CHAPTER 32: MEDICAL APPLICATIONS OF NUCLEAR PHYSICS

32.1 MEDICAL IMAGING AND DIAGNOSTICS

1. *A neutron generator uses an α source, such as radium, to bombard beryllium, inducing the reaction ${}^{4}\mathrm{He} + {}^{9}\mathrm{Be} \rightarrow {}^{12}\mathrm{C} + n$. Such neutron sources are called RaBe sources, or PuBe sources if they use plutonium to get the α s. Calculate the energy output of the reaction in MeV.c*

Solution Using $E = \Delta mc^2$, we can determine the energy output of the reaction by calculating the change in mass of the constituents in the reaction, where the masses are found either in Appendix A or Table 31.2:

$$\begin{aligned} E &= (m_\mathrm{i} - m_\mathrm{f})c^2 \\ &= (4.002603 + 9.02182 - 12.000000 - 1.008665)(931.5\ \mathrm{MeV}) = \underline{5.701\ \mathrm{MeV}} \end{aligned}$$

6. *The activities of ${}^{131}\mathrm{I}$ and ${}^{123}\mathrm{I}$ used in thyroid scans are given in Table 32.1 to be 50 and $70\ \mu\mathrm{Ci}$, respectively. Find and compare the masses of ${}^{131}\mathrm{I}$ and ${}^{123}\mathrm{I}$ in such scans, given their respective half-lives are 8.04 d and 13.2 h. The masses are so small that the radioiodine is usually mixed with stable iodine as a carrier to ensure normal chemistry and distribution in the body.*

Solution Beginning with the equation $R = \dfrac{0.693N}{t_{1/2}} = \dfrac{(0.693)(m/M)N_\mathrm{A}}{t_{1/2}}$ we can solve for the mass of the iodine isotopes, where the atomic masses and the half lives are given in the appendices:

$$m_{131}=\frac{RMt_{1/2}}{0.693N_{\text{A}}}=\frac{(5.0\times10^{-5}\ \text{Ci})(3.70\times10^{10}\ \text{Bq/Ci})(130.91\ \text{g/mol})(8.040\ \text{d})\left(86400\,\text{s/d}\right)}{(0.693)\left(6.02\times10^{23}\right)}$$
$$=\underline{4.0\times10^{-10}\ \text{g}}$$
$$m_{123}=\frac{RMt_{1/2}}{0.693N_{\text{A}}}=\frac{(7.0\times10^{-5}\ \text{Ci})(3.70\times10^{10}\ \text{Bq/Ci})(122.91\ \text{g/mol})(13.2\ \text{h})\left(3600\,\text{s/h}\right)}{(0.693)\left(6.02\times10^{23}\right)}$$
$$=\underline{3.6\times10^{-11}\ \text{g}}$$

32.2 BIOLOGICAL EFFECTS OF IONIZING RADIATION

10. *How many Gy of exposure is needed to give a cancerous tumor a dose of 40 Sv if it is exposed to α activity?*

Solution Using the equation $\text{Sv}=\text{Gy}\times\text{RBE}$ and Table 32.2, we know that $\text{RBE}=20$ for whole body exposure, so $\text{Gy}=\dfrac{\text{Sv}}{\text{RBE}}=\dfrac{40\ \text{Sv}}{20}=\underline{2\ \text{Gy}}$

32.3 THERAPEUTIC USES OF IONIZING RADIATION

21. *Large amounts of ^{65}Zn are produced in copper exposed to accelerator beams. While machining contaminated copper, a physicist ingests $50.0\ \mu\text{Ci of }^{65}\text{Zn}$. Each ^{65}Zn decay emits an average γ-ray energy of 0.550 MeV, 40.0% of which is absorbed in the scientist's 75.0-kg body. What dose in mSv is caused by this in one day?*

Solution First, we need to determine the number of decays per day:

$$\text{decays/day}=\left(5.00\times10^{-5}\ \text{Ci}\right)\left(3.70\times10^{10}\ \text{Bq/Ci}\right)\left(8.64\times10^{4}\ \text{s/d}\right)=1.598\times10^{11}\ \text{/d}$$

