CHAPTER 22: MAGNETISM

22.4 MAGNETIC FIELD STRENGTH: FORCE ON A MOVING CHARGE IN A MAGNETIC FIELD

1. *What is the direction of the magnetic force on a positive charge that moves as shown in each of the six cases shown in Figure 22.50?*

Solution Use the right hand rule-1 to solve this problem. Your right thumb is in the direction of velocity, your fingers point in the direction of magnetic field, and then your palm points in the direction of magnetic force.

(a) Your right thumb should be facing down, your fingers out of the page, and then the palm of your hand points to the left (West).

(b) Your right thumb should point up, your fingers should point to the right, and then the palm of your hand points into the page.

(c) Your right thumb should point to the right, your fingers should point into the page, and then the palm of your hand points up (North).

(d) The velocity and the magnetic field are anti-parallel, so there is no force.

(e) Your right thumb should point into the page, your fingers should point up, and then the palm of your hand points to the right (East).

(f) Your right thumb should point out of the page, your fingers should point to the left, and then the palm of your hand points down (South).

7. *What is the maximum force on an aluminum rod with a* $0.100\text{-}\mu\text{C}$ *charge that you pass between the poles of a 1.50-T permanent magnet at a speed of 5.00 m/s? In what direction is the force?*

Solution Examining the equation $F = qvB\sin\theta$, we see that the maximum force occurs when $\sin\theta = 1$, so that: $F_{\max} = qvB = (0.100\times10^{-6}\text{ C})(5.00\text{ m/s})(1.50\text{ T}) = \underline{7.50\times10^{-7}\text{ N}}$

22.5 FORCE ON A MOVING CHARGE IN A MAGNETIC FIELD: EXAMPLES AND APPLICATIONS

13. *A proton moves at* 7.50×10^{7} m/s *perpendicular to a magnetic field. The field causes the proton to travel in a circular path of radius 0.800 m. What is the field strength?*

Solution Using the equation $r = \frac{mv}{qB}$, we can solve for the magnetic field strength necessary to move the proton in a circle of radius 0.800 m:

$$B = \frac{mv}{qr} = \frac{(1.67\times10^{-27}\text{ kg})(7.50\times10^{7}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.800\text{ m})} = \underline{0.979\text{ T}}$$

19. *(a) At what speed will a proton move in a circular path of the same radius as the electron in Exercise 22.12? (b) What would the radius of the path be if the proton had the same speed as the electron? (c) What would the radius be if the proton had the same kinetic energy as the electron? (d) The same momentum?*

Solution (a) Since we know $r = \frac{mv}{qB}$, and we want the radius of the proton to equal the radius of the electron in Exercise 22.12, we can write the velocity of the proton in terms of the information we know about the electron:

$$v_{\text{p}} = \frac{q_{\text{p}}Br}{m_{\text{p}}} = \frac{q_{\text{p}}B}{m_{\text{p}}}\left(\frac{m_{\text{e}}v_{\text{e}}}{q_{\text{e}}B}\right) = \frac{m_{\text{e}}v_{\text{e}}}{m_{\text{p}}}$$
$$= \frac{(9.11\times10^{-31}\text{ kg})\,(7.50\times10^{6}\text{ m/s})}{1.67\times10^{-27}\text{ kg}} = \underline{4.09\times10^{3}\text{ m/s}}$$

(b) Now, using $r = \frac{mv}{qB}$, we can solve for the radius of the proton if the velocity equals the velocity of the electron:

$$r_{\text{p}} = \frac{mv_{\text{e}}}{qB} = \frac{(1.67\times10^{-27}\text{ kg})(7.50\times10^{6}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(1.00\times10^{-5}\text{ T})} = \underline{7.83\times10^{3}\text{ m}}$$

(c) First, we need to determine the speed of the proton if the kinetic energies were the same: $\frac{1}{2}m_{\text{e}}v_{\text{e}}^{2} = \frac{1}{2}m_{\text{p}}v_{\text{p}}^{2}$, so that

$$v_p = v_e\sqrt{\frac{m_e}{m_p}} = (7.50\times10^6\text{ m/s})\sqrt{\frac{9.11\times10^{-31}\text{ kg}}{1.67\times10^{-27}\text{ kg}}} = 1.75\times10^5\text{ m/s}$$

