CHAPTER 30: ATOMIC PHYSICS

30.1 DISCOVERY OF THE ATOM

1. *Using the given charge-to-mass ratios for electrons and protons, and knowing the magnitudes of their charges are equal, what is the ratio of the proton's mass to the electron's? (Note that since the charge-to-mass ratios are given to only three-digit accuracy, your answer may differ from the accepted ratio in the fourth digit.)*

Solution   We can calculate the ratio of the masses by taking the ratio of the charge to mass

ratios given: $\dfrac{q}{m_e} = 1.76 \times 10^{11}$ C/kg   and $\dfrac{q}{m_p} = 9.57 \times 10^{7}$ C/kg , so that

$$\frac{m_p}{m_e} = \frac{q/m_e}{q/m_p} = \frac{1.76 \times 10^{11} \text{ C/kg}}{9.57 \times 10^{7} \text{ C/kg}} = 1839 = 1.84 \times 10^{3} \ .$$

The actual mass ratio is: $\dfrac{m_p}{m_e} = \dfrac{1.6726 \times 10^{-27} \text{ kg}}{9.1094 \times 10^{-31} \text{ kg}} = 1836 = 1.84 \times 10^{3}$, so to three

digits, the mass ratio is correct.

30.3 BOHR'S THEORY OF THE HYDROGEN ATOM

12. *A hydrogen atom in an excited state can be ionized with less energy than when it is in its ground state. What is n for a hydrogen atom if 0.850 eV of energy can ionize it?*

Solution   Using $E_n = \dfrac{-13.6 \text{ eV}}{n^2}$, we can determine the value for n, given the ionization energy:

$$n = \sqrt{\frac{-13.6 \text{ eV}}{E_n}} = \left(\frac{-13.6 \text{ eV}}{-0.85 \text{ eV}}\right)^{1/2} = 4.0 = \underline{4}$$

(Remember that n must be an integer.)

18.   *(a) Which line in the Balmer series is the first one in the UV part of the spectrum? (b) How many Balmer series lines are in the visible part of the spectrum? (c) How many are in the UV?*

Solution (a) We know that the UV range is from $\lambda = 10\text{ nm}$ to approximately $\lambda = 380\text{ nm}$. Using

the equation $\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_f^2} - \dfrac{1}{n_i^2}\right)$, where $n_f = 2$ for the Balmer series, we can solve

for n_i. Finding a common denominator gives $\dfrac{1}{\lambda R} = \dfrac{n_i^2 - n_f^2}{n_i^2 n_f^2}$, so that

$n_i^2 n_f^2 = \lambda R(n_i^2 - n_f^2)$, or $n_i = n_f \sqrt{\dfrac{\lambda R}{\lambda R - n_f^2}}$. The first line will be for the lowest

energy photon, and therefore the largest wavelength, so setting $\lambda = 380\text{ nm}$

gives $n_i = 2\sqrt{\dfrac{(3.80\times10^{-7}\text{ m})(1.097\times10^7\text{ m}^{-1})}{(3.80\times10^{-7}\text{ m})(1.097\times10^7\text{ m}^{-1}) - 4}} = 9.94 \Rightarrow n_i = \underline{10}\,$will be the first.

(b) Setting $\lambda = 760\text{ nm}$ allows us to calculate the smallest value for n_i in the visible

range: $n_i = 2\sqrt{\dfrac{(7.60\times10^{-7}\text{ m})(1.097\times10^7\text{ m}^{-1})}{(7.60\times10^{-7}\text{ m})(1.097\times10^7\text{ m}^{-1}) - 4}} = 2.77 \Rightarrow n_i = 3$ so $n_i = 3\text{ to }9\,$are

visible, or <u>7 lines are in the visible range</u>.

(c) The smallest λ in the Balmer series would be for $n_i = \infty$, which corresponds to a value of:

$\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_f^2} - \dfrac{1}{n_i^2}\right) = \dfrac{R}{n_f^2} \Rightarrow \lambda = \dfrac{n_f^2}{R} = \dfrac{4}{1.097\times10^7\text{ m}^{-1}} = 3.65\times10^{-7}\text{ m} = 365\text{ nm}$, which

is in the ultraviolet. Therefore, there are <u>an infinite number of Balmer lines in the ultraviolet</u>. All lines from $n_i = 10\text{ to }\infty$ fall in the ultraviolet part of the spectrum.

23.   *1. Verify Equations* $r_n = \dfrac{n^2}{Z}a_B$ *and* $a_B = \dfrac{h^2}{4\pi^2 m_e k q_e^2} = 0.529\times10^{-10}\text{ m}$ *using the approach stated in the text. That is, equate the Coulomb and centripetal forces and then insert an expression for velocity from the condition for angular momentum quantization.*

Solution

Using $F_{\text{coulomb}} = F_{\text{centripetal}} \Rightarrow \dfrac{kZq_e^2}{r_n^2} = \dfrac{m_e v^2}{r_n}$, so that $r_n = \dfrac{kZq_e^2}{m_e v^2} = \dfrac{kZq_e^2}{m_e} \dfrac{1}{v^2}$.

