CHAPTER 29: INTRODUCTION TO QUANTUM PHYSICS

29.1 QUANTIZATION OF ENERGY

1. *A LiBr molecule oscillates with a frequency of 1.7×10^{13} Hz. (a) What is the difference in energy in eV between allowed oscillator states? (b) What is the approximate value of n for a state having an energy of 1.0 eV?*

Solution (a) $\Delta E = hf = \left(6.63 \times 10^{-34} \text{ J} \cdot \text{s}\right)\left(1.7 \times 10^{13} \text{ s}^{-1}\right) = 1.127 \times 10^{-20}$ J

so that $\left(1.127 \times 10^{-20} \text{ J}\right)\left(\dfrac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) = 7.04 \times 10^{-2}$ eV $= \underline{7.0 \times 10^{-2} \text{ eV}}$

(b) Using the equation $E = nhf$, we can solve for n:

$$n = \frac{E}{hf} - \frac{1}{2} = \frac{\left(1.0 \text{ eV}\right)\left(1.60 \times 10^{-19} \text{ J/eV}\right)}{\left(6.63 \times 10^{-34} \text{ J} \cdot \text{s}\right)\left(1.7 \times 10^{13} \text{ s}^{-1}\right)} - \frac{1}{2} = 13.7 = \underline{14}$$

29.2 THE PHOTOELECTRIC EFFECT

7. *Calculate the binding energy in eV of electrons in aluminum, if the longest-wavelength photon that can eject them is 304 nm.*

Solution The longest wavelength corresponds to the shortest frequency, or the smallest energy. Therefore, the smallest energy is when the kinetic energy is zero. From the equation $\text{KE} = hf - \text{BE} = 0$, we can calculate the binding energy (writing the frequency in terms of the wavelength):

$$\text{BE} = hf = \frac{hc}{\lambda} \Rightarrow$$

$$\text{BE} = \frac{hc}{\lambda} = \frac{\left(6.63 \times 10^{-34} \text{ J} \cdot \text{s}\right)\left(3.00 \times 10^{8} \text{ m/s}\right)}{\left(3.04 \times 10^{-7} \text{ m}\right)}$$

$$= 6.543 \times 10^{-19} \text{ J} \times \left(\frac{1.000 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) = \underline{4.09 \text{ eV}}$$

13.   *Find the wavelength of photons that eject 0.100-eV electrons from potassium, given that the binding energy is 2.24 eV. Are these photons visible?*

Solution   Using the equations $KE = hf - BE$ and $c = \lambda f$ we see that $hf = \dfrac{hc}{\lambda} = BE + KE$ so that we can calculate the wavelength of the photons in terms of energies:

$$\lambda = \frac{hc}{BE + KE} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{2.24 \text{ eV} + 0.100 \text{ eV}} \times \left(\frac{1.000 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right)$$
$$= 5.313 \times 10^{-7} \text{ m} = \underline{531 \text{ nm}}.$$

Yes, these photons are visible.

19.   ***Unreasonable Results*** *(a) What is the binding energy of electrons to a material from which 4.00-eV electrons are ejected by 400-nm EM radiation? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?*

Solution   (a) We want to use the equation $KE = hf - BE$ to determine the binding energy, so we first need to determine an expression of hf. Using $E = hf$, we know:

$$hf = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{4.00 \times 10^{-7} \text{ m}}$$
$$= (4.966 \times 10^{-19} \text{ J}) \left(\frac{1 \text{ eV}}{(1.602 \times 10^{-19} \text{ J})} \right) = 3.100 \text{ eV}$$

and since $KE = hf - BE$: $BE = hf - KE = 3.100 \text{ eV} - 4.00 \text{ eV} = \underline{-0.90 \text{ eV}}$

(b) The binding energy is too large for the given photon energy.

(c) The electron's kinetic energy is too large for the given photon energy; it cannot be greater than the photon energy.