Then using $r = \frac{mv}{qB}$, we can determine the radius:

$$r = \frac{mv}{qB} = \frac{(1.67\times10^{-27}\text{ kg})\,(1.752\times10^5\text{ m/s})}{(1.60\times10^{-19}\text{ C})\,(1.00\times10^{-5}\text{ T})} = \underline{1.83\times10^2\text{ m}}$$

(d) First, we need to determine the speed of the proton if the momentums are the same: $m_e v_e = m_p v_p$, so that

$$v_p = v_e\left(\frac{m_e}{m_p}\right) = (7.50\times10^6\text{ m/s})\left(\frac{9.11\times10^{-31}\text{ kg}}{1.67\times10^{-27}\text{ kg}}\right) = \underline{4.09\times10^3\text{ m/s}}$$

Then using $r = \frac{mv}{qB}$, we can determine the radius:

$$r = \frac{mv}{qB} = \frac{(1.67\times10^{-27}\text{ kg})(4.091\times10^3\text{ m/s})}{(1.60\times10^{-19}\text{ C})(1.00\times10^{-5}\text{ T})} = \underline{4.27\text{ m}}$$

22.6 THE HALL EFFECT

25. *A nonmechanical water meter could utilize the Hall effect by applying a magnetic field across a metal pipe and measuring the Hall voltage produced. What is the average fluid velocity in a 3.00-cm-diameter pipe, if a 0.500-T field across it creates a 60.0-mV Hall voltage?*

Solution Using the equation $E = Blv$, we can determine the average velocity of the fluid. Note that the width is actually the diameter in this case:

$$v = \frac{E}{Bl} = \frac{60.0\times10^{-3}\text{ V}}{(0.500\text{ T})(0.0300\text{ m})} = \underline{4.00\text{ m/s}}$$

29. *Show that the Hall voltage across wires made of the same material, carrying identical currents, and subjected to the same magnetic field is inversely proportional to their diameters. (Hint: Consider how drift velocity depends on wire diameter.)*

Solution Using the equation $E = Blv$, where the width is twice the radius, $l = 2r$, and using the equation $I = nqAv_d$, we can get an expression for the drift velocity:

$v_d = \frac{I}{nqA} = \frac{I}{nq\pi q^2}$, so substituting into $E = Blv$, gives:

$$E = B \times 2r \times \frac{I}{n\,q\,\pi\,q^2} = \frac{2IB}{nq\,\pi q} \propto \frac{1}{r} \propto \frac{1}{d}.$$

So, the Hall voltage is inversely proportional to the diameter of the wire.

22.7 MAGNETIC FORCE ON A CURRENT-CARRYING CONDUCTOR

36. *What force is exerted on the water in an MHD drive utilizing a 25.0-cm-diameter tube, if 100-A current is passed across the tube that is perpendicular to a 2.00-T magnetic field? (The relatively small size of this force indicates the need for very large currents and magnetic fields to make practical MHD drives.)*

Solution Using $F = IlB\sin\theta$, where l is the diameter of the tube, we can find the force on the water: $F = IlB\sin\theta = (100\,\text{A})(0.250\,\text{m})(2.00\,\text{T})(1) = \underline{50.0\,\text{N}}$

22.8 TORQUE ON A CURRENT LOOP: MOTORS AND METERS

42. *(a) What is the maximum torque on a 150-turn square loop of wire 18.0 cm on a side that carries a 50.0-A current in a 1.60-T field? (b) What is the torque when θ is $10.9°$?*

Solution (a) Using the equation $\tau_{\max} = NIAB\sin\phi$ we see that the maximum torque occurs when $\sin\phi = 1$, so the maximum torque is:

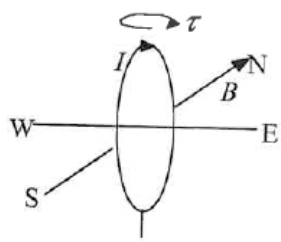
$$\tau_{\max} = NIAB\sin\phi = (150)(50.0\,\text{A})(0.180\,\text{m})^2(1.60\,\text{T})(1) = \underline{389\,\text{N}\cdot\text{m}}$$

