) = 3.6\times10^{6}\ \text{N/m}^2 = \underline{4\times10^{6}\ \text{N/m}^2} = \underline{4\times10^{2}\ \text{N/cm}^2}$$

Since $1\ \text{atm} = 1.013\times10^{5}\ \text{N/m}^2$, the pressure is about 36 atmospheres, far greater than the average jar is designed to withstand.