29.3 PHOTON ENERGIES AND THE ELECTROMAGNETIC SPECTRUM

21.   *(a) Find the energy in joules and eV of photons in radio waves from an FM station that has a 90.0-MHz broadcast frequency. (b) What does this imply about the number of photons per second that the radio station must broadcast?*

Solution (a) Using the equation $E = hf$ we can determine the energy of photons:

$$E = hf = (6.63 \times 10^{-34} \text{ J/s})(9.00 \times 10^8 \text{ s}^{-1}) = \underline{5.97 \times 10^{-26} \text{ J}}$$

$$= 5.97 \times 10^{-26} \text{ J} \left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right) = \underline{3.73 \times 10^{-7} \text{ eV}}$$

(b) This implies that a tremendous number of photons must be broadcast per second. In order to have a broadcast power of, say 50.0 kW, it would take

$$\frac{5.00 \times 10^4 \text{ J/s}}{5.97 \times 10^{-26} \text{ J/photon}} = \underline{8.38 \times 10^{29} \text{ photon/sec}}$$

24. *Do the unit conversions necessary to show that $hc = 1240 \text{ eV} \cdot \text{nm}$, as stated in the text.*

Solution Using the conversion for joules to electron volts and meters to nanometers gives:

$$hc = (6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s}) \left(\frac{10^9 \text{ nm}}{1 \text{ m}} \right) \left(\frac{1.00 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right) = \underline{1240 \text{ eV} \cdot \text{nm}}$$

33. *(a) If the power output of a 650-kHz radio station is 50.0 kW, how many photons per second are produced? (b) If the radio waves are broadcast uniformly in all directions, find the number of photons per second per square meter at a distance of 100 km. Assume no reflection from the ground or absorption by the air.*

Solution (a) We can first calculate the energy of each photon:

$$E_\gamma = hf = (6.63 \times 10^{-34} \text{ Js})(6.50 \times 10^5 \text{ s}^{-1}) = \underline{4.31 \times 10^{-28} \text{ J}}$$

Then using the fact that the broadcasting power is 50.0 kW, we can calculate the number of photons per second:

$$N = \frac{5.00 \times 10^4 \text{ J/s}}{4.31 \times 10^{-28} \text{ J/photon}} = 1.16 \times 10^{32} \text{ photon/s} = \underline{1.16 \times 10^{32} \text{ photon/s}}$$

(b) To calculate the flux of photons, we assume that the broadcast is uniform in all directions, so the area is the surface area of a sphere giving:

$$\Phi_N = \frac{N}{4\pi r^2} = \frac{1.16 \times 10^{32} \text{ photons/s}}{4\pi(1.00 \times 10^5 \text{ m})^2} = \underline{9.23 \times 10^{20} \text{ photons/s} \cdot \text{m}^2}$$

29.4 PHOTON MOMENTUM

40.   *(a) What is the wavelength of a photon that has a momentum of 5.00×10^{-29} kg·m/s?*
           (b) Find its energy in eV.

Solution   (a) Using the equation $p = \frac{h}{\lambda}$, we can solve for the wavelength of the photon:

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34} \text{ J.s}}{5.00 \times 10^{-29} \text{ kg} \cdot \text{m/s}} = 1.326 \times 10^{-5} \text{ m} = \underline{13.3 \ \mu\text{m}}$$

(b) Using the equation $p = \frac{E}{c}$, we can solve for the energy and then convert the units
to electron volts:

$$E = pc = (5.00 \times 10^{-29} \text{ kg} \cdot \text{m/s})(3.00 \times 10^8 \text{ m/s})$$
$$= 1.50 \times 10^{-20} \text{ J} \times \left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right) = \underline{9.38 \times 10^{-2} \text{ eV}}$$

46.   *Take the ratio of relativistic rest energy, $E = \gamma mc^2$, to relativistic momentum,*
           $p = \gamma mu$, and show that in the limit that mass approaches zero, you find $E/p = c$.

Solution   Beginning with the two equations $E = \gamma mc^2$ and, $p = \gamma mu$ gives $\frac{E}{p} = \frac{\gamma mc^2}{\gamma mu} = \frac{c^2}{u}$

As the mass of the particle approaches zero, its velocity u will approach c so that the
ratio of energy to momentum approaches $\lim_{m \to 0} \frac{E}{p} = \frac{c^2}{c} = c$, which is consistent

with the equation $p = \frac{E}{c}$ for photons.