(b) Now, use $\tau_{\max} = NIAB\sin\phi$, and set $\phi = 20.0°$, so that the torque is:

$$\tau = NIAB\sin\phi = (150)(50.0\,\text{A})(0.180\,\text{m})^2(1.60\,\text{T})\sin 10.9° = \underline{73.5\,\text{N}\cdot\text{m}}$$

48. *(a) A 200-turn circular loop of radius 50.0 cm is vertical, with its axis on an east-west line. A current of 100 A circulates clockwise in the loop when viewed from the east. The Earth's field here is due north, parallel to the ground, with a strength of* $3.00\times10^{-5}\ \text{T}$. *What are the direction and magnitude of the torque on the loop? (b) Does this device have any practical applications as a motcor?*

Solution



(a) The torque, τ, is clockwise as seen from directly above since the loop will rotate clockwise as seen from directly above. Using the equation $\tau_{\max} = NIAB\sin\phi$, we find the maximum torque to be:

$$\tau = NIAB = (200)(100\ \text{A})\ \pi\ (0.500\ \text{m})^2\ (3.00\times10^{-5}\ \text{T}) = \underline{0.471\ \text{N}\cdot\text{m}}$$

(b) If the loop was connected to a wire, this is an example of a simple motor (see Figure 22.30). When current is passed through the loops, the magnetic field exerts a torque on the loops, which rotates a shaft. Electrical energy is converted to mechanical work in the process.

22.10 MAGNETIC FORCE BETWEEN TWO PARALLEL CONDUCTORS

50. *(a) The hot and neutral wires supplying DC power to a light-rail commuter train carry 800 A and are separated by 75.0 cm. What is the magnitude and direction of the force between 50.0 m of these wires? (b) Discuss the practical consequences of this force, if any.*

Solution

(a) Using the equation $\frac{\text{F}}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$, we can calculate the force on the wires:

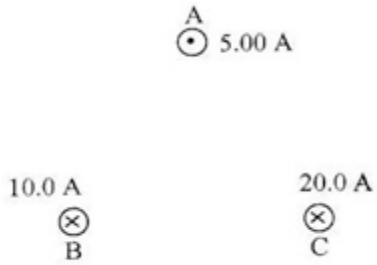
$$F = \frac{l\mu_0 I^2}{2\pi r} = \frac{(50.0)\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(800\ \text{A})^2}{2\pi(0.750\ \text{m})} = \underline{8.53\ \text{N}}$$

The force is <u>repulsive</u> because the currents are in opposite directions.

(b) This force is repulsive and therefore there is never a risk that the two wires will touch and short circuit.

56. *Find the direction and magnitude of the force that each wire experiences in Figure 22.58(a) by using vector addition.*

Solution



Opposites repel, likes attract, so we need to consider each wire's relationship with the other two wires. Let *f* denote force per unit length, then by

$$f = \frac{\mu_0 I_1 I_2}{2\pi r}$$

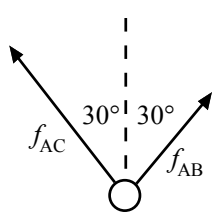
$$f_{AB} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(5.00\ \text{A}\right)\left(10.0\ \text{A}\right)}{2\pi\left(0.100\ \text{m}\right)} = 1.00\times 10^{-4}\ \text{N/m}$$

$$f_{BC} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(10.0\ \text{A}\right)\left(20.0\ \text{A}\right)}{2\pi\left(0.100\ \text{m}\right)} = 4.00\times 10^{-4}\ \text{N/m} = 4 f_{AB}$$

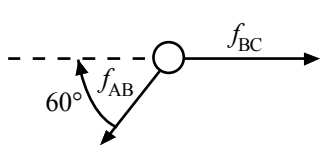
$$f_{AC} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(5.00\ \text{A}\right)\left(20.0\ \text{A}\right)}{2\pi\left(0.100\ \text{m}\right)} = 2.00\times 10^{-4}\ \text{N/m} = 2 f_{AB}$$

