

CHAPTER 23: ELECTROMAGNETIC INDUCTION, AC CIRCUITS, AND ELECTRICAL TECHNOLOGIES

23.1 INDUCED EMF AND MAGNETIC FLUX

1. What is the value of the magnetic flux at coil 2 in *Figure 23.56* due to coil 1?

Solution Using the equation $\Phi = BA \cos \theta$, we can calculate the flux through coil 2, since the coils are perpendicular: $\Phi = BA \cos \theta = BA \cos 90^\circ = \underline{0}$

23.2 FARADAY'S LAW OF INDUCTION: LENZ'S LAW

7. Verify that the units of $\Delta\Phi / \Delta t$ are volts. That is, show that $1 \text{ T} \cdot \text{m}^2 / \text{s} = 1 \text{ V}$.

Solution The units of $\frac{\Delta\Phi}{\Delta t}$ will be:

$$\frac{[\Delta\Phi]}{[\Delta t]} = \frac{\text{T} \cdot \text{m}^2}{\text{s}} = \left(\text{N/A} \cdot \text{m} \right) \left(\frac{\text{m}^2}{\text{s}} \right) = \frac{\text{N} \cdot \text{m}}{\text{A} \cdot \text{s}} = \frac{\text{N} \cdot \text{m}}{\text{C}} = \text{V} \text{ so that } \underline{1 \text{ T} \cdot \text{m}^2 / \text{s} = 1 \text{ V}}$$

14. A lightning bolt produces a rapidly varying magnetic field. If the bolt strikes the earth vertically and acts like a current in a long straight wire, it will induce a voltage in a loop aligned like that in *Figure 23.57(b)*. What voltage is induced in a 1.00 m diameter loop 50.0 m from a $2.00 \times 10^6 \text{ A}$ lightning strike, if the current falls to zero in $25.0 \mu\text{s}$? (b) Discuss circumstances under which such a voltage would produce noticeable consequences.

Solution (a) We know $E_0 = -\frac{N\Delta\Phi}{\Delta t}$, where the minus sign means that the emf creates the current and magnetic field that opposes the change in flux, and $\Phi = BA = (\pi r^2)B$.

Since the only thing that varies in the magnetic flux is the magnetic field, we can then say that $\Delta\Phi = \pi r^2 \Delta B$. Now, since $B = \frac{\mu_0 I}{2\pi d}$ the change in magnetic field occurs because of a change in the current, or $\Delta B = \frac{\mu_0 \Delta I}{2\pi d}$. Finally, substituting these into the equation $E_0 = -\frac{N\Delta\Phi}{\Delta t}$ gives:

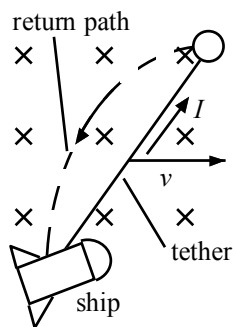
$$\begin{aligned} E_0 &= -\frac{N\pi r^2 \mu_0 \Delta I}{2\pi d \Delta t} = -\frac{\mu_0 N r^2 \Delta I}{2d \Delta t} \\ &= -\frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1)(0.500 \text{ m})^2(-2.00 \times 10^6 \text{ A})}{2(50.0 \text{ m})(2.50 \times 10^{-5} \text{ s})} = \underline{251 \text{ V}} \end{aligned}$$

- (b) An example is the alternator in your car. If you were driving during a lightning storm and this large bolt of lightning hit at 50.0 m away, it is possible to fry the alternator of your battery because of this large voltage surge. In addition, the hair at the back of your neck would stand on end because it becomes statically charged.

23.3 MOTIONAL EMF

16. If a current flows in the Satellite Tether shown in **Figure 23.12**, use Faraday's law, Lenz's law, and RHR-1 to show that there is a magnetic force on the tether in the direction opposite to its velocity.

Solution The flux through the loop (into the page) is increasing because the loop is getting larger and enclosing more magnetic field.



Thus, a magnetic field (out of the page) is induced to oppose the change in flux from the original field. Using RHR-2, point your fingers out of the page within the loop, then your thumb points in the counterclockwise direction around the loop, so the induced magnetic field is produced by the induction of a counterclockwise current in

the circuit. Finally, using RHR-1, putting your right thumb in the direction of the current and your fingers into the page (in the direction of the magnetic field), your palm points to the left, so the magnetic force on the wire is to the left (in the direction opposite to its velocity).

