CHAPTER 31: RADIOACTIVITY AND NUCLEAR PHYSICS

31.2 RADIATION DETECTION AND DETECTORS

1. *The energy of 30.0* eV *is required to ionize a molecule of the gas inside a Geiger tube, thereby producing an ion pair. Suppose a particle of ionizing radiation deposits 0.500 MeV of energy in this Geiger tube. What maximum number of ion pairs can it create?*

Solution To calculate the number of pairs created, simply divide the total energy by energy needed per pair: $\#\text{pairs} = \dfrac{(0.500\text{ MeV})(1.00\times10^6\text{ eV/MeV})}{30.0\text{ eV/pair}} = \underline{1.67\times10^4\text{ pairs}}$. This is the maximum number of ion pairs because it assumes all the energy goes to creating ion pairs and that there are no energy losses.

31.3 SUBSTRUCTURE OF THE NUCLEUS

9. *(a) Calculate the radius of* ^{58}Ni*, one of the most tightly bound stable nuclei. (b) What is the ratio of the radius of* ^{58}Ni *to that of* ^{258}Ha*, one of the largest nuclei ever made? Note that the radius of the largest nucleus is still much smaller than the size of an atom.*

Solution (a) Using the equation $r = r_0A^{1/3}$ we can approximate the radius of ^{58}Ni:

$$r_{\text{Ni}} = r_0A_{\text{Ni}}^{1/3} = (1.2\times10^{-15}\text{ m})(58)^{1/3} = 4.6\times10^{-15}\text{ m} = \underline{4.6\text{ fm}}$$

(b) Again using this equation this time we can approximate the radius of ^{258}Ha:

$$r_{\text{Ha}} = (1.2\times10^{-15}\text{ m})(258)^{1/3} = 7.6\times10^{-15}\text{ m} = \underline{7.6\text{ fm}}.$$

Finally, taking the ratio of Ni to Ha gives: $\frac{\mathrm{r_{Ni}}}{\mathrm{r_{Ha}}} = \frac{4.645\times10^{-15}\ \mathrm{m}}{7.639\times10^{-15}\ \mathrm{m}} = \underline{0.61\ \mathrm{to}\ 1}$

15. *What is the ratio of the velocity of a 5.00-MeV β ray to that of an α particle with the same kinetic energy? This should confirm that β s travel much faster than α s even when relativity is taken into consideration. (See also Exercise 31.11.)*

Solution We know that the kinetic energy for a relativistic particle is given by the equation $\mathrm{KE_{rel}} = (\gamma-1)mc^2$, and that since $\gamma = \left(1-\frac{v^2}{c^2}\right)^{-1/2}$, we can get an expression for the speed $\frac{v^2}{c^2} = \left(1-\frac{1}{\gamma^2}\right)$, or $v = c\sqrt{1-\frac{1}{\gamma^2}}$

For the β particle: $\mathrm{KE} = 5.00\ \mathrm{MeV} = (\gamma-1)(0.511\ \mathrm{MeV})$, so that $(\gamma-1) = 9.785$ or $\gamma = 10.785$. Thus, the velocity for the β particle is:

$$v_\beta = c\sqrt{1-\frac{1}{\gamma^2}} = (2.998\times10^8\ \mathrm{m/s})\sqrt{1-\frac{1}{(10.785)^2}} = 2.985\times10^8\ \mathrm{m/s}$$

For the α particle: $\mathrm{KE} = 5.00\ \mathrm{MeV} = (\gamma-1)(4.0026\ \mathrm{u})\left(\frac{931.5\ \mathrm{MeV/u}}{c^2}\right)c^2$ so that $\gamma = 1.00134$. Thus, the velocity for the α particle is:

$v_\alpha = (2.998\times10^8\ \mathrm{m/s})\sqrt{1-\frac{1}{(1.00134)^2}} = 1.551\times10^7\ \mathrm{m/s}$. Finally, the ratio of the velocities is given by: $\frac{v_\beta}{v_\alpha} = \frac{2.985\times10^8\ \mathrm{m/s}}{1.55\times10^7\ \mathrm{m/s}} = \underline{19.3\ \mathrm{to}\ 1}$.

