CHAPTER 3: TWO-DIMENSIONAL KINEMATICS

3.2 VECTOR ADDITION AND SUBTRACTION: GRAPHICAL METHODS

1. *Find the following for path A in Figure 3.54: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.*

Solution (a) To measure the total distance traveled, we take a ruler and measure the length of Path A to the north, and add to it to the length of Path A to the east. Path A travels 3 blocks north and 1 block east, for a total of four blocks. Each block is 120 m, so the distance traveled is $d = (4 \times 120\,\text{m}) = \underline{480\,\text{m}}$

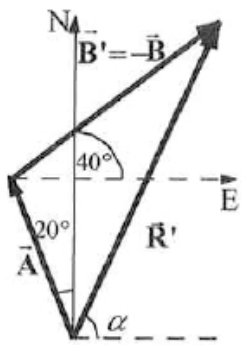
(b) Graphically, measure the length and angle of the line from the start to the arrow of Path A. Use a protractor to measure the angle, with the center of the protractor at the start, measure the angle to where the arrow is at the end of Path A. In order to do this, it may be necessary to extend the line from the start to the arrow of Path A, using a ruler. The length of the displacement vector, measured from the start to the arrow of Path A, along the line you just drew.

$\underline{\mathbf{S} = 379\,\text{m}, 18.4°\ \text{E of N}}$

7. *Repeat the problem two problems prior, but for the second leg you walk 20.0 m in a direction* $40.0°$ *north of east (which is equivalent to subtracting* $\mathbf{B}$ *from* $\mathbf{A}$ *—that is, to finding* $\mathbf{R'} = \mathbf{A} - \mathbf{B}$ *). (b) Repeat the problem two problems prior, but now you first walk 20.0 m in a direction* $40.0°$ *south of west and then 12.0 m in a direction* $20.0°$ *east of south (which is equivalent to subtracting* $\mathbf{A}$ *from* $\mathbf{B}$ *—that is, to finding* $\mathbf{R}'' = \mathbf{B} - \mathbf{A} = -\mathbf{R}'$ *). Show that this is the case.*

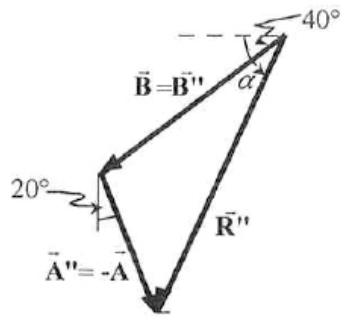
Solution (a) To do this problem, draw the two vectors **A** and **B'** = **–B** tip to tail as shown below. The vector **A** should be 12.0 units long and at an angle of $20°$ to the left of the *y*-axis. Then at the arrow of vector **A**, draw the vector **B'** = **–B**, which should be 20.0 units long and at an angle of $40°$ above the *x*-axis. The resultant vector, **R'**, goes from the tail of vector **A** to the tip of vector **B**, and therefore has an angle of α above the *x*-axis. Measure the length of the resultant vector using your ruler, and use a protractor with center at the tail of the resultant vector to get the angle.

$\underline{\mathrm{R}' = 26.6\,\mathrm{m}, \text{and}\ \alpha = 65.1°\ \mathrm{N\ of\ E}}$



(b) To do this problem, draw the two vectors **B** and **A"** = **–A** tip to tail as shown below. The vector **B** should be 20.0 units long and at an angle of $40°$ below the *x*-axis. Then at the arrow of vector **B**, draw the vector **A"** = **–A**, which should be 12.0 units long and at an angle of $20°$ to the right of the negative *y*-axis. The resultant vector, **R"**, goes from the tail of vector **B** to the tip of vector **A"**, and therefore has an angle of α below the *x*-axis. Measure the length of the resultant vector using your ruler, and use a protractor with center at the tail of the resultant vector to get the angle.

$\underline{\mathrm{R}'' = 26.6\,\mathrm{m}, \text{and}\ \alpha = 65.1°\ \mathrm{S\ of\ W}}$



So the length is the same, but the direction is reversed from part (a).