23.4 EDDY CURRENTS AND MAGNETIC DAMPING

27. *A coil is moved through a magnetic field as shown in Figure 23.59. The field is uniform inside the rectangle and zero outside. What is the direction of the induced current and what is the direction of the magnetic force on the coil at each position shown?*

Solution (a) The magnetic field is zero and not changing, so there is no current and therefore no force on the coil.

(b) The magnetic field is increasing out of the page, so the induced magnetic field is into the page, created by an induced clockwise current. This current creates a force to the left.

(c) The magnetic field is not changing, so there is no current and therefore no force on the coil.

(d) The magnetic field is decreasing out of the page, so the induced magnetic field is out of the page, created by an induced counterclockwise current. This current creates a force to the right.

(e) The magnetic field is zero and not changing, so there is no current and therefore no force on the coil.

23.5 ELECTRIC GENERATORS

31. *What is the peak emf generated by a 0.250 m radius, 500-turn coil is rotated one-fourth of a revolution in 4.17 ms, originally having its plane perpendicular to a uniform magnetic field. (This is 60 rev/s.)*

Solution Using the information given in **Exercise 23.12**:

$$\Delta\theta = \frac{1}{4} \text{ rev} = \frac{1}{4}(2\pi \text{ rad}) \text{ and } \Delta t = 4.17 \times 10^{-3} \text{ s},$$

$$N = 500; A = \pi r^2 = \pi(0.250 \text{ m})^2; \text{ and } B = 0.425 \text{ T},$$

we get: $\omega = \frac{\Delta\theta}{\Delta t} = \frac{(1/4)(2\pi)\text{ rad}}{4.17 \times 10^{-3} \text{ s}} = 376.7 \text{ rad/s}$. Therefore,

$$E_0 = NAB\omega = (500)(\pi)(0.250 \text{ m})^2(0.425 \text{ T})(376.7 \text{ rad/s}) = 1.57 \times 10^4 \text{ V} = \underline{15.7 \text{ kV}}$$

23.6 BACK EMF

39. Suppose a motor connected to a 120 V source draws 10.0 A when it first starts. (a) What is its resistance? (b) What current does it draw at its normal operating speed when it develops a 100 V back emf?

Solution (a) Using the equation $I = \frac{V}{R}$, we can determine given the voltage and the current:

$$R = \frac{V}{I} = \frac{120 \text{ V}}{20.0 \text{ A}} = \underline{6.00 \Omega}$$

(b) Again, using $I = \frac{V}{R}$, we can now determine the current given that the net voltage is the difference between the source voltage and the back emf:

$$I = \frac{V}{R} = \frac{120 \text{ V} - 100 \text{ V}}{6.00 \Omega} = \underline{3.33 \text{ A}}$$

43. The motor in a toy car is powered by four batteries in series, which produce a total emf of 6.00 V. The motor draws 3.00 A and develops a 4.50 V back emf at normal speed. Each battery has a 0.100Ω internal resistance. What is the resistance of the motor?

Solution Since the resistors are in series, we know the total internal resistance of the batteries is $R = 4(0.100 \Omega)$. Therefore,

$$I = \frac{E - V}{R + R'}, \text{ so that } R + R' = \frac{E - V}{I} \Rightarrow R' = \frac{6.00 \text{ V} - 4.50 \text{ V}}{3.00 \text{ A}} - 4(0.100 \Omega) = \underline{0.100 \Omega}$$

23.7 TRANSFORMERS

46. A cassette recorder uses a plug-in transformer to convert 120 V to 12.0 V, with a maximum current output of 200 mA. (a) What is the current input? (b) What is the power input? (c) Is this amount of power reasonable for a small appliance?

Solution

(a) Using the equations $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ and $\frac{V_s}{V_p} = \frac{I_p}{I_s}$, we can determine the primary

current: $\frac{I_p}{I_s} = \frac{N_p}{N_s} = \frac{V_p}{V_s}$, so that

$$I_p = I_s \left(\frac{V_p}{V_s} \right) = (0.200 \text{ A}) \left(\frac{12.0 \text{ V}}{120 \text{ V}} \right) = 2.00 \times 10^{-2} \text{ A} = \underline{20.0 \text{ mA}}$$

$$(b) P_{\text{in}} = I_p V_p = (0.200 \text{ A})(120 \text{ V}) = \underline{2.40 \text{ W}}$$

(c) Yes, this amount of power is quite reasonable for a small appliance.