In other words, when the β and α particles have the same kinetic energy , the β particle is approximately 19 times faster than the α particle.

31.4 NUCLEAR DECAY AND CONSERVATION LAWS

22. *Electron capture by* $^{106}\mathrm{In}$.

Solution Referring to the electron capture equation, ${}^{A}_{Z}X_{N} + e^{-} \rightarrow {}^{A}_{Z-1}Y_{N+1} + v_{e}$, we need to calculate the values of Z and N. From the periodic table we know that indium has $Z = 49$ and the element with $Z = 48$ is cadmium. Using the equation $A = N + Z$ we know that $N = A\text{-}Z = 106\text{-}49 = 57$ for indium and $N = 58$ for cadmium. Putting this all together gives $\underline{{}^{106}_{49}\mathrm{In}_{57} + e^{-} \rightarrow {}^{106}_{48}\mathrm{Cd}_{58} + v_{e}}$

28. α *decay producing* $^{208}\mathrm{Pb}$. *The parent nuclide is in the decay series produced by* $^{232}\mathrm{Th}$, *the only naturally occurring isotope of thorium.*

Solution Since we know that $^{208}\mathrm{Pb}$ is the product of an α decay, ${}^{A}_{Z}X_{N} \rightarrow {}^{A-4}_{Z-2}Y_{N-2} + {}^{4}_{2}\mathrm{He}_{2}$ tells us that $A - 4 = 208$, and since $Z - 2 = 82$ from the periodic table, we then know that $N - 2 = 208 - 82 = 126.$ So for the parent nucleus we have $A = 212, Z = 84$ and $N = 128$. Therefore from the periodic table the parent nucleus is $^{212}\mathrm{Po}$ and the decay is $\underline{{}^{212}_{84}\mathrm{Po}_{128} \rightarrow {}^{208}_{82}\mathrm{Pb}_{126} + {}^{4}_{2}\mathrm{He}_{2}}$

34. *A rare decay mode has been observed in which* $^{222}\mathrm{Ra}$ *emits a* $^{14}\mathrm{C}$ *nucleus. (a) The decay equation is* $^{222}\mathrm{Ra} \rightarrow {}^{A}\mathrm{X} + {}^{14}\mathrm{C}$. *Identify the nuclide* $^{A}\mathrm{X}$. *(b) Find the energy emitted in the decay. The mass of* $^{222}\mathrm{Ra}$ *is 222.015353 u.*

Solution (a) The decay is ${}^{222}_{88}\mathrm{Ra}_{134} \rightarrow {}^{A}_{Z}\mathrm{X}_{N} + {}^{14}_{6}\mathrm{C}_{8}$, so we know that:
$A = 222 - 14 = 208; Z = 88 - 6 = 82$ and $N = A - Z = 208 - 82 = 126,$ so from the periodic table the element is lead and $\underline{\mathrm{X} = {}^{208}_{82}\mathrm{Pb}_{126}}$

(b) $\Delta m = m\left({}^{222}_{88}\text{Ra}_{134}\right) - m\left({}^{208}_{82}\text{Pb}_{126}\right) - m\left({}^{14}_{6}\text{C}_{8}\right)$

$= 222.015353\text{ u} - 207.976627\text{ u} - 14.003241\text{ u} = 3.5485\times10^{-2}\text{ u}$

$$E = \Delta mc^2 = \left(3.5485\times10^{-2}\text{ u}\right)\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right)c^2 = \underline{33.05\text{ MeV}}$$

40. *(a) Write the complete β^+ decay equation for ${}^{11}\text{C}$. (b) Calculate the energy released in the decay. The masses of ${}^{11}\text{C}$ and ${}^{11}\text{B}$ are 11.011433 and 11.009305 u, respectively.*

Solution (a) Using ${}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z-1}Y_{N+1} + \beta^+ + v_e$ and the periodic table we can get the complete decay equation: $\underline{{}^{11}_{6}\text{C}_5 \rightarrow {}^{11}_{5}\text{B}_6 + \beta^+ + v_e}$

(b) To calculate the energy emitted we first need to calculate the change in mass. The change in mass is the mass of the parent minus the mass of the daughter and the positron it created. The mass given for the parent and the daughter, however, are given for the neutral atoms. So the carbon has one additional electron than the boron and we must subtract an additional mass of the electron to get the correct change in mass.

