CHAPTER 34: FRONTIERS OF PHYSICS

34.1 COSMOLOGY AND PARTICLE PHYSICS

1. *Find the approximate mass of the luminous matter in the Milky Way galaxy, given it has approximately* 10^{11} *stars of average mass 1.5 times that of our Sun.*

Solution The approximate mass of the luminous matter in the Milky Way galaxy can be found by multiplying the number of stars times 1.5 times the mass of our Sun:

$$M = (10^{11})(1.5)m_S = (10^{11})(1.5)(1.99\times10^{30}\text{ kg}) = \underline{3\times10^{41}\text{ kg}}$$

7. *(a) What is the approximate velocity relative to us of a galaxy near the edge of the known universe, some 10 Gly away? (b) What fraction of the speed of light is this? Note that we have observed galaxies moving away from us at greater than* $0.9c$.

Solution (a) Using $v = H_0 d$ and the Hubble constant, we can calculate the approximate velocity of the near edge of the known universe:

$$v = H_0 d = (20\text{ km/s}\cdot\text{Mly})(10\times10^{3}\text{ Mly}) = \underline{2.0\times10^{5}\text{ km/s}}$$

(b) To calculate the fraction of the speed of light, divide this velocity by the speed of light: $\dfrac{v}{c} = \dfrac{(2.0\times10^{5}\text{ km/s})(10^{3}\text{ m/km})}{3.00\times10^{8}\text{ m/s}} = 0.67$, so that $\underline{v = 0.67c}$

11. *Andromeda galaxy is the closest large galaxy and is visible to the naked eye. Estimate its brightness relative to the Sun, assuming it has luminosity* 10^{12} *times that of the Sun and lies 2 Mly away.*

Solution The relative brightness of a star is going to be proportional to the ratio of surface areas times the luminosity, so:

$$\text{Relative Brightness} = (\text{luminosity})\frac{4\pi r^2}{4\pi R_{\text{Andromeda}}^2} = \left(10^{12}\right)\left(\frac{r_{\text{Sun}}}{R_{\text{Andromeda}}}\right)^2.$$

From Appendix C, we know the average distance to the sun is $1.496\times10^{11}\ \text{m}$, and we are told the average distance to Andromeda, so:

$$\text{Relative Brightness} = \frac{\left(10^{12}\right)\left(1.496\times10^{11}\ \text{m}\right)^2}{\left[\left(2\times10^{6}\ \text{ly}\right)\left(9.46\times10^{15}\ \text{m/ly}\right)\right]^2} = \underline{6\times10^{-11}}.$$

Note: this is an overestimate since some of the light from Andromeda is blocked by its own gas and dust clouds.

15. *(a) What Hubble constant corresponds to an approximate age of the universe of* 10^{10} *y? To get an approximate value, assume the expansion rate is constant and calculate the speed at which two galaxies must move apart to be separated by 1 Mly (present average galactic separation) in a time of* 10^{10} *y. (b) Similarly, what Hubble constant corresponds to a universe approximately* 2×10^{10}*-y old?*

Solution (a) Since the Hubble constant has units of $\text{km/s}\cdot\text{Mly}$, we can calculate its value by considering the age of the universe and the average galactic separation. If the universe is 10^{10} years old, then it will take 10^{10} years for the galaxies to travel 1 Mly. Now, to determine the value for the Hubble constant, we just need to determine the average velocity of the galaxies from the equation $d = v\times t$:

$$v = \frac{d}{t}, \text{ so that } v = \frac{1\ \text{Mly}}{10^{10}\ \text{y}} = \frac{1\times10^{6}\ \text{ly}}{10^{10}\ \text{y}}\times\frac{9.46\times10^{12}\ \text{km}}{1\ \text{ly}}\times\frac{1\ \text{y}}{3.156\times10^{7}\ \text{s}} = 30\ \text{km/s}.$$

$$\text{Thus, } H_0 = \frac{30\ \text{km/s}}{1\ \text{Mly}} = \underline{30\ \text{km/s}\cdot\text{Mly}}$$

(b) Now, the time is 2×10^{10} years, so the velocity becomes:

$$v=\frac{1\,\text{Mly}}{2\times10^{10}\ \text{y}}=\frac{1\times10^{6}\ \text{ly}}{2\times10^{10}\ \text{y}}\times\frac{9.46\times10^{12}\ \text{km}}{1\,\text{ly}}\times\frac{1\,\text{y}}{3.156\times10^{7}\ \text{s}}=15\ \text{km/s}.$$

Thus, the Hubble constant would be approximately $\underline{H_0=\frac{15\ \text{km/s}}{1\,\text{Mly}}=15\,\text{km/s}\cdot\text{Mly}}$

16. *Show that the velocity of a star orbiting its galaxy in a circular orbit is inversely proportional to the square root of its orbital radius, assuming the mass of the stars inside its orbit acts like a single mass at the center of the galaxy. You may use an equation from a previous chapter to support your conclusion, but you must justify its use and define all terms used.*

Solution A star orbiting its galaxy in a circular orbit feels the gravitational force acting toward the center, which is the centripetal force (keeping the star orbiting in a circle). So, from $F=G\frac{mM}{r^2}$, we get an expression for the gravitational force acting on the star, and from $F_c=m\frac{v^2}{r}$, we get an expression for the centripetal force keeping the star orbiting in a circle. Setting the two forces equal gives: $F=\frac{mv^2}{r}=\frac{GMm}{r^2}$, where m is the mass of the star, M is the mass of the galaxy (assumed to be concentrated at the center of the rotation), G is the gravitational constant, v is the velocity of the star, and r is the orbital radius. Solving the equation for the velocity gives: $\underline{v=\sqrt{\frac{GM}{r}}}$ so that the velocity of a star orbiting its galaxy in a circular orbit is indeed inversely proportional to the square root of its orbital radius.