23.9 INDUCTANCE

55. Two coils are placed close together in a physics lab to demonstrate Faraday's law of induction. A current of 5.00 A in one is switched off in 1.00 ms, inducing a 9.00 V emf in the other. What is their mutual inductance?

Solution

Using the equation $E_2 = -M \frac{\Delta I_1}{\Delta t}$, where the minus sign is an expression of Lenz's law, we can calculate the mutual inductance between the two coils:

$$M = E_2 \frac{\Delta t}{\Delta I_1} = (9.00 \text{ V}) \frac{(1.00 \times 10^{-3} \text{ s})}{5.00 \text{ A}} = \underline{1.80 \text{ mH}}$$

61. A large research solenoid has a self-inductance of 25.0 H. (a) What induced emf opposes shutting it off when 100 A of current through it is switched off in 80.0 ms? (b) How much energy is stored in the inductor at full current? (c) At what rate in watts must energy be dissipated to switch the current off in 80.0 ms? (d) In view of the answer to the last part, is it surprising that shutting it down this quickly is difficult?

Solution

(a) Using the equation $E = L \frac{\Delta I}{\Delta t}$, we have

$$E = L \frac{\Delta I}{\Delta t} = (25.0 \text{ H}) \left(\frac{100 \text{ A}}{8.00 \times 10^{-2} \text{ s}} \right) = 3.125 \times 10^4 \text{ V} = \underline{31.3 \text{ kV}}$$

(b) Using $E_{\text{ind}} = \frac{1}{2} LI^2 = \left(\frac{1}{2} \right) (25.0 \text{ H})(100 \text{ A})^2 = \underline{1.25 \times 10^5 \text{ J}}$

(c) Using the equation $P = \frac{\Delta E}{\Delta t}$, we have

$$P = \frac{\Delta E}{\Delta t} = \frac{1.25 \times 10^5 \text{ J}}{8.00 \times 10^{-2} \text{ s}} = 1.563 \times 10^6 \text{ W} = \underline{1.56 \text{ MW}}$$

(d) No, it is not surprising since this power is very high.

68. **Unreasonable Results** A 25.0 H inductor has 100 A of current turned off in 1.00 ms. (a) What voltage is induced to oppose this? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

Solution (a) $|E| = L \frac{\Delta I}{\Delta t} = (25.0 \text{ H}) \frac{(100 \text{ A})}{1.00 \times 10^{-3} \text{ s}} = \underline{2.50 \times 10^6 \text{ V}}$

(b) The voltage is so extremely high that arcing would occur and the current would not be reduced so rapidly.

(c) It is not reasonable to shut off such a large current in such a large inductor in such an extremely short time.

23.10 RL CIRCUITS

69. If you want a characteristic RL time constant of 1.00 s, and you have a 500 Ω resistor, what value of self-inductance is needed?

Solution Using the equation $\tau = \frac{L}{R}$, we know $L = \tau R = (1.00 \text{ s})(500 \Omega) = \underline{500 \text{ H}}$

75. What percentage of the final current I_0 flows through an inductor L in series with a resistor R , three time constants after the circuit is completed?

Solution We use the equation $I = I_0 \left(1 - e^{-t/\tau}\right)$, because the problem says, “after the circuit is completed.” Thus, the final current is given by: $I = I_0 \left(1 - e^{-t/\tau}\right)$, where $t = 3\tau$ so that:

$$\frac{I}{I_0} = \left(1 - e^{-t/\tau}\right) = 1 - e^{-3} = 0.9502$$

The current is 95.0% of the final current after 3 time constants.

23.11 REACTANCE, INDUCTIVE AND CAPACITIVE

81. What capacitance should be used to produce a $2.00 \text{ M}\Omega$ reactance at 60.0 Hz ?

Solution Using the equation $X_C = \frac{1}{2\pi f C}$, we can determine the necessary capacitance:

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi (60.0 \text{ Hz})(2.00 \times 10^6 \Omega)} = 1.326 \times 10^{-9} \text{ F} = \underline{1.33 \text{ nF}}$$

87. (a) An inductor designed to filter high-frequency noise from power supplied to a personal computer is placed in series with the computer. What minimum inductance should it have to produce a $2.00 \text{ k}\Omega$ reactance for 15.0 kHz noise? (b) What is its reactance at 60.0 Hz ?

