# CHAPTER 27: WAVE OPTICS

## 27.1 THE WAVE ASPECT OF LIGHT: INTERFERENCE

1.      *Show that when light passes from air to water, its wavelength decreases to 0.750 times its original value.*

Solution    Using the equation $\lambda_n = \dfrac{\lambda}{n}$, we can calculate the wavelength of light in water. The

index of refraction for water is given in Table 25.1, so that $\lambda_n = \dfrac{\lambda}{n} = \dfrac{\lambda}{1.333} = \underline{0.750\,\lambda}$.

The wavelength of light in water is 0.750 times the wavelength in air.

## 27.3 YOUNG'S DOUBLE SLIT EXPERIMENT

7.      *Calculate the angle for the third-order maximum of 580-nm wavelength yellow light falling on double slits separated by 0.100 mm.*

Solution    Using the equation $d \sin\theta = m\lambda$ for $m = 0,1,2,3,......$ we can calculate the angle for $m = 3$, given the wavelength and the slit separation:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{(3)(5.80 \times 10^{-7}\ \text{m})}{1.00 \times 10^{-4}\ \text{m}}\right] = \underline{0.997°}$$

13.     *What is the highest-order maximum for 400-nm light falling on double slits separated by $25.0\,\mu\text{m}$ ?*

Solution    Using the equation $d \sin\theta = m\lambda$, we notice that the highest order occurs when

$\sin\theta = 1$, so the highest order is: $m = \dfrac{d}{\lambda} = \dfrac{2.50 \times 10^{-5}\ \text{m}}{4.00 \times 10^{-7}\ \text{m}} = 62.5$

Since *m* must be an integer, the highest order is then $\underline{m = 62.}$

19.        *Using the result of the problem above, calculate the distance between fringes for 633-nm light falling on double slits separated by 0.0800 mm, located 3.00 m from a screen as in Figure 27.56.*

Solution    From Exercise 25.18, we have an expression for the distance between the fringes , so that: $\Delta y = \dfrac{x\lambda}{d} = \dfrac{(3.00 \text{ m})(6.33 \times 10^{-7} \text{ m})}{8.00 \times 10^{-5} \text{ m}} = 2.37 \times 10^{-2} \text{ m} = \underline{2.37 \text{ cm}}$

## 27.4 MULTIPLE SLIT DIFFRACTION

25.        *Calculate the wavelength of light that has its second-order maximum at $45.0°$ when falling on a diffraction grating that has 5000 lines per centimeter.*

Solution    The second order maximum is constructive interference, so for diffraction gratings we use the equation $d \sin \theta = m\lambda$ for m = 0,1,2,3,... where the second order maximum has $m = 2$. Next, we need to determine the slit separation by using the fact that there are 5000 lines per centimeter: $d = \dfrac{1}{5000 \text{ slits/cm}} \times \dfrac{1 \text{ m}}{100 \text{ cm}} = 2.00 \times 10^{-6} \text{ m}$

So, since $\theta = 45.0°$, we can determine the wavelength of the light:

$\lambda = \dfrac{d \sin \theta}{m} = \dfrac{(2.00 \times 10^{-6} \text{ m})(\sin 45.0°)}{2} = 7.07 \times 10^{-3} \text{ m} = \underline{707 \text{ nm}}$

34.        *Show that a diffraction grating cannot produce a second-order maximum for a given wavelength of light unless the first-order maximum is at an angle less than $30.0°$.*

Solution    The largest possible second order occurs when $\sin \theta_2 = 1$. Using the equation $d \sin \theta_m = m\lambda$, we see that the value for the slit separation and wavelength are the same for the first and second order maximums, so that:

$d \sin \theta_1 = \lambda$ and $d \sin \theta_2 = 2\lambda$, so that: $\dfrac{\sin \theta_1}{\sin \theta_2} = \dfrac{1}{2}$

Now, since we know the maximum value for $\sin \theta_2$, we can solve for the maximum value for $\theta_1$: $\theta_1 = \sin\left(\dfrac{1}{2}\sin \theta_2\right)^{-1}$ so that $\theta_{1,max} = \sin\left(\dfrac{1}{2}\right)^{-1} = \underline{30.0°}$

37.        *(a) Show that a 30,000-line-per-centimeter grating will not produce a maximum for visible light. (b) What is the longest wavelength for which it does produce a first-order maximum? (c) What is the greatest number of lines per centimeter a diffraction grating can have and produce a complete second-order spectrum for visible light?*

Solution    (a) First we need to calculate the slit separation:

$$d = \frac{1\,\text{line}}{N} = \frac{1\,\text{line}}{30,000\,\text{lines/cm}} \times \frac{1\,\text{m}}{100\,\text{cm}} = 3.333 \times 10^{-7}\,\text{m} = 333.3\,\text{nm}.$$

Next, using the equation $d\sin\theta = m\lambda$, we see that the longest wavelength will be for $\sin\theta = 1$ and $m = 1$, so in that case, $d = \lambda = 333.3\,\text{nm}$, which is not visible.