3.3 VECTOR ADDITION AND SUBTRACTION: ANALYTICAL METHODS

13. *Find the following for path C in Figure 3.58: (a) the total distance traveled and (b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.*

Solution (a) To solve this problem analytically, add up the distance by counting the blocks traveled along the line that is Path C:

$$d = (1\times 120\,\text{m}) + (5\times 120\,\text{m}) + (2\times 120\,\text{m}) + (1\times 120\,\text{m}) + (1\times 120\,\text{m}) + (3\times 120\,\text{m})$$
$$= \underline{1.56\times 10^3\ \text{m}}$$

(b) To get the displacement, calculate the displacements in the *x*- and *y*- directions separately, then use the formulas for adding vectors. The displacement in the *x*-direction is calculated by adding the *x*-distance traveled in each leg, being careful to subtract values when they are negative:

$$s_x = (0 + 600 + 0 - 120 + 0 - 360)\text{m} = 120\,\text{m}$$

Using the same method, calculate the displacement in the *y*-direction:

$$s_y = (120 + 0 - 240 + 0 + 120 + 0)\text{m} = 0\,\text{m}$$

Now using the equations $R = \sqrt{R_x^2 + R_y^2}$ and $\theta = \tan^{-1}\left(R_x / R_y\right)$, calculate the total displacement vectors:

$$s = \sqrt{s^2{}_x + s^2{}_y} = \sqrt{(120\text{ m})^2 + (0\text{ m})^2} = 120\text{ m}$$

$$\theta = \tan^{-1}\left(S_y / S_x\right) = \tan^{-1}\left(0\text{ m} / 120\text{ m}\right)$$

$0° \Rightarrow$ east, so that **S** = 120 m, east

19. *Do Problem 3.16 again using analytical techniques and change the second leg of the walk to* 25.0 m *straight south. (This is equivalent to subtracting* $\mathbf{B}$ *from* $\mathbf{A}$ *—that is, finding* $\mathbf{R'} = \mathbf{A} - \mathbf{B}$ *) (b) Repeat again, but now you first walk* 25.0 m *north and then* 18.0 m *east. (This is equivalent to subtract* $\mathbf{A}$ *from* $\mathbf{B}$ *—that is, to find* $\mathbf{A} = \mathbf{B} + \mathbf{C}$*. Is that consistent with your result?)*

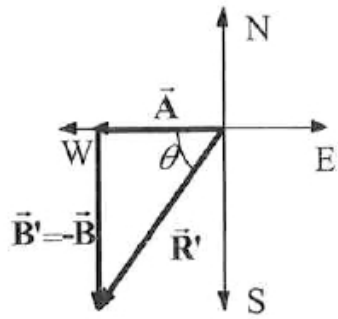
Solution (a) We want to calculate the displacement for walk 18.0 m to the west, followed by 25.0 m to the south. First, calculate the displacement in the *x*- and *y*-directions, using the equations $R_x = A_x + B_x$ and $R_y = A_y + B_y$: (the angles are measured from due east).

$$R_x = -18.0\text{ m}, R_y = -25.0\text{ m}$$

Then, using the equations $R = \sqrt{R_x^2 + R_y^2}$ and $\theta = \tan^{-1}\left(R_x / R_y\right)$, calculate the total displacement vectors:

$$R' = \sqrt{R_x{}^2 + R_y{}^2} = \sqrt{(18.0\text{ m})^2 + (25.0\text{ m})^2} = \underline{30.8\text{ m}}$$

$$\theta = \tan^{-1}\frac{\text{opp}}{\text{adj}} = \tan^{-1}\left(\frac{25.0\text{ m}}{18.0\text{ m}}\right) = \underline{54.2°\text{ S of W}}$$



(b) Now do the same calculation, except walk 25.0 m to the north, followed by 18.0 m to the east. Use the equations $R_x = A_x + B_x$ and $R_y = A_y + B_y$:

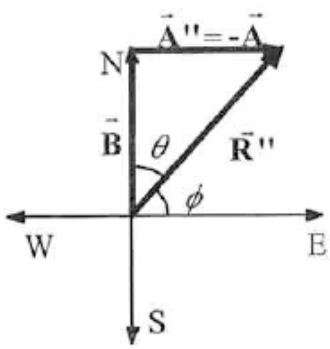
$R_x = 18.0\,\text{m}, R_y = 25.0\,\text{m}$

Then, use the equations $R = \sqrt{R_x^2 + R_y^2}$ and $\theta = \tan^{-1}\left(R_x / R_y\right)$.

$$R'' = \sqrt{R_x^{\,2} + R_y^{\,2}} = \sqrt{(18.0\,\text{m})^2 + (25.0\,\text{m})^2} = \underline{30.8\,\text{m}}$$

$$\theta = \tan^{-1}\frac{\text{opp}}{\text{adj}} = \tan^{-1}\left(\frac{25.0\,\text{m}}{18.0\,\text{m}}\right) = \underline{54.2°\text{ N of E}}$$

which is consistent with part (a).