Solution (a) Using the equation $X_L = 2\pi f L$,

$$X_L = 2\pi f L, \text{ or } L = \frac{X_L}{2\pi f} = \frac{(2.00 \times 10^3 \Omega)}{2\pi (1.50 \times 10^4 \text{ Hz})} = 2.122 \times 10^{-2} \text{ H} = \underline{21.2 \text{ mH}}$$

(b) Again using $X_L = 2\pi f L$,

$$X_L = 2\pi f L = 2\pi (60.0 \text{ Hz})(2.122 \times 10^{-2} \text{ H}) = \underline{8.00 \Omega}$$

23.12 RLC SERIES AC CIRCUITS

95. What is the resonant frequency of a 0.500 mH inductor connected to a $40.0 \mu\text{F}$ capacitor?

Solution Using the equation $f_0 = \frac{1}{2\pi\sqrt{LC}}$, we can determine the resonant frequency for the circuit: $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(5.00 \times 10^{-4} \text{ H})(4.00 \times 10^{-5} \text{ F})}} = 1.125 \times 10^3 \text{ Hz} = \underline{1.13 \text{ kHz}}$

101. *An RLC series circuit has a 2.50Ω resistor, a $100 \mu\text{H}$ inductor, and an $80.0 \mu\text{F}$ capacitor. (a) Find the circuit's impedance at 120 Hz . (b) Find the circuit's impedance at 5.00 kHz . (c) If the voltage source has $V_{\text{rms}} = 5.60 \text{ V}$, what is I_{rms} at each frequency? (d) What is the resonant frequency of the circuit? (e) What is I_{rms} at resonance?*

Solution (a) The equation $X_L = 2\pi fL$ gives the inductive reactance:

$$X_L = 2\pi fL = 2\pi(120 \text{ Hz})(1.00 \times 10^{-4} \text{ H}) = 7.540 \times 10^{-2} \Omega$$

The equation $X_C = \frac{1}{2\pi fC}$ gives the capacitive reactance:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(120 \text{ Hz})(8.00 \times 10^{-5} \text{ F})} = 16.58 \Omega$$

Finally, the equation $Z = \sqrt{R^2 + (X_L - X_C)^2}$ gives the impedance of the RLC circuit:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(2.50 \Omega)^2 + (7.54 \times 10^{-2} \Omega - 16.58 \Omega)^2} = \underline{16.7 \Omega}$$

(b) Again, $X_L = 2\pi fL$ gives the inductive reactance:

$$X_L = 2\pi(5.00 \times 10^3 \text{ Hz})(1.00 \times 10^{-4} \text{ H}) = 3.142 \Omega$$

$X_C = \frac{1}{2\pi fC}$ gives the capacitive reactance:

$$X_C = \frac{1}{2\pi(5.00 \times 10^3 \text{ Hz})(8.00 \times 10^{-4} \text{ F})} = 3.979 \times 10^{-1} \Omega$$

And $Z = \sqrt{R^2 + (X_L - X_C)^2}$ gives the impedance:

$$Z = \sqrt{(2.50 \Omega)^2 + (3.142 \Omega - 3.979 \times 10^{-1} \Omega)^2} = \underline{3.71 \Omega}$$

(c) The rms current is found using the equation $I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$. For $f = 120 \text{ Hz}$,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{5.60 \text{ V}}{16.69 \Omega} = \underline{0.336 \text{ A}}$$

$$\text{and for } f = 5.00 \text{ kHz}, I_{\text{rms}} = \frac{5.60 \text{ V}}{3.712 \Omega} = \underline{1.51 \text{ A}}$$

(d) The resonant frequency is found using the equation $f_0 = \frac{1}{2\pi\sqrt{LC}}$:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1.00 \times 10^{-7} \text{ H})(8.00 \times 10^{-5} \text{ F})}} = 5.63 \times 10^4 \text{ Hz} = \underline{56.3 \text{ kHz}}$$

(e) At resonance, $X_L = X_R$, so that $Z = R$ and $I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$ reduces to:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{5.60 \text{ V}}{2.50 \Omega} = \underline{2.24 \text{ A}}$$