Since $m_e v r_n = n\dfrac{h}{2\pi}$, we can substitute for the velocity giving: $r_n = \dfrac{kZq_e^2}{m_e} \cdot \dfrac{4\pi^2 m_e^2 r_n^2}{n^2 h^2}$

so that $r_n = \dfrac{n^2}{Z} \dfrac{h^2}{4\pi^2 m_e kq_e^2} = \dfrac{n^2}{Z} a_B$, where $a_B = \dfrac{h^2}{4\pi^2 m_e kq_e^2}$.

30.4 X RAYS: ATOMIC ORIGINS AND APPLICATIONS

26.   *A color television tube also generates some x rays when its electron beam strikes the screen. What is the shortest wavelength of these x rays, if a 30.0-kV potential is used to accelerate the electrons? (Note that TVs have shielding to prevent these x rays from exposing viewers.)*

Solution

Using the equations $E = qV$ and $E = \dfrac{hc}{\lambda}$ gives $E = qV = \dfrac{hc}{\lambda}$, which allows us to calculate the wavelength:

$$\lambda = \frac{hc}{qV} = \frac{\left(6.626 \times 10^{-34}\ \text{J} \cdot \text{s}\right)\left(2.998 \times 10^8\ \text{m/s}\right)}{\left(1.602 \times 10^{-19}\ \text{C}\right)\left(3.00 \times 10^4\ \text{V}\right)} = \underline{4.13 \times 10^{-11}\ \text{m}}$$

30.5 APPLICATIONS OF ATOMIC EXCITATIONS AND DE-EXCITATIONS

33.   *(a) What energy photons can pump chromium atoms in a ruby laser from the ground state to its second and third excited states? (b) What are the wavelengths of these photons? Verify that they are in the visible part of the spectrum.*

Solution   (a) From Figure 30.64, we see that it would take 2.3 eV photons to pump chromium atoms into the second excited state. Similarly, it would take 3.0 eV photons to pump chromium atoms into the third excited state.

(b) $\lambda_2 = \dfrac{hc}{E} = \dfrac{1.24 \times 10^{-6} \text{ eV} \cdot \text{m}}{2.3 \text{ eV}} = 5.39 \times 10^{-7} \text{ m} = \underline{5.4 \times 10^2 \text{ nm}}$, which is yellow-green.

$\lambda_2 = \dfrac{hc}{E} = \dfrac{1.24 \times 10^{-6} \text{ eV} \cdot \text{m}}{3.0 \text{ eV}} = 4.13 \times 10^{-7} \text{ m} = \underline{4.1 \times 10^2 \text{ nm}}$, which is blue-violet.

30.8 QUANTUM NUMBERS AND RULES

40. *(a) What is the magnitude of the angular momentum for an $l = 1$ electron? (b) Calculate the magnitude of the electron's spin angular momentum. (c) What is the ratio of these angular momenta?*

Solution (a) Using the equation $L = \sqrt{\ell(\ell+1)} \; \dfrac{h}{2\pi}$, we can calculate the angular momentum of

an $\ell = 1$ electron: $L = \sqrt{\ell(\ell+1)} \; \dfrac{h}{2\pi} = \sqrt{1(2)} \left(\dfrac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi} \right) = \underline{1.49 \times 10^{-34} \text{ J} \cdot \text{s}}$

(b) Using the equation $S = \sqrt{s(s+1)} \; \dfrac{h}{2\pi}$, we can determine the electron's spin

angular momentum, since $s = \frac{1}{2}$:

$S = \sqrt{s(s+1)} \; \dfrac{h}{2\pi} = \sqrt{\dfrac{1}{2}\left(\dfrac{3}{2}\right)} \; \dfrac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi} = \underline{9.13 \times 10^{-35} \text{ J} \cdot \text{s}}$

(c) $\dfrac{L}{S} = \dfrac{\sqrt{\ell(\ell+1)} \; h/2\pi}{\sqrt{s(s+1)} \; h/2\pi} = \dfrac{\sqrt{2}}{\sqrt{3/4}} = \underline{1.63}$

30.9 THE PAULI EXCLUSION PRINCIPLE

55. ***Integrated Concepts*** *Calculate the velocity of a star moving relative to the earth if you observe a wavelength of 91.0 nm for ionized hydrogen capturing an electron directly into the lowest orbital (that is, a $n_i = \infty$ to $n_f = 1$, or a Lyman series transition).*