Next, we can calculate the energy because each decay emits an average of 0.550 MeV of energy:

$$E/day=\left(\frac{1.598\times10^{11}\ \text{decays}}{\text{d}}\right)(0.400)\left(\frac{0.550\,\text{MeV}}{\text{decay}}\right)\left(\frac{1.602\times10^{-13}\ \text{J}}{\text{MeV}}\right)$$
$$=5.633\times10^{-3}\ \text{J/d}$$

Then, dividing by the mass of tissue gives the dose:

$$\text{Dose in rad/d} = \left(\frac{5.633\times10^{-3}\ \text{J/d}}{75.0\ \text{kg}}\right)\frac{1\ \text{rad}}{0.0100\ \text{J/kg}} = 7.51\times10^{-3}\ \text{rad/d}$$

Finally, from Table 32.2, we see that the RBE is 1 for γ radiation, so:

$$\text{rem/d} = \text{rad}\times\text{RBE} = \left(7.51\times10^{-3}\ \text{rad/d}\right)\times(1) = 7.51\times10^{-3}\ \text{rem/d}\frac{\text{mSv}}{0.1\ \text{rem}}$$

$$= \underline{7.51\times10^{-4}\ \text{mSv/d}}$$

This dose is approximately $2700\ \text{mrem/y}$, which is larger than background radiation sources, but smaller than doses given for cancer treatments.

32.5 FUSION

30. *The energy produced by the fusion of a 1.00-kg mixture of deuterium and tritium was found in Example Calculating Energy and Power from Fusion. Approximately how many kilograms would be required to supply the annual energy use in the United States?*

Solution From Table 7.6, we know $E = 1.05\times10^{20}\ \text{J}$ and from Example 32.2, we know that a 1.00 kg mixture of deuterium and tritium releases $3.37\times10^{14}\ \text{J}$ of energy, so:

$$M = \left(1.05\times10^{20}\ \text{J}\right)\left(\frac{1.00\ \text{kg}}{3.37\times10^{14}\ \text{J}}\right) = \underline{3.12\times10^{5}\ \text{kg}}$$

35. *The power output of the Sun is $4\times10^{26}\ \text{W}$. (a) If 90% of this is supplied by the proton-proton cycle, how many protons are consumed per second? (b) How many neutrinos per second should there be per square meter at the Earth from this process? This huge number is indicative of how rarely a neutrino interacts, since large detectors observe very few per day.*

Solution (a) Four protons are needed for each cycle to occur. The energy released by a proton-proton cycle is $26.7\ \text{MeV}$, so that

$$\#\ \text{protons/s} = (0.90)\left(4\times10^{26}\ \text{J/s}\right)\left(\frac{4\ \text{protons}}{26.7\ \text{MeV}}\right)\left(\frac{1\ \text{MeV}}{1.602\times10^{-13}\ \text{J}}\right)$$

$$= \underline{3\times10^{38}\ \text{protons/s}}$$

(b) For each cycle, two neutrinos are created and four protons are destroyed. To determine the number of neutrinos at Earth, we need to determine the number of neutrinos leaving the Sun and divide that by the surface area of a sphere with radius from the Sun to Earth:

$$\frac{\#}{\text{area}}=\frac{\#}{4\pi R^2}=\frac{(2v_e/4\,\text{protons})(3.37\times10^{38}\,\text{protons/s})}{4\pi(1.50\times10^{11}\text{ m})^2}=\underline{6\times10^{14}\text{ neutrinos/m}^2\cdot\text{s}}$$

32.6 FISSION

45. *(a) Calculate the energy released in the neutron-induced fission reaction* $n+{}^{239}\text{Pu}\rightarrow{}^{96}\text{Sr}+{}^{140}\text{Ba}+4n$, *given* $m({}^{96}\text{Sr})=95.921750\text{ u}$ *and* $m({}^{140}\text{Ba})=139.910581\text{ u}$. *(b) Confirm that the total number of nucleons and total charge are conserved in this reaction.*