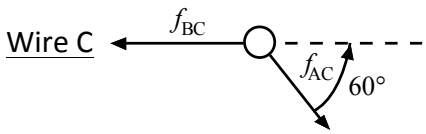
Look at each wire separately:

<u>Wire A</u>



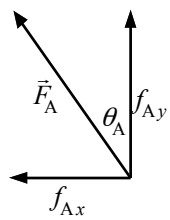
<u>Wire B</u>





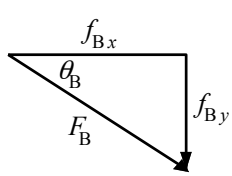
For wire A:

$$f_{Ax} = f_{AB}\sin 30° - f_{AC}\sin 30°$$
$$= \left(f_{AB} - 2f_{AB}\right)\sin 30° = -f_{AB}\left(\cos 30°\right) = -0.500\times 10^{-4}\ \text{N/m}$$
$$f_{Ay} = f_{AB}\cos 30° - f_{AC}\cos 30° = \left(f_{AB} - 2f_{AB}\right)\cos 30°$$
$$= -3f_{AB}\left(\cos 30°\right) = 2.60\times 10^{-4}\ \text{N/m}$$
$$F_A = \sqrt{f_{Ax}^2 + f_{Ay}^2} = \underline{2.65\times 10^{-4}\ \text{N/m}}$$
$$\theta_A = \tan^{-1}\left(\frac{|f_{Ax}|}{f_{Ay}}\right) = \underline{10.9°}$$



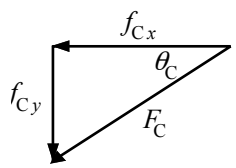
For Wire B:

$$f_{Bx} = f_{BC} - f_{AB}\cos 60°$$
$$= 4.00\times 10^{-4}\ \text{N/m} - \left(1.00\times 10^{-4}\ \text{N/m}\right)\cos 60° = 3.50\times 10^{-4}\ \text{N/m}$$
$$f_{By} = -f_{AB}\sin 60° = -\left(1.00\times 10^{-4}\ \text{N/m}\right)\sin 60° = -0.866\times 10^{-4}\ \text{N/m}$$
$$F_B = \sqrt{f_{Bx}^2 + f_{By}^2} = \underline{3.61\times 10^{-4}\ \text{N/m}}$$
$$\theta_B = \tan^{-1}\left(\frac{|f_{Bx}|}{f_{By}}\right) = \underline{13.9°}$$



For Wire C:

$$f_{Cx} = f_{AC}\cos 60° - f_{BC}$$
$$= \left(2.00\times10^{-4}\ \text{N/m}\right)\sin 60° - 4.00\times10^{-4}\ \text{N/m} = \underline{3.00\times10^{-4}\ \text{N/m}}$$
$$f_{Cy} = -f_{AC}\sin 60° - f_{BC}$$
$$= -\left(2.00\times10^{-4}\ \text{N/m}\right)\sin 60° - 4.00\times10^{-4}\ \text{N/m} = -1.73\times10^{-4}\ \text{N/m}$$
$$F_C = \sqrt{f_{Cx}^2 + f_{Cy}^2} = \underline{3.46\times10^{-4}\ \text{N/m}}$$
$$\theta_C = \tan^{-1}\left(\frac{f_{Cy}}{f_{Cx}}\right) = \underline{30.0°}$$



22.11 MORE APPLICATIONS OF MAGNETISM

77. ***Integrated Concepts*** *(a) Using the values given for an MHD drive in Exercise 22.36, and assuming the force is uniformly applied to the fluid, calculate the pressure created in* N/m^2. *(b) Is this a significant fraction of an atmosphere?*

Solution (a) Using $P = \dfrac{F}{A}$, we can calculate the pressure:

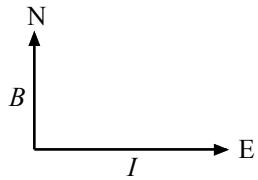
$$P = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{50.0\ \text{N}}{\pi\left(0.125\ \text{m}\right)^2} = \underline{1.02\times10^3\ \text{N/m}^2}$$