3.4 PROJECTILE MOTION

30. *A rugby player passes the ball 7.00 m across the field, where it is caught at the same height as it left his hand. (a) At what angle was the ball thrown if its initial speed was 12.0 m/s, assuming that the smaller of the two possible angles was used? (b) What other angle gives the same range, and why would it not be used? (c) How long did this pass take?*

Solution (a) Find the range of a projectile on level ground for which air resistance is negligible:

$R = \frac{v_0^{\ 2}\ \sin 2\theta_0}{g}$, where v_0 is the initial speed and θ_0 is the initial angle relative to the horizontal. Solving for initial angle gives:

$\theta_0 = \frac{1}{2}\sin^{-1}\left(\frac{\text{gR}}{v_0^{\ 2}}\right)$, where: $R = 7.0\text{ m}, v_0 = 12.0\text{ m/s}, \text{and } g = 9.8\text{ m/s}^2$.

Therefore, $\theta_0 = \frac{1}{2}\sin^{-1}\left(\frac{(9.80\text{ m/s}^2)(7.0\text{ m})}{(12.0\text{ m/s})^2}\right) = \underline{14.2°}$

(b) Looking at the equation $R = \frac{v_0^{\ 2}\ \sin 2\theta_0}{g}$, we see that range will be same for another angle, θ_0', where $\theta_0 + \theta_0' = 90°$ or $\theta_0' = 90° - 14.2° = \underline{75.8°}$.

This angle is not used as often, because the time of flight will be longer. In rugby that means the defense would have a greater time to get into position to knock down or intercept the pass that has the larger angle of release.

40. *An eagle is flying horizontally at a speed of 3.00 m/s when the fish in her talons wiggles loose and falls into the lake 5.00 m below. Calculate the velocity of the fish relative to the water when it hits the water.*

Solution <u>*x*-direction (horizontal)</u>

Given: $v_{0,x} = 3.00\text{ m/s}, a_x = 0\text{ m/s}^2$.

Calculate v_x.

$$v_x = v_{0,x} = \text{constant} = 3.00\ \text{m/s}$$

<u>*y*-direction (vertical)</u>

Given: $v_{0,y} = 0.00\ \text{m/s}, a_y = -g = -9.80\ \text{m/s}^2, \Delta y = (y - y_0) = -5.00\ \text{m}$

Calculate v_y.

$$v_y^2 = v_{0,y}^2, -2g(y\text{-}y_0)$$
$$v_y = \sqrt{(0\ \text{m/s})^2 - 2(9.80\ \text{m/s}^2)(-5.00\ \text{m})} = -9.90\ \text{m/s}$$

Now we can calculate the final velocity:

$$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{(3.00\ \text{m/s})^2 + (-9.90\ \text{m/s})^2} = 10.3\ \text{m/s}$$

and $\theta = \tan^{-1}\left(\dfrac{v_y}{v_x}\right) = \tan^{-1}\left(\dfrac{-9.90\ \text{m/s}}{3.00\ \text{m/s}}\right) = -73.1°$

so that $v = \underline{10.3\ \text{m/s}, 73.1°\ \text{below the horizontal}}$

46. *A basketball player is running at* $5.00\ \text{m/s}$ *directly toward the basket when he jumps into the air to dunk the ball. He maintains his horizontal velocity. (a) What vertical velocity does he need to rise 0.750 m above the floor? (b) How far from the basket (measured in the horizontal direction) must he start his jump to reach his maximum height at the same time as he reaches the basket?*

Solution (a) Given: $v_x = 5.00\ \text{m/s}, y - y_0 = 0.75\ \text{m}, v_y = 0\ \text{m/s}, a_y = -g = -9.80\ \text{m/s}^2$.

