

CHAPTER 28: SPECIAL RELATIVITY

28.2 SIMULTANEITY AND TIME DILATION

1. (a) What is γ if $v = 0.250c$? (b) If $v = 0.500c$?

Solution (a) Using the definition of γ , where $v = 0.250c$:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \left[1 - \frac{(0.250c)^2}{c^2}\right]^{-1/2} = \underline{1.0328}$$

(b) Again using the definition of γ , now where $v = 0.500c$:

$$\gamma = \left[1 - \frac{(0.500c)^2}{c^2}\right]^{-1/2} = 1.1547 = \underline{1.15}$$

Note that γ is unitless, and the results are reported to three digits to show the difference from 1 in each case.

6. A neutron lives 900 s when at rest relative to an observer. How fast is the neutron moving relative to an observer who measures its life span to be 2065 s?

Solution

Using $\Delta t = \gamma \Delta t_0$, where $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$, we see that:

$$\gamma = \frac{\Delta t}{\Delta t_0} = \frac{2065 \text{ s}}{900 \text{ s}} = 2.2944 = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}.$$

Squaring the equation gives $\gamma^2 = \left(\frac{c^2 - v^2}{c^2}\right)^{-1} = \frac{c^2}{c^2 - v^2}$

Cross-multiplying gives $c^2 - v^2 = \frac{c^2}{\gamma^2}$, and solving for speed finally gives:

$$v = \sqrt{c^2 - \frac{c^2}{\gamma^2}} = c \left(1 - \frac{1}{\gamma^2} \right)^{1/2} = c \left[1 - \frac{1}{(2.2944)^2} \right]^{1/2} = 0.90003c = \underline{0.900c}$$

11. **Unreasonable Results** (a) Find the value of γ for the following situation. An Earth-bound observer measures 23.9 h to have passed while signals from a high-velocity space probe indicate that 24.0 h have passed on board. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Solution (a) Using the equation $\Delta t = \gamma \Delta t_0$, we can solve for γ :

$$\gamma = \frac{\Delta t}{\Delta t_0} = \frac{23.9 \text{ h}}{24.0 \text{ h}} = 0.9958 = \underline{0.996}$$

(b) γ cannot be less than 1.

(c) The earthbound observer must measure a longer time in the observer's rest frame than the ship bound observer. The assumption that time is longer in the moving ship is unreasonable.

28.3 LENGTH CONTRACTION

14. (a) How far does the muon in **Example 28.1** travel according to the Earth-bound observer? (b) How far does it travel as viewed by an observer moving with it? Base your calculation on its velocity relative to the Earth and the time it lives (proper time). (c) Verify that these two distances are related through length contraction $\gamma = 3.20$.

Solution Using the values given in **Example 28.1**:

$$\begin{aligned} \text{(a) } L_0 &= v\Delta t = (0.950c)(4.87 \times 10^{-6} \text{ s}) \\ &= (0.950c)(2.998 \times 10^8 \text{ m/s})(4.87 \times 10^{-6} \text{ s}) = 1.386 \times 10^3 \text{ m} = \underline{1.39 \text{ km}} \end{aligned}$$

$$\text{(b) } L_0 = v\Delta t = (0.950c)(2.998 \times 10^8 \text{ m/s})(1.52 \times 10^{-6} \text{ s}) = 4.329 \times 10^2 \text{ m} = \underline{0.433 \text{ km}}$$

$$\text{(c) } L = \frac{L_0}{\gamma} = \frac{1.386 \times 10^3 \text{ m}}{3.20} = 4.33 \times 10^2 \text{ m} = \underline{0.433 \text{ km}}$$

28.4 RELATIVISTIC ADDITION OF VELOCITIES

22. *If a spaceship is approaching the Earth at $0.100c$ and a message capsule is sent toward it at $0.100c$ relative to the Earth, what is the speed of the capsule relative to the ship?*

Solution Using the equation $u = \frac{v + u'}{1 + (vu'/c^2)}$, we can add the relativistic velocities:

$$u = \frac{v + u'}{1 + (vu'/c^2)} = \frac{0.100c + 0.100c}{1 + [(0.100c)(0.100c)/c^2]} = \underline{0.198c}$$