(b) <u>No</u>, this is not a significant fraction of an atmosphere.

$$\frac{P}{P_{atm}} = \frac{1.02\times10^3\ \text{N/m}^2}{1.013\times10^5\ \text{N/m}^2} = 1.01\%$$

83. ***Integrated Concepts*** *(a) What is the direction of the force on a wire carrying a current due east in a location where the Earth's field is due north? Both are parallel to the ground. (b) Calculate the force per meter if the wire carries 20.0 A and the field strength is* $3.00\times10^{-5}\ \text{T}$. *(c) What diameter copper wire would have its weight supported by this force? (d) Calculate the resistance per meter and the voltage per meter needed.*

Solution

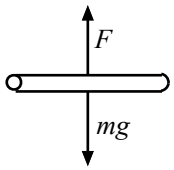


(a) Use the right hand rule-1. Put your right thumb to the east and your fingers to the north, then your palm points in the direction of the force, or <u>up from the ground</u> (out of the page).

(b) Using $F = IlB\sin\theta$, where $\theta = 90°$, so that $F = IlB\sin\theta$, or

$$\frac{F}{l} = IB\sin\theta = (20.0\text{ A})(3.00\times10^{-5}\text{ T})(1) = \underline{6.00\times10^{-4}\text{ N/m}}$$

(c)



We want the force of the magnetic field to balance the weight force, so $F = mg$.

Now, to calculate the mass, recall $\rho = \frac{m}{V}$, where the volume is $\text{V} = \pi\,\text{r}^2 L$, so $\text{m} = \rho\text{V} = \rho\pi\,\text{r}^2 L$ and $F = \rho\pi\,\text{r}^2 Lg$, or

$$r = \sqrt{\frac{F/L}{\rho\pi g}} = \sqrt{\frac{6.00\times10^{-4}\text{ N/m}}{(8.80\times10^{3}\text{ kg/m}^3)(\pi)(9.80\text{ m/s}^2)}} = 4.71\times10^{-5}\text{ m}$$

$$\Rightarrow d = 2r = \underline{9.41\times10^{-5}\text{ m}}$$

(d) From $R = \frac{\rho L}{A} = \frac{\rho L}{\pi\,\text{r}^2}$, where ρ is the resistivity:

$$\frac{R}{L} = \frac{\rho}{\pi\,\text{r}^2} = \frac{1.72\times10^{-8}\ \Omega.\text{m}}{\pi(4.71\times10^{-5}\text{ m})^2} = \underline{2.47\ \Omega/\text{m}}.$$

Also, using the equation $I = \frac{V}{R}$, we find that

$$\frac{V}{L} = I\frac{R}{L} = (20.0\text{ A})(2.47\ \Omega/\text{m}) = \underline{49.4\text{ V/m}}$$

89. ***Unreasonable Results*** *A surveyor 100 m from a long straight 200-kV DC power line suspects that its magnetic field may equal that of the Earth and affect compass readings. (a) Calculate the current in the wire needed to create a* $5.00\times10^{-5}\ \text{T}$ *field at this distance. (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?*

Solution

(a) Using the equation $B=\dfrac{\mu_0 I}{2\pi\,\text{r}}$, we can calculate the current required to get the desired magnetic field strength:

$$I=\frac{(2\pi\,\text{r})B}{\mu_0}=\frac{2\pi(100\,\text{m})(5.00\times10^{-5}\ \text{T})}{4\pi\times10^{-7}\ \text{T.m/A}}=2.50\times10^{4}\ \text{A}=\underline{25.0\,\text{kA}}$$

(b) This current is unreasonably high. It implies a total power delivery in the line of $P=IV=(25.0\times10^{3}\ \text{A})(200\times10^{3}\ \text{V})=50.0\times10^{9}\ \text{W}=50.0\,\text{GW}$, which is much too high for standard transmission lines.

(c) 100 meters is a long distance to obtain the required field strength. Also coaxial cables are used for transmission lines so that there is virtually no field for DC power lines, because of cancellation from opposing currents. The surveyor's concerns are not a problem for his magnetic field measurements.