CHAPTER 20: ELECTRIC CURRENT, RESISTANCE, AND OHM'S LAW

20.1 CURRENT

1. *What is the current in milliamperes produced by the solar cells of a pocket calculator through which 4.00 C of charge passes in 4.00 h?*

Solution Using the equation $I = \frac{\Delta Q}{\Delta t}$, we can calculate the current given in the charge and the time, remembering that $1\,\text{A} = 1\,\text{C/s}$:

$$I = \frac{\Delta Q}{\Delta t} = \frac{4.00\,\text{C}}{4.00\,\text{h}}\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = 2.778\times10^{-4}\ \text{A} = \underline{0.278\,\text{mA}}$$

7. *(a) A defibrillator sends a 6.00-A current through the chest of a patient by applying a 10,000-V potential as in Figure 20.38. What is the resistance of the path? (b) The defibrillator paddles make contact with the patient through a conducting gel that greatly reduces the path resistance. Discuss the difficulties that would ensue if a larger voltage were used to produce the same current through the patient, but with the path having perhaps 50 times the resistance. (Hint: The current must be about the same, so a higher voltage would imply greater power. Use this equation for power: $P = I^2R$.)*

Solution (a) Using the equation $I = \frac{V}{R}$, we can calculate the resistance of the path given the current and the potential: $I = \frac{V}{R}$, so that

$$R = \frac{V}{I} = \frac{10{,}000\,\text{V}}{6.00\,\text{A}} = 1.667\times10^{3}\ \Omega = \underline{1.67\,\text{k}\Omega}$$

(b) If a 50 times larger resistance existed, keeping the current about the same, the

power would be increased by a factor of about 50, causing much more energy to be transferred to the skin, which could cause serious burns. The gel used reduces the resistance, and therefore reduces the power transferred to the skin.

13. *A large cyclotron directs a beam of* He^{++} *nuclei onto a target with a beam current of 0.250 mA. (a) How many* He^{++} *nuclei per second is this? (b) How long does it take for 1.00 C to strike the target? (c) How long before 1.00 mol of* He^{++} *nuclei strike the target?*

Solution (a) Since we know that a He^{++} ion has a charge of twice the basic unit of charge, we can convert the current, which has units of C/s, into the number of He^{++} ions per second: $(2.50\times10^{-4}\ \mathrm{C/s})\dfrac{1\ \mathrm{He}^{++}}{2(1.60\times10^{-19}\ \mathrm{C})} = \underline{7.81\times10^{14}\ \mathrm{He}^{++}\mathrm{nuclei/s}}$

(b) Using the equation $I = \dfrac{\Delta Q}{\Delta t}$, we can determine the time it takes to transfer 1.00 C of charge, since we know the current: $I = \dfrac{\Delta Q}{\Delta t}$, so that

$$\Delta t = \frac{\Delta Q}{I} = \frac{1.00\ \mathrm{C}}{2.50\times10^{-4}\ \mathrm{A}} = \underline{4.00\times10^{3}\ \mathrm{s}}$$

(c) Using the result from part (a), we can determine the time it takes to transfer 1.00 mol of He^{++} ions by converting units:

$$(1.00\ \mathrm{mol\ He}^{++})\left(\frac{6.02\times10^{23}\ \mathrm{ions}}{\mathrm{mol}}\right)\left(\frac{1\ \mathrm{s}}{7.81\times10^{14}\ \mathrm{He}^{++}\mathrm{ions}}\right) = \underline{7.71\times10^{8}\ \mathrm{s}}$$

20.2 OHM'S LAW: RESISTANCE AND SIMPLE CIRCUITS

19. *Calculate the effective resistance of a pocket calculator that has a 1.35-V battery and through which 0.200 mA flows.*

Solution Using the equation $I = \frac{V}{R}$, given the voltage and the current, we can determine the resistance: $I = \frac{V}{R}$, so that $R = \frac{V}{I} = \frac{1.35\text{ V}}{2.00\times10^{-4}\text{ A}} = 6.75\times10^{3}\,\Omega = \underline{6.75\text{ k}\Omega}$

20.3 RESISTANCE AND RESISTIVITY

25. *The diameter of 0-gauge copper wire is 8.252 mm. Find the resistance of a 1.00-km length of such wire used for power transmission.*

Solution We know we want to use the equation $R = \frac{\rho L}{A}$, so we need to determine the radius for the cross-sectional area of $A = \pi r^2$. Since we know the diameter of the wire is 8.252 mm, we can determine the radius of the wire:

$$r = \frac{d}{2} = \frac{8.252\times10^{-3}\text{ m}}{2} = 4.126\times10^{-3}\text{ m}.$$