$\Delta m = m\left({}^{11}\text{C}\right) - \left[m\left({}^{11}\text{B}\right) + 2m_e\right]$

$= 11.011433\text{ u} - \left[11.009305\text{ u} + 2\left(0.00054858\text{ u}\right)\right] = 1.031\times10^{-3}\text{ u}$

$$E = \Delta mc^2 = \left(1.031\times10^{-3}\text{ u}\right)\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right)c^2 = \underline{0.9602\text{ MeV}}$$

31.5 HALF-LIFE AND ACTIVITY

46. *(a) Calculate the activity R in curies of 1.00 g of ${}^{226}\text{Ra}$. (b) Discuss why your answer is not exactly 1.00 Ci, given that the curie was originally supposed to be exactly the activity of a gram of radium.*

Solution (a) First we must determine the number of atoms for radium. We use the molar mass of 226 g/mol to get: $N = (1.00\ \text{g})\left(\dfrac{\text{mol}}{226\ \text{g}}\right)\dfrac{6.022\times10^{23}\ \text{atoms}}{\text{mol}} = 2.6646\times10^{21}\ \text{atoms}$

Then using the equation $R = \dfrac{0.693N}{t_{1/2}}$, where we know the half life of ^{226}Ra is $1.6\times10^{3}\ \text{y}$,

$$R = \frac{(0.693)(2.6646\times10^{21})}{1.6\times10^{3}\ \text{y}}\times\left(\frac{1\ \text{y}}{3.156\times10^{7}\ \text{s}}\right) = 3.66\times10^{10}\ \text{Bq}$$

$$\left(\frac{\text{Ci}}{3.70\times10^{16}\ \text{Bq}}\right) = \underline{0.988\ \text{Ci}}$$

(b) The half life of ^{226}Ra is more accurately known than it was when the Ci unit was established.

52. *^{50}V has one of the longest known radioactive half-lives. In a difficult experiment, a researcher found that the activity of 1.00 kg of ^{50}V is 1.75 Bq. What is the half-life in years?*

Solution Using the equation $R = \dfrac{0.693N}{t_{1/2}}$, we can write the activity in terms of the half-life, the molar mass, M, and the mass of the sample, m:

$$R = \frac{0.693N}{t_{1/2}} = \frac{(0.693)\left[(6.02\times10^{23}\ \text{atoms/mol})/M\right]m}{t_{1/2}}$$

From the periodic table, $M = 50.94\ \text{g/mol}$, so

$$t_{1/2} = \frac{(0.693)(6.02\times10^{23}\ \text{atoms/mol})(1000\ \text{g})}{(50.94\ \text{g/mol})(1.75\ \text{Bq})}$$

$$= 4.681\times10^{24}\ \text{s}\left(\frac{1\ \text{y}}{3.156\times10^{7}\ \text{s}}\right) = \underline{1.48\times10^{17}\ \text{y}}$$

58. *The β^- particles emitted in the decay of ^3H (tritium) interact with matter to create light in a glow-in-the-dark exit sign. At the time of manufacture, such a sign contains 15.0 Ci of ^3H. (a) What is the mass of the tritium? (b) What is its activity 5.00 y after manufacture?*

Solution Using the equation $R=\dfrac{0.693N}{t_{1/2}}$, we can write the activity in terms of the half-life, the atomic mass, M and the mass of the sample m:

(a) $R=\dfrac{0.693N}{t_{1/2}}=\dfrac{(0.693)(m/M)}{t_{1/2}}$. The atomic mass of tritium (from Appendix A) is