28. *When a missile is shot from one spaceship towards another, it leaves the first at $0.950c$ and approaches the other at $0.750c$. What is the relative velocity of the two ships?*

Solution We are given: $u = 0.750c$ and $u' = 0.950c$. We want to find v , starting with the equation $u = \frac{v + u'}{1 + (vu'/c^2)}$. First multiply both sides by the denominator:

$$u + v \frac{uu'}{c^2} = v + u', \text{ then solving for } v \text{ gives:}$$

$$v = \frac{u' - u}{(uu'/c^2) - 1} = \frac{0.950c - 0.750c}{[(0.750c)(0.950c)/c^2] - 1} = \underline{-0.696c}$$

The velocity v is the speed measured by the second spaceship, so the minus sign indicates the ships are moving apart from each other (v is in the opposite direction as u).

34. *(a) All but the closest galaxies are receding from our own Milky Way Galaxy. If a galaxy 12.0×10^9 ly away is receding from us at $0.900c$, at what velocity relative to us must we send an exploratory probe to approach the other galaxy at $0.990c$, as measured from that galaxy? (b) How long will it take the probe to reach the other galaxy as measured from the Earth? You may assume that the velocity of the other galaxy remains constant. (c) How long will it then take for a radio signal to be beamed back? (All of this is possible in principle, but not practical.)*

Solution Note that all answers to this problem are reported to 5 significant figures, to distinguish the results.

(a) We are given $v = -0.900c$ and $u = 0.990c$. Starting with the equation

$$u = \frac{v + u'}{1 + (vu'/c^2)}, \text{ we now want to solve for } u'. \text{ First multiply both sides by the}$$

denominator: $u + u' \frac{uv}{c^2} = v + u'$; then solving for the probe's speed gives:

$$u' = \frac{u - v}{1 - (uv/c^2)} = \frac{0.990c - (-0.900c)}{1 - [(0.990c)(-0.900c)/c^2]} = \underline{0.99947c}$$

(b) When the probe reaches the other galaxy, it will have traveled a distance (as seen on Earth) of $d = x_0 + vt$ because the galaxy is moving away from us. As seen from

$$\text{Earth, } u' = 0.9995c, \text{ so } t = \frac{d}{u'} = \frac{x_0 + vt}{u'}.$$

$$\text{Now, } u't = x_0 + vt \text{ and } t = \frac{x_0}{u' - v} = \frac{12.0 \times 10^9 \text{ ly}}{0.99947c - 0.900c} \times \frac{(1 \text{ y})c}{1 \text{ ly}} = \underline{1.2064 \times 10^{11} \text{ y}}$$

(c) The radio signal travels at the speed of light, so the return time t' is given by

$t' = \frac{d}{c} = \frac{x_0 + vt}{c}$, assuming the signal is transmitted as soon as the probe reaches the other galaxy. Using the numbers, we can determine the time:

$$t' = \frac{1.20 \times 10^{10} \text{ ly} + (0.900c)(1.2064 \times 10^{11} \text{ y})}{c} = \underline{1.2058 \times 10^{11} \text{ y}}$$

28.5 RELATIVISTIC MOMENTUM

39. *What is the velocity of an electron that has a momentum of $3.04 \times 10^{-21} \text{ kg} \cdot \text{m/s}$? Note that you must calculate the velocity to at least four digits to see the difference from c .*

Solution Beginning with the equation $p = \gamma mu = \frac{mu}{[1 - (u^2/c^2)]^{1/2}}$ we can solve for the speed u .

First cross-multiply and square both sides, giving $1 - \frac{u^2}{c^2} = \frac{m^2}{p^2} u^2$

Then, solving for u^2 gives $u^2 = \frac{1}{(m^2/p^2) + (1/c^2)} = \frac{p^2}{m^2 + (p^2/c^2)}$

Finally, taking the square root gives $u = \frac{p}{\sqrt{m^2 + (p^2/c^2)}}$.