Solution   We will use the equation $\Delta E = E_\mathrm{f} - E_\mathrm{i}$ to determine the speed of the star, since we
are given the observed wavelength. We first need the source wavelength:

$$\Delta E = E_\mathrm{f} - E_\mathrm{i} = \frac{hc}{\lambda_\mathrm{s}} = \left(-\frac{Z^2}{n_\mathrm{f}^2}E_0\right) - \left(-\frac{Z^2}{n_\mathrm{i}^2}E_0\right) = 0 - \left[-\frac{1^2}{1^2}(13.6\,\mathrm{eV})\right] = 13.6\,\mathrm{eV},$$

so that $\lambda_\mathrm{s} = \dfrac{hc}{\Delta E} = \dfrac{1.24\times10^3\,\mathrm{eV\cdot nm}}{13.6\,\mathrm{eV}} = 91.2\,\mathrm{nm}$. Therefore, using $\lambda_\mathrm{obs} = \lambda_\mathrm{s}\sqrt{\dfrac{1+v/c}{1-v/c}}$,

we have $\dfrac{1+v/c}{1-v/c} = \dfrac{\lambda_\mathrm{obs}^2}{\lambda_\mathrm{s}^2}$, so that $1 + \dfrac{v}{c} = \dfrac{\lambda_\mathrm{obs}^2}{\lambda_\mathrm{s}^2}\left(1-\dfrac{v}{c}\right)$ and thus,

$$\frac{v}{c} = \frac{\lambda_\mathrm{obs}^2/\lambda_\mathrm{s}^2 - 1}{\lambda_\mathrm{obs}^2/\lambda_\mathrm{s}^2 + 1} = \frac{(91.0\,\mathrm{nm}/91.2\,\mathrm{nm})^2 - 1}{(91.0\,\mathrm{nm}/91.2\,\mathrm{nm})^2 + 1} = -2.195\times10^{-3}.$$

So, $v = (-2.195\times10^{-3})(2.998\times10^8\,\mathrm{m/s}) = -6.58\times10^5\,\mathrm{m/s}$. Since v is negative, the star
is moving toward the earth at a speed of $\underline{6.58\times10^5\,\mathrm{m/s.}}$

59.   ***Integrated Concepts** Find the value of l, the orbital angular momentum quantum
number, for the moon around the earth. The extremely large value obtained implies
that it is impossible to tell the difference between adjacent quantized orbits for
macroscopic objects.*

Solution   From the definition of velocity, $v = \dfrac{d}{t}$, we can get an expression for the velocity in

terms of the period of rotation of the moon: $v = \dfrac{2\pi R}{T}$. Then, from $L = I\omega$ for a point

object we get the angular momentum: $L = I\omega = mR^2\omega = mR^2\dfrac{v}{R} = mRv$. Substituting

for the velocity and setting equal to $L = \sqrt{\ell(\ell+1)}\,\dfrac{h}{2\pi}$ gives:

$L = mvR = \dfrac{2\pi\,mR^2}{T} = \sqrt{\ell(\ell+1)}\,\dfrac{h}{2\pi}$. Since l is large : $\dfrac{2\pi\,mR^2}{T} \approx \dfrac{\ell h}{2\pi}$, so

$$\ell = \frac{4\pi^2\,mR^2}{Th} = \frac{4\pi^2\,(7.35\times10^{22}\,\mathrm{kg})(3.84\times10^8\,\mathrm{m})^2}{(2.36\times10^6\,\mathrm{s})(6.63\times10^{-34}\,\mathrm{J.s})} = \underline{2.73\times10^{68}}\ .$$

66.   ***Integrated Concepts*** *A pulsar is a rapidly spinning remnant of a supernova. It rotates on its axis, sweeping hydrogen along with it so that hydrogen on one side moves toward us as fast as 50.0 km/s, while that on the other side moves away as fast as 50.0 km/s. This means that the EM radiation we receive will be Doppler shifted over a range of* $\pm 50.0 \text{ km/s}$. *What range of wavelengths will we observe for the 91.20-nm line in the Lyman series of hydrogen? (Such line broadening is observed and actually provides part of the evidence for rapid rotation.)*

Solution   We will use the Doppler shift equation to determine the observed wavelengths for the Doppler shifted hydrogen line. First, for the hydrogen moving away from us, we use $u = +50.0 \text{ km/s},$ so that:

$$\lambda_{obs} = (91.20 \text{ nm})\sqrt{\frac{1+\left(5.00\times10^4 \text{ m/s}/2.998\times10^8 \text{ m/s}\right)}{1-\left(5.00\times10^4 \text{ m/s}/2.998\times10^8 \text{ m/s}\right)}} = 91.22 \text{ nm}$$

Then, for the hydrogen moving towards us, we use $u = -50.0 \text{ km/s},$ so that:

$$\lambda_{obs} = (91.20 \text{ nm})\sqrt{\frac{1-\left(5.00\times10^4 \text{ m/s}/2.998\times10^8 \text{ m/s}\right)}{1+\left(5.00\times10^4 \text{ m/s}/2.998\times10^8 \text{ m/s}\right)}} = 91.18 \text{ nm}$$

The range of wavelengths is from $\underline{91.18 \text{ nm}}$ to $\underline{91.22 \text{ nm}}$.