$M=3.016050\ \text{u}\left(\dfrac{1.6605\times10^{-27}\ \text{kg}}{1\ \text{u}}\right)=5.0082\times10^{-27}\ \text{kg/atom}$, and the half-life is 12.33 y (from Appendix B), so we can determine the original mass of tritium:

$m=\dfrac{Rt_{1/2}M}{0.693}$ or

$$m=\frac{(15.0\ \text{Ci})(12.33\ \text{y})(5.0082\times10^{-27}\ \text{kg})M}{0.693}\left(\frac{3.70\times10^{10}\ \text{Bq}}{\text{Ci}}\right)\left(\frac{3.156\times10^{7}\ \text{s}}{\text{y}}\right)$$

$$=1.56\times10^{-6}\ \text{kg}=\underline{1.56\ \text{mg}}$$

(b) $R=R_0e^{-\lambda t}=R_0\exp\left(-\dfrac{0.693}{t_{1/2}}t\right)=(15.0\ \text{Ci})\exp\left(-\dfrac{0.693(5.00\ \text{y})}{12.33\ \text{y}}\right)=\underline{11.3\ \text{Ci}}$

31.6 BINDING ENERGY

71. *^{209}Bi is the heaviest stable nuclide, and its BE/A is low compared with medium-mass nuclides. Calculate BE/A, the binding energy per nucleon, for ^{209}Bi and compare it with the approximate value obtained from the graph in Figure 31.27.*

Solution Dividing $\text{BE}=\{[Zm(^1\text{H})+Nm_n]-m(^AX)\}c^2$ by A gives the binding energy per nucleon: $\frac{\text{BE}}{A}=\frac{\left[Zm\left({}^{1}\text{H}\right)+Nm_n-m\left({}^{209}_{83}\text{Bi}_{126}\right)\right]c^2}{A}$.

We know that $Z=83$ (from the periodic table), $N=A-Z=126$ and the mass of the ${}^{209}\text{Bi}$ nuclide is 208.908374 u (from Appendix A) so that:

$$\frac{\text{BE}}{A}=\frac{\left[83(1.007825\text{ u})+126(1.008665\text{ u})-208.980374\text{ u}\right]}{209}\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right)c^2$$
$$=\underline{7.848\text{ MeV/nucleon}}$$

This binding energy per nucleon is approximately the value given in the graph.

76. ***Unreasonable Results*** *A particle physicist discovers a neutral particle with a mass of 2.02733 u that he assumes is two neutrons bound together. (a) Find the binding energy. (b) What is unreasonable about this result? (c) What assumptions are unreasonable or inconsistent?*

Solution (a) $\text{BE}=\left[2m_n-m(\text{particle})\right]c^2=\left[2(1.008665\text{ u})-2.02733\text{ u}\right]\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right)c^2$

$$=\underline{-9.315\text{ MeV}}$$

(b) The binding energy cannot be negative; the nucleons would not stay together.

(c) The particle cannot be made from two neutrons.

31.7 TUNNELING

78. ***Integrated Concepts*** *A 2.00-T magnetic field is applied perpendicular to the path of charged particles in a bubble chamber. What is the radius of curvature of the path of a 10 MeV proton in this field? Neglect any slowing along its path.*

Solution Using the equation $r=\frac{mv}{qB}$, we can determine the radius of a moving charge in a magnetic field. First, we need to determine the velocity of the proton. Since the energy of the proton (10.0 MeV) is substantially less than the rest mass energy of the proton (938 MeV), we know the velocity is non-relativistic and that $E = \frac{1}{2}mv^2$.

Therefore,

$$v = \left(\frac{2E}{m}\right)^{1/2} = \left(\frac{2 \cdot 10.0\text{ MeV}}{938.27\text{ MeV}/\text{c}^2}\right)^{1/2} = (0.1460)(2.998 \times 10^7\text{ m/s}).\text{ So,}$$

$$r = \frac{mv}{qB} = \frac{(1.6726 \times 10^{-27}\text{ kg})(4.377 \times 10^7\text{ m/s})}{(1.602 \times 10^{-19}\text{ C})(2.00\text{ T})} 0.228\text{ m} = \underline{22.8\text{ cm}}$$