Find: $v_{0,y}$.

Using the equation ${v_y}^2 = {v_y}^2 - 2g(y - y_0)$ gives:

$$v_{0,y} = \sqrt{v_y^{\ 2} + 2g(y - y_0)} = \sqrt{(0\,\text{m/s})^2 + 2(9.80\,\text{m/s}^2)(0.75\,\text{m})} = \underline{3.83\,\text{m/s}}$$

(b) To calculate the x-direction information, remember that the time is the same in the x- and y-directions. Calculate the time from the y-direction information, then use it to calculate the x-direction information:

Calculate the time:

$v_y = v_{0,y} - gt$, so that

$$t = \frac{v_{0,y} - v_y}{g} = \frac{(3.83\,\text{m/s}) - (0\,\text{m/s})}{9.80\,\text{m/s}^2} = 0.391\,\text{s}$$

Now, calculate the horizontal distance he travels to the basket:

$x = x_0 + v_x t$, so that $(x - x_0) = v_x t = (5.00\,\text{m/s})(0.391\,\text{s}) = \underline{1.96\,\text{m}}$

So, he must leave the ground 1.96 m before the basket to be at his maximum height when he reaches the basket.

3.5 ADDITION OF VELOCITIES

54. *Near the end of a marathon race, the first two runners are separated by a distance of 45.0 m. The front runner has a velocity of 3.50 m/s, and the second a velocity of 4.20 m/s. (a) What is the velocity of the second runner relative to the first? (b) If the front runner is 250 m from the finish line, who will win the race, assuming they run at constant velocity? (c) What distance ahead will the winner be when she crosses the finish line?*

Solution (a) To keep track of the runners, let's label F for the first runner and S for the second runner. Then we are given: $v_F = 3.50\,\text{m/s}, v_S = 4.20\,\text{m/s}$. To calculate the velocity of the second runner relative to the first, subtract the velocities:

$v_{SF} = v_S - v_F = 4.20\,\text{m/s} - 3.50\,\text{m/s} = \underline{0.70\,\text{m/s}}$ faster than first runner

(b) Use the definition of velocity to calculate the time for each runner separately. For

the first runner, she runs 250 m at a velocity of 3.50 m/s:

$$t_F = \frac{x_F}{v_F} = \frac{250\text{ m}}{3.50\text{ m/s}} = 71.43\text{ s}$$

For the second runner, she runs 45 m father than the first runner at a velocity of 4.20 m/s: $t_S = \frac{x_S}{v_S} = \frac{250 + 45\text{ m}}{4.20\text{ m/s}} = 70.24\text{ s}$

So, since $t_S < t_F$, <u>the second runner will win</u>.

(c) We can calculate their relative position, using their relative velocity and time of travel. Initially, the second runner is 45 m behind, the relative velocity was found in part (a), and the time is the time for the second runner, so:

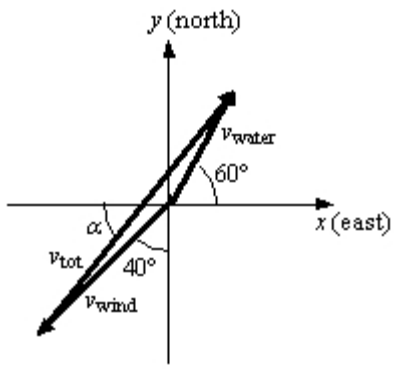
$$x_{SF} = x_{O,SF} + v_{SF}t_S = -45.0\text{ m} + (0.70\text{ m/s})(70.24\text{ s}) = \underline{4.17\text{ m}}$$

62. *The velocity of the wind relative to the water is crucial to sailboats. Suppose a sailboat is in an ocean current that has a velocity of 2.20 m/s in a direction* $30.0°$ *east of north relative to the Earth. It encounters a wind that has a velocity of 4.50 m/s in a direction of* $50.0°$ *south of west relative to the Earth. What is the velocity of the wind relative to the water?*

Solution In order to calculate the velocity of the wind relative to the ocean, we need to add the vectors for the wind and the ocean together, being careful to use vector addition. The velocity of the wind relative to the ocean is equal to the velocity of the wind relative to the earth plus the velocity of the earth relative to the ocean. Now,

$$\mathbf{v}_{WO} = \mathbf{v}_{WE} + \mathbf{v}_{EO} = \mathbf{v}_{WE} - \mathbf{v}_{OE}.$$

The first subscript is the object, the second is what it is relative to. In other words the velocity of the earth relative to the ocean is the opposite of the velocity of the ocean relative to the earth.



