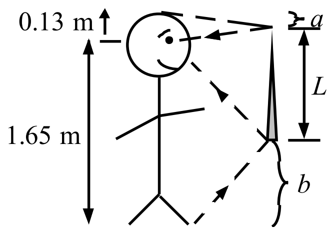
CHAPTER 25: GEOMETRIC OPTICS

25.1 THE RAY ASPECT OF LIGHT

1. *Suppose a man stands in front of a mirror as shown in Figure 25.50. His eyes are 1.65 m above the floor, and the top of his head is 0.13 m higher. Find the height above the floor of the top and bottom of the smallest mirror in which he can see both the top of his head and his feet. How is this distance related to the man's height?*

Solution



From ray-tracing and the law of reflection, we know that the angle of incidence is equal to the angle of reflection, so the top of the mirror must extend to at least halfway between his eyes and the top of his head. The bottom must go down to halfway between his eyes and the floor. This result is independent of how far he stands from the wall. Therefore, $a=\dfrac{0.13\text{ m}}{2}=0.065\text{ m}, b=\dfrac{1.65\text{ m}}{2}=0.825\text{ m}$ and

$$L=1.65\text{ m}+0.13\text{ m}-a-b=1.78\text{ m}-\frac{0.13\text{ m}}{2}-\frac{1.65\text{ m}}{2}=0.89\text{ m}$$

The bottom is $b=\underline{0.825\text{ m}}$ from the floor and the top is $b+L=0.825\text{ m}+0.89\text{ m}=\underline{1.715\text{ m}}$ from the floor.

25.3 THE LAW OF REFRACTION

7. *Calculate the index of refraction for a medium in which the speed of light is* $2.012\times10^8\text{ m/s}$, *and identify the most likely substance based on Table 25.1.*

Solution Use the equation $n=\dfrac{c}{v}=\dfrac{2.997\times10^8\text{ m/s}}{2.012\times10^8\text{ m/s}}=\underline{1.490}$. From Table 25.1, the substance is polystyrene.

13. *Suppose you have an unknown clear substance immersed in water, and you wish to identify it by finding its index of refraction. You arrange to have a beam of light enter it at an angle of* $45.0°$*, and you observe the angle of refraction to be* $40.3°$*. What is the index of refraction of the substance and its likely identity?*

Solution Using the equation $n_1 \sin\theta_1 = n_2 \sin\theta$, we can solve for the unknown index of refraction: $n_2 = n_1 = \dfrac{\sin\theta_1}{\sin\theta_2} = \dfrac{(1.33)(\sin 45.0°)}{\sin 40.3°} = \underline{1.46}$

From Table 25.1, the most likely solid substance is <u>fused quartz</u>.

25.4 TOTAL INTERNAL REFLECTION

22. *An optical fiber uses flint glass clad with crown glass. What is the critical angle?*

Solution Using the equation $\theta_c = \sin^{-1}\left(\dfrac{n_2}{n_1}\right)$ and the indices of refraction from Table 25.1 gives a critical angle of $\theta_c = \sin^{-1}\left(\dfrac{n_2}{n_1}\right) = \sin^{-1}\left(\dfrac{1.52}{1.66}\right) = \underline{66.3°}$

25.5 DISPERSION: THE RAINBOW AND PRISMS

33. *A ray of 610 nm light goes from air into fused quartz at an incident angle of* $55.0°$*. At what incident angle must 470 nm light enter flint glass to have the same angle of refraction?*

Solution Using Snell's law, we have $n_1 \sin\theta_1 = n_2 \sin\theta_2$ and $n'_1 \sin\theta'_1 = n'_2 \sin\theta'_2$. We can set θ_2 equal to θ'_2, because the angles of refraction are equal. Combining the equations gives $\dfrac{n_1 \sin\theta_1}{n_2} = \dfrac{n'_1 \sin\theta'_1}{n'_2}$. We know that $n_1 = n'_1 = 1.00$ because the light is entering from air. From Table 25.2, we find the 610 nm light in fused quartz has $n_2 = 1.456$ and the 470 nm light in flint glass has $n'_2 = 1.684$. We can solve for the incident angle θ_1:

$$\theta'_1 = \sin^{-1}\left(\frac{n_1 n'_2}{n_2 n'_1}\sin\theta_1\right) = \sin^{-1}\left[\frac{(1)(1.684)}{(1.456)(1)}\sin 55.0°\right] = \underline{71.3°}$$