Taking the values for the mass of the electron and the speed of light to five significant figures gives:

$$u = \frac{3.04 \times 10^{-21} \text{ kg} \cdot \text{m/s}}{\left\{ (9.1094 \times 10^{-31} \text{ kg})^2 + \left[(3.34 \times 10^{-21} \text{ kg} \cdot \text{m/s}) / (2.9979 \times 10^8 \text{ kg})^2 \right]^2 \right\}^{1/2}}$$

$$= \underline{2.988 \times 10^8 \text{ m/s}}$$

28.6 RELATIVISTIC ENERGY

48. (a) Using data from **Table 7.1**, calculate the mass converted to energy by the fission of 1.00 kg of uranium. (b) What is the ratio of mass destroyed to the original mass, $\Delta m / m$?

Solution (a) From **Table 7.1**, the energy released from the nuclear fission of 1.00 kg of uranium is $\Delta E = 8.0 \times 10^{13} \text{ J}$. So, $\Delta E_0 = E_{\text{released}} = \Delta mc^2$, we get

$$\Rightarrow \Delta m = \frac{\Delta E}{c^2} = \frac{(8.0 \times 10^{13} \text{ J})}{(3.00 \times 10^8 \text{ m/s})^2} = 8.89 \times 10^{-4} \text{ kg} = \underline{0.89 \text{ g}}$$

(b) To calculate the ratio, simply divide by the original mass:

$$\frac{\Delta m}{m} = \frac{(8.89 \times 10^{-4} \text{ kg})}{(1.00 \text{ kg})} = 8.89 \times 10^{-4} = \underline{8.9 \times 10^{-4}}$$

52. A π -meson is a particle that decays into a muon and a massless particle. The π -meson has a rest mass energy of 139.6 MeV, and the muon has a rest mass energy of 105.7 MeV. Suppose the π -meson is at rest and all of the missing mass goes into the muon's kinetic energy. How fast will the muon move?

Solution Using the equation $\text{KE}_{\text{rel}} = \Delta mc^2$, we can determine the kinetic energy of the muon by determining the missing mass: $\text{KE}_{\text{rel}} = \Delta mc^2 = (m_\pi - m_\mu)c^2 = (\gamma - 1)m_\mu c^2$

Solving for γ will give us a way of calculating the speed of the muon. From the

equation above, we see that:

$$\gamma = \frac{m_\pi - m_\mu}{m_\mu} + 1 = \frac{m_\pi - m_\mu - m_\mu}{m_\mu} = \frac{m_\pi}{m_\mu} = \frac{139.6 \text{ MeV}}{105.7 \text{ MeV}} = 1.32072.$$

Now, use $\gamma = \left[1 - (v^2/c^2)\right]^{-1/2}$ or $1 - \frac{v^2}{c^2} = \frac{1}{\gamma^2}$, so

$$v = c \sqrt{1 - \frac{1}{\gamma^2}} = c \sqrt{1 - \frac{1}{(1.32072)^2}} = \underline{0.6532c}$$

58. *Find the kinetic energy in MeV of a neutron with a measured life span of 2065 s, given its rest energy is 939.6 MeV, and rest life span is 900s.*

Solution From **Exercise 28.6**, we know that $\gamma = 2.2944$, so we can determine the kinetic energy of the neutron: $\text{KE}_{\text{rel}} = (\gamma - 1)mc^2 = (2.2944 - 1)(939.6 \text{ MeV}) = \underline{1216 \text{ MeV}}$.

64. *(a) Calculate the energy released by the destruction of 1.00 kg of mass. (b) How many kilograms could be lifted to a 10.0 km height by this amount of energy?*

Solution (a) Using the equation $E = mc^2$, we can calculate the rest mass energy of a 1.00 kg mass. This rest mass energy is the energy released by the destruction of that amount of mass:

$$E_{\text{released}} = mc^2 = (1.00 \text{ kg})(2.998 \times 10^8 \text{ m/s})^2 = 8.988 \times 10^{16} \text{ J} = \underline{8.99 \times 10^{16} \text{ J}}$$

(b) Using the equation $\text{PE} = mgh$, we can determine how much mass can be raised to

$$\text{a height of 10.0 km: } m = \frac{\text{PE}}{gh} = \frac{8.99 \times 10^{16} \text{ J}}{(9.80 \text{ m/s}^2)(10.0 \times 10^3 \text{ m})} = \underline{9.17 \times 10^{11} \text{ kg}}$$