(b) From part (a), we know that the longest wavelength is equal to the slit separation, or <u>333 nm</u>.

(c) To get the largest number of lines per cm and still produce a complete spectrum, we want the smallest slit separation that allows the longest wavelength of visible light to produce a second order maximum, so $\lambda_{max} = 760\,\text{nm}$ (see Example 27.3). For there to be a second order spectrum, $m = 2$ and $\sin\theta = 1$, so

$$d = 2\lambda_{max} = 2(760\,\text{nm}) = 1.52 \times 10^{-6}\,\text{m}$$

Now, using the technique in step (a), only in reverse:

$$N = \frac{1\,\text{line}}{d} = \frac{1\,\text{line}}{1.52 \times 10^{-6}\,\text{m}} \times \frac{1\,\text{m}}{100\,\text{cm}} = \underline{6.58 \times 10^{3}\,\text{lines/cm}}$$

41.        ***Unreasonable Results** (a) What visible wavelength has its fourth-order maximum at an angle of $25.0°$ when projected on a 25,000-line-per-centimeter diffraction grating? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?*

Solution    (a) For diffraction gratings, we use the equation $d\sin\theta = m\lambda$, for $m = 0,1,2,3,4,...$ where the fourth order maximum has $m = 4$. We first need to determine the slit separation by using the fact that there are 25,000 lines per centimeter:

$$d = \frac{1}{25,000\,\text{lines /cm}} \times \frac{1\,\text{m}}{100\,\text{cm}} = 4.00 \times 10^{-7}\,\text{m}$$

So, since $\theta = 25.0°$, we can determine the wavelength of the light:

$$\lambda = \frac{d\sin\theta}{m} = \frac{(4.00 \times 10^{-7}\,\text{m})(\sin 25.0°)}{4} = 4.226 \times 10^{-8}\,\text{m} = \underline{42.3\,\text{nm}}$$

(b) This wavelength is not in the visible spectrum.

(c) The number of slits in this diffraction grating is too large. Etching in integrated circuits can be done to a resolution of 50 nm, so slit separations of 400 nm are at the limit of what we can do today. This line spacing is too small to produce diffraction of light.

## 27.5 SINGLE SLIT DIFFRACTION

48.     *Calculate the wavelength of light that produces its first minimum at an angle of $36.9°$ when falling on a single slit of width $1.00\ \mu m$.*

Solution   Using the equation $D \sin \theta = m\lambda$, where $D$ is the slit width, we can determine the wavelength for the first minimum,

$$\lambda = \frac{D \sin \theta}{m} = \frac{\left(1.00 \times 10^{-6}\ \text{m}\right)\left(\sin 36.9°\right)}{1} = 6.004 \times 10^{-7}\ \text{m} = \underline{600\ \text{nm}}$$

54.     *A double slit produces a diffraction pattern that is a combination of single and double slit interference. Find the ratio of the width of the slits to the separation between them, if the first minimum of the single slit pattern falls on the fifth maximum of the double slit pattern. (This will greatly reduce the intensity of the fifth maximum.)*

Solution   The problem is asking us to find the ratio of $D$ to $d$. For the single slit, using the equation $D \sin \theta = n\lambda$, we have $n = 1$. For the double slit using the equation $d \sin \theta = m\lambda$ (because we have a maximum), we have $m = 5$. Dividing the single slit equation by double slit equation, where the angle and wavelength are the same gives: $\dfrac{D}{d} = \dfrac{n}{m} = \dfrac{1}{5} \Rightarrow \dfrac{D}{d} = \underline{0.200}$

So, the slit separation is five times the slit width.

55.     ***Integrated Concepts*** *A water break at the entrance to a harbor consists of a rock barrier with a 50.0-m-wide opening. Ocean waves of 20.0-m wavelength approach the opening straight on. At what angle to the incident direction are the boats inside the harbor most protected against wave action?*

Solution    We are looking for the first minimum for single slit diffraction because the 50.0 m
            wide opening acts as a single slit. Using the equation $D\sin\theta = m\lambda$, where $m = 1$, we
            can determine the angle for first minimum:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{D}\right) = \sin^{-1}\left[\frac{(1)(20.0\text{ m})}{50.0\text{ m}}\right] = 23.58° = \underline{23.6°}$$

Since the main peak for single slit diffraction is the main problem, a boat in the
harbor at an angle greater than this first diffraction minimum will feel smaller waves.
At the second minimum, the boat will not be affected by the waves at all:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{D}\right) = \sin^{-1}\left[\frac{(2)(20.0\text{ m})}{50.0\text{ m}}\right] = 53.13° = \underline{53.1°}$$