25.6 IMAGE FORMATION BY LENSES

39. *You note that your prescription for new eyeglasses is –4.50 D. What will their focal length be?*

Solution Using the equation $P=\frac{1}{f}$, we can solve for the focal length of your eyeglasses, recalling that $1\,\text{D}=1/\text{m}$:

$$f=\frac{1}{P}=\frac{1}{-4.50\ \text{D}}=-0.222\ \text{m}=\underline{-22.2\ \text{cm}}.$$

43. *How far from a piece of paper must you hold your father's 2.25 D reading glasses to try to burn a hole in the paper with sunlight?*

Solution Using the equation $P=\frac{1}{f}$, we can solve for the focal length for your father's reading glasses: $f=\frac{1}{P}=\frac{1}{2.25\ \text{D}}=0.444\ \text{m}=\underline{44.4\ \text{cm}}$. In order to burn a hole in the paper, you want to have the glasses exactly one focal length from the paper, so the glasses should be 44.4 cm from the paper.

49. *In Example 25.7, the magnification of a book held 7.50 cm from a 10.0 cm focal length lens was found to be 3.00. (a) Find the magnification for the book when it is held 8.50 cm from the magnifier. (b) Do the same for when it is held 9.50 cm from the magnifier. (c) Comment on the trend in m as the object distance increases as in these two calculations.*

Solution (a) Using the equation $\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$, we can first determine the image distance:

$d_i=\left(\frac{1}{f}-\frac{1}{d_o}\right)^{-1}=\left(\frac{1}{10.0\ \text{cm}}-\frac{1}{8.50\ \text{cm}}\right)^{-1}=-56.67\,\text{cm}$. Then we can determine the magnification using the equation $m=-\frac{d_i}{d_o}=\frac{56.67\ \text{cm}}{8.50\ \text{cm}}=\underline{6.67}$.

(b) Using this equation again gives: $d_i=\left(\frac{1}{10.0\ \text{cm}}-\frac{1}{9.50\ \text{cm}}\right)^{-1}=-190\ \text{cm}$

And a magnification of $m = -\frac{d_i}{d_o} = \frac{190\ \text{cm}}{9.5\ \text{cm}} = \underline{+20.0}$

(c) The magnification, m, increases rapidly as you increase the object distance toward the focal length.

25.7 IMAGE FORMATION BY MIRRORS

57. *What is the focal length of a makeup mirror that produces a magnification of 1.50 when a person's face is 12.0 cm away? Explicitly show how you follow the steps in the Problem-Solving Strategy for Mirrors.*

Solution *Step 1*: Image formation by a mirror is involved.

Step 2: Draw the problem set up when possible.

Step 3: Use the thin lens equations to solve this problem.

Step 4: Find f.

Step 5: Given: $m = 1.50, d_o = 0.120\ \text{m}$.

Step 6: No ray tracing is needed.

Step 7: Using the equation $m = -\frac{d_i}{d_o}$, we know that

$d_i = -md_o = -(1.50)(0.120\ \text{m}) = -0.180\ \text{m}$. Then, using the equation $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$, we can determine the focal length: $f = \left(\frac{1}{d_i} + \frac{1}{d_o}\right)^{-1} = \left(\frac{1}{-0.180\ \text{m}} + \frac{1}{0.120\ \text{m}}\right)^{-1} = \underline{0.360\ \text{m}}$

Step 8: Since the focal length is greater than the object distance, we are dealing with case 2. For case 2, we should have a virtual image, a negative image distance and a positive (greater than one) magnification. Our answer is consistent with these expected properties, so it is reasonable.