To solve this vector equation, we need to add the *x*- and *y*-components separately.

$$v_{\mathrm{WO}x} = v_{\mathrm{WE}x} - v_{\mathrm{OE}x} = (-4.50\ \mathrm{m/s})\cos 50° - (2.20\ \mathrm{m/s})\cos 60° = -3.993\ \mathrm{m/s}$$
$$v_{\mathrm{WO}y} = v_{\mathrm{WE}y} - v_{\mathrm{OE}y} = (-4.50\ \mathrm{m/s})\sin 50° - (2.20\ \mathrm{m/s})\sin 60° = -5.352\ \mathrm{m/s}$$

Finally, we can use the equations below to calculate the velocity of the water relative to the ocean:

$$v = \sqrt{v_x^{\ 2} + v_y^{\ 2}} = \sqrt{(\text{-}3.993\ \mathrm{m/s})^2 + (\text{-}5.352\ \mathrm{m/s})^2} = \underline{6.68\ \mathrm{m/s}}$$
$$\alpha = \tan^{-1}\frac{|v_y|}{|v_x|} = \tan^{-1}\left(\frac{5.352\ \mathrm{m/s}}{3.993\ \mathrm{m/s}}\right) = \underline{53.3°\ \mathrm{S\ of\ W}}$$

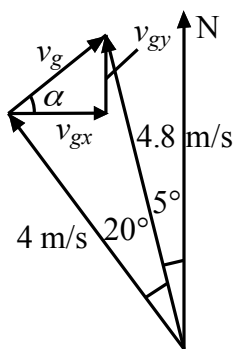
66. *A ship sailing in the Gulf Stream is heading* $25.0°$ *west of north at a speed of 4.00 m/s relative to the water. Its velocity relative to the Earth is* $4.80\ \mathrm{m/s}$ $5.00°$ *west of north. What is the velocity of the Gulf Stream? (The velocity obtained is typical for the Gulf Stream a few hundred kilometers off the east coast of the United States.)*

Solution To calculate the velocity of the water relative to the earth, we need to add the vectors. The velocity of the water relative to the earth is equal to the velocity of the water relative to the ship plus the velocity of the ship relative to the earth.

$$\mathbf{v}_{\mathbf{WE}} = \mathbf{v}_{\mathbf{WS}} + \mathbf{v}_{\mathbf{SE}} = -\mathbf{v}_{\mathbf{SW}} + \mathbf{v}_{\mathbf{SE}}$$

Now, we need to calculate the *x*- and *y*-components separately:

$$v_{\mathrm{WE}x} = -v_{\mathrm{SW}x} + v_{\mathrm{SE}x} = -(4.00\ \mathrm{m/s})\cos 115° + (4.80\ \mathrm{m/s})\cos 95° = 1.272\ \mathrm{m/s}$$
$$v_{\mathrm{WE}y} = -v_{\mathrm{SW}y} + v_{\mathrm{SE}y} = -(4.00\ \mathrm{m/s})\sin 115° + (4.80\ \mathrm{m/s})\sin 95° = 1.157\ \mathrm{m/s}$$



Finally, we use the equations below to calculate the velocity of the water relative to the earth:

$$v_{\mathrm{WE}} = \sqrt{v^2{}_{\mathrm{WE},x} + v^2{}_{\mathrm{WE},y}} = \sqrt{(1.272\ \mathrm{m/s})^2 + (1.157\ \mathrm{m/s})^2} = \underline{1.72\ \mathrm{m/s}}$$

$$\alpha = \tan^{-1}\left(\frac{v_{\mathrm{WE},y}}{v_{\mathrm{WE},x}}\right) = \tan^{-1}\frac{1.157\ \mathrm{m/s}}{1.272\ \mathrm{m/s}} = \underline{42.3°\ \mathrm{N\ of\ E.}}$$