## 27.6 LIMITS OF RESOLUTION: THE RAYLEIGH CRITERION

62.         *The limit to the eye's acuity is actually related to diffraction by the pupil. (a) What is
            the angle between two just-resolvable points of light for a 3.00-mm-diameter pupil,
            assuming an average wavelength of 550 nm? (b) Take your result to be the practical
            limit for the eye. What is the greatest possible distance a car can be from you if you
            can resolve its two headlights, given they are 1.30 m apart? (c) What is the distance
            between two just-resolvable points held at an arm's length (0.800 m) from your eye?
            (d) How does your answer to (c) compare to details you normally observe in everyday
            circumstances?*

Solution    (a) Using Rayleigh's Criterion, we can determine the angle (in radians) that is just

resolvable : $\theta = 1.22\dfrac{\lambda}{D} = (1.22)\left(\dfrac{550\times10^{-9}\text{ m}}{3.00\times10^{-3}\text{ m}}\right) = 2.237\times10^{-4}\text{ rad} = \underline{2.24\times10^{-4}\text{ rad}}$

(b) The distance $s$ between two objects, a distance $r$ away, separated by an angle $\theta$
is $s = r\theta$, so $r = \dfrac{s}{\theta} = \dfrac{1.30\text{ m}}{2.237\times10^{-4}\text{ rad}} = 5.811\times10^{3}\text{ m} = \underline{5.81\text{ km}}$

(c) Using the same equation as in part (b):

$$s = r\theta = (0.800\text{ m})(2.237\times10^{-4}\text{ rad}) = 1.789\times10^{-4}\text{ m} = \underline{0.179\text{ mm}}$$

(d) Holding a ruler at arm's length, you can easily see the millimeter divisions; so you
can resolve details 1.0 mm apart. Therefore, you probably can resolve details 0.2
mm apart at arm's length.

## 27.7 THIN FILM INTERFERENCE

**73.**      *Find the minimum thickness of a soap bubble that appears red when illuminated by white light perpendicular to its surface. Take the wavelength to be 680 nm, and assume the same index of refraction as water.*

**Solution**   The minimum thickness will occur when there is one phase change, so for light incident perpendicularly, constructive interference first occurs when $2t = \dfrac{\lambda_n}{2} = \dfrac{\lambda}{2n}$.

So, using the index of refraction for water from Table 25.1:

$$t = \frac{\lambda}{4n} = \frac{6.80 \times 10^{-7}\ \text{m}}{(4)(1.33)} = 1.278 \times 10^{-7}\ \text{m} = \underline{128\ \text{nm}}$$

**79.**      *Figure 27.34 shows two glass slides illuminated by pure-wavelength light incident perpendicularly. The top slide touches the bottom slide at one end and rests on a 0.100-mm-diameter hair at the other end, forming a wedge of air. (a) How far apart are the dark bands, if the slides are 7.50 cm long and 589-nm light is used? (b) Is there any difference if the slides are made from crown or flint glass? Explain.*

**Solution**
(a)
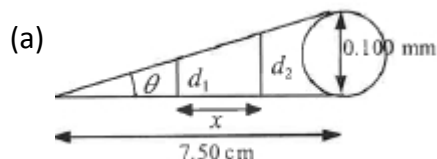


Two adjacent dark bands will have thickness differing by one wavelength, i.e.,

$$\lambda = d_2 - d_1,\ \text{and}\ \tan\theta = \frac{\text{hair diameter}}{\text{slide length}}\ \text{or}\ \theta = \tan^{-1}\!\left(\frac{1.00 \times 10^{-4}\ \text{m}}{0.075\ \text{m}}\right) = 0.076394°..$$

So, since $x\tan\theta = d_2 - d_1 = \lambda$, we see that

$$x = \frac{\lambda}{\tan\theta} = \frac{5.89 \times 10^{-7}\ \text{m}}{\tan(0.076394°)} = 4.418 \times 10^{-4}\ \text{m} = \underline{0.442\ \text{mm}}$$

(b) The material makeup of the slides is irrelevant because it is the path difference in the air between the slides that gives rise to interference.

## 27.8 POLARIZATION

**86.**      *If you have completely polarized light of intensity $150 \, \text{W} / \text{m}^2$, what will its intensity be after passing through a polarizing filter with its axis at an $89.0°$ angle to the light's polarization direction?*

**Solution**      Using the equation $I = I_0 \cos^2 \theta$, we can calculate the intensity:

$$I = I_0 \cos^2 \theta = \left(150 \, \text{W}/\text{m}^2 \right)\cos^2 \left(89.0°\right) = 4.57 \times 10^{-2} \, \text{W}/\text{m}^2 = \underline{45.7 \, \text{m} \, \text{W}/\text{m}^2}$$

**92.**      *What is Brewster's angle for light traveling in water that is reflected from crown glass?*

**Solution**      Using the equation $\tan \theta_b = \dfrac{n_2}{n_1}$, where $n_2$ is for crown glass and $n_1$ is for water (see

Table 25.1), Brewster's angle is $\theta_b = \tan^{-1}\left(\dfrac{n_2}{n_1}\right) = \tan^{-1}\left(\dfrac{1.52}{1.333}\right) = 48.75° = \underline{48.8°}$

At $48.8°$ (Brewster's angle) the reflected light is completely polarized.