CHAPTER 24: ELECTROMAGNETIC WAVES

24.1 MAXWELL'S EQUATIONS: ELECTROMAGNETIC WAVES PREDICTED AND OBSERVED

1.   *Verify that the correct value for the speed of light c is obtained when numerical values for the permeability and permittivity of free space (μ_0 and ε_0) are entered into the equation $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$.*

Solution   We know that $\mu_0 = 4\pi \times 10^{-7}$ T·m / A, and from Section 19.1, we know that $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, so that the equation becomes:

$$c = \frac{1}{\sqrt{\left(4\pi \times 10^{-7}\text{ T·m / A}\right)\left(8.8542 \times 10^{-12}\text{ F/m}\right)}} = 2.998 \times 10^8\text{ m/s} = \underline{3.00 \times 10^8\text{ m/s}}$$

The units work as follows:

$$[c] = \frac{1}{\sqrt{\text{T·F/A}}} = \sqrt{\frac{\text{A}}{\text{T·F}}} = \sqrt{\frac{\text{C/s}}{\left(\text{N·s/C·m}\right)\times\left(\text{C}^2/\text{J}\right)}} = \sqrt{\frac{\text{J·m}}{\text{N·s}^2}} = \sqrt{\frac{\left(\text{N·m}\right)\text{m}}{\text{N·s}^2}} = \sqrt{\frac{\text{m}^2}{\text{s}^2}} = \text{m/s}$$

24.3 THE ELECTROMAGNETIC SPECTRUM

8.   *A radio station utilizes frequencies between commercial AM and FM. What is the frequency of a 11.12-m-wavelength channel?*

Solution   Using the equation $c = f\lambda$, we can solve for the frequency since we know the speed of light and are given the wavelength;

$$f = \frac{c}{\lambda} = \frac{2.998 \times 10^8\text{ m/s}}{11.2\text{ m}} = 2.696 \times 10^7\text{ s}^{-1} = \underline{26.96\text{ MHz}}$$

17.   *If the Sun suddenly turned off, we would not know it until its light stopped coming. How long would that be, given that the Sun is 1.50×10^{11} m away?*

Solution   We know that $v = \dfrac{d}{t}$, and since we know the speed of light and the distance from the

sun to the earth, we can calculate the time: $t = \dfrac{d}{c} = \dfrac{1.50 \times 10^{11} \text{ m}}{3.00 \times 10^{8} \text{ m/s}} = \underline{500 \text{ s}}$

23.   *(a) What is the frequency of the 193-nm ultraviolet radiation used in laser eye surgery? (b) Assuming the accuracy with which this EM radiation can ablate the cornea is directly proportional to wavelength, how much more accurate can this UV be than the shortest visible wavelength of light?*

Solution   (a) Using the equation $c = f\lambda$ we can calculate the frequency given the speed of light and the wavelength of the radiation:

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8} \text{ m/s}}{1.93 \times 10^{-7} \text{ m}} = 1.55 \times 10^{15} \text{ s}^{-1} = \underline{1.55 \times 10^{15} \text{ Hz}}$$

(b) The shortest wavelength of visible light is 380 nm, so that: $\dfrac{\lambda_{\text{visible}}}{\lambda_{\text{UV}}} = \dfrac{380 \text{ nm}}{193 \text{ nm}} = 1.97.$

In other words, the UV radiation is 97% more accurate than the shortest wavelength of visible light, or almost twice as accurate.

24.4 ENERGY IN ELECTROMAGNETIC WAVES

31.   *Find the intensity of an electromagnetic wave having a peak magnetic field strength of $4.00 \times 10^{-9} \text{ T}$.*

Solution   Using the equation $I_{\text{ave}} = \dfrac{cB_0^{\;2}}{2\mu_0}$ we see that:

$$I_{\text{ave}} = \frac{cB_0^{\;2}}{2\mu_0} = \frac{\left(3.00 \times 10^{8} \text{ m/s}\right)\left(4.00 \times 10^{-9} \text{ T}\right)^2}{2\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)} = \underline{1.91 \times 10^{-3} \text{ W/m}^2}$$

The units work as follows:

$$[I] = \frac{(\text{m/s})\text{T}^2}{\text{T} \cdot \text{m/A}} = \frac{\text{T} \cdot \text{A}}{\text{s}} = \frac{(\text{N/A} \cdot \text{m})(\text{A})}{\text{s}} = \frac{\text{N}}{\text{s.m}} = \frac{\text{J/m}}{\text{s} \cdot \text{m}} = \frac{\text{W}}{\text{m}^2}$$

36. *Lasers can be constructed that produce an extremely high intensity electromagnetic wave for a brief time—called pulsed lasers. They are used to ignite nuclear fusion, for example. Such a laser may produce an electromagnetic wave with a maximum electric field strength of 1.00×10^{11} V / m for a time of 1.00 ns. (a) What is the maximum magnetic field strength in the wave? (b) What is the intensity of the beam? (c) What energy does it deliver on a 1.00 - mm^2 area?*

Solution (a) Using the equation $\dfrac{E}{B} = c$, we can determine the maximum magnetic field strength given the maximum electric field strength:

$$B_0 = \frac{E_0}{c} = \frac{1.00 \times 10^{11} \text{ N/C}}{3.00 \times 10^8 \text{ m/s}} = \underline{333 \text{ T}}, \text{ recalling that 1 V/m = 1 N/C.}$$