Solution (a) To calculate the energy released, we use $E=\Delta mc^2$ to calculate the difference in energy before and after the reaction:

$$\begin{aligned}E&=\left(m_n+m\left({}^{239}\text{Pu}\right)-m\left({}^{96}\text{Sr}\right)-m\left({}^{140}\text{Ba}\right)-4m_n\right)c^2\\&=\left(m\left({}^{239}\text{Pu}\right)-m\left({}^{96}\text{Sr}\right)-m\left({}^{140}\text{Ba}\right)-3m_n\right)c^2\\&=\left[239.052157-95.921750-139.910581-(3)(1.008665)\right](931.5\text{ MeV})\\&=\underline{180.6\text{ MeV}}\end{aligned}$$

(b) Writing the equation in full form gives ${}^1_0n_1+{}^{239}_{94}\text{Pu}_{145}\rightarrow{}^{96}_{38}\text{Sr}_{84}+4{}^1_0n_1$ so we can determine the total number of nucleons before and after the reaction and the total charge before and after the reaction:

$$\begin{aligned}A_\text{i}&=1+239=240=96+140+4=A_\text{f};\\Z_\text{i}&=0+94=94=56+38+4(0)=Z_\text{f}.\end{aligned}$$

Therefore, both the total number of nucleons and the total charge are conserved.

32.7 NUCLEAR WEAPONS

51. *Find the mass converted into energy by a 12.0-kT bomb.*

Solution Using $E=mc^2$, we can calculate the mass converted into energy for a 12.0 kT bomb:

$$m=\frac{E}{c^2}=\frac{(12.0\text{ kT})(4.2\times10^{12}\text{ J/kT})}{(3.00\times10^8\text{ m/s})^2}=5.60\times10^{-4}\text{ kg}=\underline{0.56\text{ g}}$$

57. *Assume one-fourth of the yield of a typical 320-kT strategic bomb comes from fission reactions averaging 200 MeV and the remainder from fusion reactions averaging 20 MeV. (a) Calculate the number of fissions and the approximate mass of uranium and plutonium fissioned, taking the average atomic mass to be 238. (b) Find the number of fusions and calculate the approximate mass of fusion fuel, assuming an average total atomic mass of the two nuclei in each reaction to be 5. (c) Considering the masses found, does it seem reasonable that some missiles could carry 10 warheads? Discuss, noting that the nuclear fuel is only a part of the mass of a warhead.*

Solution (a) Given that for fission reactions, the energy produced is 200 MeV per fission, we can convert the $1/4$ of 320 kT yield into the number of fissions:

$$\#\text{ of fissions} = \frac{(1/4)(320\text{ kT})(4.2\times10^{12}\text{ J/kT})}{(200\text{ MeV/fission})(1.60\times10^{-13}\text{ J/MeV})} = \underline{1.1\times10^{25}\text{ fissions}}$$

Then, $m = \left(1.1\times10^{25}\text{ nuclei}\right)\left(\frac{1\text{ mol}}{6.022\times10^{23}\text{ nuclei}}\right)(238\text{ g/mol}) = 4.35\times10^{3}\text{ g} = \underline{4.3\text{ kg}}$

(b) Similarly, given that for fusion reactions, the energy produced is 20 MeV per fusion, we convert the $3/4$ of 320 kT yield into the number of fusions:

$$\#\text{ of fusions} = \frac{(3/4)(320\text{ kT})(4.2\times10^{12}\text{ J/kT})}{(200\text{ MeV/fission})(1.60\times10^{-13}\text{ J/MeV})} = \underline{3.2\times10^{26}\text{ fusions}}$$

Then:

$$m = \left(3.2\times10^{26}\text{ fusions}\right)\left(\frac{1\text{ mol}}{6.022\times10^{23}\text{ nuclei}}\right)(5\text{ g LiD fuel/mol}) = 2.66\times10^{3}\text{ g} = \underline{2.7\text{ kg}}$$

(c) The nuclear fuel totals only 6 kg, so it is quite reasonable that some missiles carry 10 overheads. The mass of the fuel would only be 60 kg and therefore the mass of the 10 warheads, weighing about 10 times the nuclear fuel, would be only 1500 lbs. If the fuel for the missiles weighs 5 times the total weight of the warheads, the missile would weigh about 9000 lbs or 4.5 tons. This is not an unreasonable weight for a missile.