29.6 THE WAVE NATURE OF MATTER

54.   *Experiments are performed with ultracold neutrons having velocities as small as 1.00 m/s. (a) What is the wavelength of such a neutron? (b) What is its kinetic energy in eV?*

Solution   (a) Using the equations $p = \dfrac{h}{\lambda}$ and $p = mv$ we can calculate the wavelength of the neutron:

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(1.675 \times 10^{-27} \text{ kg})(1.00 \text{ m/s})} = 3.956 \times 10^{-7} \text{ m} = \underline{396 \text{ nm}}$$

(b) Using the equation $KE = \dfrac{1}{2}mv^2$ we can calculate the kinetic energy of the neutron:

$$KE = \frac{1}{2}mv^2 = \frac{1}{2} \times (1.675 \times 10^{-27} \text{ kg})(1.00 \text{ m/s})^2$$

$$= 8.375 \times 10^{-28} \text{ J} \times \left(\frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} \right) = \underline{5.23 \times 10^{-9} \text{ eV}}$$

29.7 PROBABILITY: THE HEISENBERG UNCERTAINTY PRINCIPLE

66.   *A relatively long-lived excited state of an atom has a lifetime of 3.00 ms. What is the minimum uncertainty in its energy?*

Solution   Using the equation $\Delta E \Delta t = \dfrac{h}{4\pi}$, we can determine the minimum uncertainty for its energy:

$$\Delta E \geq \frac{h}{4\pi \Delta t} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{4\pi (3.00 \times 10^{-3} \text{ s})} = 1.759 \times 10^{-32} \text{ J} \times \left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J.}} \right) = \underline{1.10 \times 10^{-13} \text{ eV}}$$

29.8 THE PARTICLE-WAVE DUALITY REVIEWED

72.   ***Integrated Concepts*** *The 54.0-eV electron in* Example 29.7 *has a 0.167-nm wavelength. If such electrons are passed through a double slit and have their first maximum at an angle of* $25.0°$*, what is the slit separation* d *?*

Solution   Using the equation $d \sin \theta = m\lambda, m = 0,1,2....$, we can solve for the slit separation

when $m = 1$ for the first order maximum: $d = \dfrac{m\lambda}{\sin \theta} = \dfrac{(1)(0.167 \, nm)}{\sin 25.0°} = \underline{0.395 \, nm}$

78.   ***Integrated Concepts*** *(a) What is γ for a proton having an energy of 1.00 TeV, produced by the Fermilab accelerator? (b) Find its momentum. (c) What is the proton's wavelength?*

Solution   (a) Using the equation $E = \gamma mc^2$, we can find γ for 1.00 TeV proton:

$$\gamma = \frac{E}{mc^2} = \frac{\left(1.00 \times 10^{12} \, eV\right)\left(1.60 \times 10^{-19} \, J/eV\right)}{\left(1.6726 \times 10^{-27} \, kg\right)\left(3.00 \times 10^{8} \, m/s^2\right)^2} = 1.063 \times 10^3 = \underline{1.06 \times 10^3}$$

(b) $p = \gamma mc = \dfrac{E}{c} = \dfrac{(1.00 \times 10^{12} \, eV)(1.60 \times 10^{-19} \, J/eV)}{3.00 \times 10^8 \, m/s} = \underline{5.33 \times 10^{-16} \, kg \cdot m/s}$

(c) Using the equation $p = \dfrac{h}{\lambda}$, we can calculate the proton's wavelength:

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34} \, kg \cdot m/s}{5.33 \times 10^{-16} \, kg \cdot m/s} = \underline{1.24 \times 10^{-18} \, m}$$

83.   ***Integrated Concepts*** *One problem with x rays is that they are not sensed. Calculate the temperature increase of a researcher exposed in a few seconds to a nearly fatal accidental dose of x rays under the following conditions. The energy of the x-ray photons is 200 keV, and 4.00×10^{13} of them are absorbed per kilogram of tissue, the specific heat of which is $0.830 \, kcal/kg \cdot °C$. (Note that medical diagnostic x-ray machines* cannot *produce an intensity this great.)*

Solution   First, we know the amount of heat absorbed by 1.00 kg of tissue is equal to the number of photons times the energy each one carry, so:

$$Q = NE_\gamma = \left(4.00 \times 10^{13}\right)\left(2.00 \times 10^5 \, eV\right)\left(\frac{1.60 \times 10^{-19} \, J}{1 \, eV}\right) = \underline{1.28 \, J}$$

Next, using the equation $Q = mc\Delta t$, we can determine how much 1.00 kg tissue is

heated: $\Delta t = \dfrac{Q}{mc} = \dfrac{1.282 \, J}{(1.00 \, kg)(0.830 \, kcal/kg \cdot °C)(4186 \, J/kcal)} = \underline{3.69 \times 10^{-4} \, °C}$