(b) Using the equation $I_{ave} = \dfrac{c \varepsilon_0 E_0^2}{2}$, we can calculate the intensity without using the result from part (a):

$$I = \frac{c \varepsilon_0 E_0^2}{2}$$

$$= \frac{\left(3.00 \times 10^8 \text{ m/s}\right)\left(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}\right)\left(1.00 \times 10^{11} \text{ N/C}\right)^2}{2} = \underline{1.33 \times 10^{19} \text{ W/m}^2}$$

(c) We can get an expression for the power in terms of the intensity: $P = IA$, and from the equation $E = Pt$, we can express the energy in terms of the power provided. Since we are told the time of the laser pulse, we can calculate the energy delivered to a 1.00 mm^2 area per pulse:

$$E = P \Delta t = IA \Delta t$$

$$= \left(1.328 \times 10^{19} \text{ W/m}^2\right)\left(1.00 \text{ mm}^2\right)\left(\frac{1 \text{ m}}{1000 \text{ mm}}\right)^2 \left(1.00 \times 10^{-9} \text{ s}\right)$$

$$= 1.33 \times 10^4 \text{ J} = \underline{13.3 \text{ kJ}}$$

40. ***Integrated Concepts*** *What capacitance is needed in series with an 800 - μH inductor to form a circuit that radiates a wavelength of 196 m?*

Solution Using the equation $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$, we can find the capacitance in terms of the

resonant frequency: $C = \dfrac{1}{4\pi^2 L f_0^2}$. Substituting for the frequency, using the equation

$c = f\lambda$ gives: $C = \dfrac{\lambda^2}{4\pi^2 Lc^2} \dfrac{(196\,\text{m})^2}{4\pi^2(8.00\times10^{-4}\,\text{H})(3.00\times10^8\,\text{m/s})} = 1.35\times10^{-11}\,\text{F} = \underline{13.5\,\text{pF}}$

The units work as follows: $[C] = \dfrac{\text{m}^2}{\text{H}(\text{m/s})^2} = \dfrac{\text{s}^2}{\text{H}} = \dfrac{\text{s}^2}{\Omega\cdot\text{s}} = \dfrac{\text{s}}{\Omega} = \dfrac{\text{s}}{\text{V/A}} = \dfrac{\text{A}\cdot\text{s}}{\text{V}} = \dfrac{\text{C}}{\text{V}} = \text{F}$

44.   ***Integrated Concepts*** *Electromagnetic radiation from a 5.00-mW laser is concentrated on a* 1.00 - mm^2 *area. (a) What is the intensity in* W/m^2*? (b) Suppose a 2.00-nC static charge is in the beam. What is the maximum electric force it experiences? (c) If the static charge moves at 400 m/s, what maximum magnetic force can it feel?*

Solution

(a) From the equation $I = \dfrac{P}{A}$, we know: $I = \dfrac{P}{A} = \dfrac{5.00\times10^{-3}\,\text{W}}{1.00\times10^{-6}\,\text{m}^2} = \underline{5.00\times10^3\,\text{W/m}^2}$

(b) Using the equation $I_{\text{ave}} = \dfrac{c\varepsilon_0 E_0^{\,2}}{2}$, we can solve for the maximum electric field:

$E_0 = \sqrt{\dfrac{2I}{c\varepsilon_0}} = \sqrt{\dfrac{2(5.00\times10^3\,\text{W/m}^2)}{(3.00\times10^8\,\text{m/s})(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)}} = 1.94\times10^3\,\text{N/C}.$

So, using the equation $E = \dfrac{F}{q}$ we can calculate the force on a 2.00 nC charges:

$F = qE_0 = (2.00\times10^{-9}\,\text{C})(1.94\times10^3\,\text{N/C}) = \underline{3.88\times10^{-6}\,\text{N}}$

(c) Using the equations $F = qvB\sin\theta$ and $\dfrac{E}{B} = c$, we can write the maximum magnetic force in terms of the electric field, since the electric and magnetic fields are related for electromagnetic radiation:

$F_{B,\text{max}} = qvB_0 = \dfrac{qvE_0}{c} = \dfrac{(2.00\times10^{-9}\,\text{C})(400\,\text{m/s})(1.94\times10^3\,\text{N/C})}{3.00\times10^8\,\text{m/s}} = \underline{5.18\times10^{-12}\,\text{N}}$

So the electric force is approximately 6 orders of magnitude stronger than the magnetic force.

50.   ***Unreasonable Results*** *An LC circuit containing a 1.00-pF capacitor oscillates at such a frequency that it radiates at a 300-nm wavelength. (a) What is the inductance of the circuit? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?*

Solution

(a) Using the equations $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$ and $c = f\lambda$, we can solve for the inductance:

$$f = \frac{c}{\lambda} = \frac{1}{2\pi\sqrt{LC}}, \text{ so that}$$

$$L = \frac{\lambda^2}{4\pi^2 C c^2} = \frac{\left(3.00\times10^{-7}\,\text{m}\right)^2}{4\pi^2\left(1.00\times10^{-12}\,\text{F}\right)\left(3.00\times10^8\,\text{m/s}\right)^2} = \underline{2.53\times10^{-20}\,\text{H}}$$

(b) This inductance is unreasonably small.

(c) The wavelength is too small.