We also know from Table 20.1 that the resistivity of copper is $1.72\times10^{-8}\ \Omega\text{m}$. These values give a resistance of: $R = \frac{\rho L}{A} = \frac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(1.00\times10^{3}\text{ m})}{\pi(4.126\times10^{-3}\text{ m})^2} = \underline{0.322\,\Omega}$

31. *Of what material is a resistor made if its resistance is 40.0% greater at* $100°\text{C}$ *than at* $20.0°\text{C}$*?*

Solution We can use the equation $R = R_0(1+\alpha\Delta T)$ to determine the temperature coefficient of resistivity of the material. Then, by examining Table 20.2, we can determine the type of material used to make the resistor. Since $R = R_0(1+\alpha\Delta T) = 1.400R_0$, for a temperature change of $80.0°\text{C}$, we can determine α:

$$\alpha\Delta T = 1.400 - 1 \Rightarrow \alpha = \frac{0.400}{\Delta T} = \frac{0.400}{80.0°\text{ C}} = \underline{5.00\times10^{-3}\ /°\text{C}}$$

So, based on the values of in Table 20.2, the resistor is made of iron.

37. *(a) Digital medical thermometers determine temperature by measuring the resistance of a semiconductor device called a thermistor (which has $\alpha = -0.0600/°\text{C}$) when it is at the same temperature as the patient. What is a patient's temperature if the thermistor's resistance at that temperature is 82.0% of its value at $37.0°\text{C}$ (normal body temperature)? (b) The negative value for α may not be maintained for very low temperatures. Discuss why and whether this is the case here. (Hint: Resistance can't become negative.)*

Solution (a) $R = R_0\left[1+\alpha(T-37.0°\text{ C})\right] = 0.820\,R_0$, where $\alpha = -0.600/°\text{C}$.

Dividing by R_0, $1+\alpha(T-37.0°\text{ C}) = 0.820$, so that $0.180 = -\alpha(T-37.0°\text{ C})$,

giving $(T-37.0°\text{ C}) = \dfrac{-0.180}{\alpha} = \dfrac{-0.180}{-0.0600/°\text{C}} = 3.00°\text{ C}$.

Finally, $T = 37.0°\text{ C} + 3.00°\text{ C} = \underline{40.0°\text{ C}}$

(b) If α is negative at low temperatures, then the term $\left[1+\alpha(T-37.0°\text{ C})\right]$ can become negative, which implies that the resistance has the opposite sign of the initial resistance , or it has become negative. Since it is not possible to have a negative resistance, the temperature coefficient of resistivity cannot remain negative to low temperatures. In this example the magnitude is $\underline{\alpha \rangle \dfrac{1}{37.0°\text{ C} - T}}$

39. ***Unreasonable Results*** *(a) To what temperature must you raise a resistor made of constantan to double its resistance, assuming a constant temperature coefficient of resistivity? (b) To cut it in half? (c) What is unreasonable about these results? (d) Which assumptions are unreasonable, or which premises are inconsistent?*

Solution (a) Using the equation $R = R_0(1+\alpha\Delta T)$ and setting the resistance equal to twice the initial resistance, we can solve for the final temperature:

$$R = R_0(1+\alpha\Delta T) = 2R_0 \Rightarrow \alpha\Delta T = \alpha(T-T_0) = 1.\ \left(T_0 = 20°\text{C}\right)$$

So the final temperature will be:

$$T - T_0 = \frac{1}{\alpha} = \frac{1}{2\times10^{-6}/°\text{C}} = 5\times10^5\ °\text{C} \Rightarrow T = \underline{5\times10^5\ °\text{C}}$$

(b) Again, using the equation $R = R_0(1+\alpha\Delta T)$, we can solve for the final temperature when the resistance is half the initial resistance:

$R = R_0(1+\alpha\Delta T) = \frac{R_0}{2} \Rightarrow \alpha(T - T_0) = -\frac{1}{2}$, so the final temperature will be:

$$T - T_0 = \frac{-0.5}{2\times10^{-6}/°\text{C}} = -2.5\times10^5\ °\text{C or } T = \underline{-2.5\times10^5\ °\text{C}}.$$

(c) In part (a), the temperature is above the melting point of any metal. In part (b) the temperature is far below $0\,\text{K}$, which is impossible.

(d) The assumption that the resistivity for constantan will remain constant over the derived temperature ranges in part (a) and (b) above is wrong. For large temperature changes, α may vary, or a non-linear equation may be needed to find ρ.

20.4 ELECTRIC POWER AND ENERGY

45. *Verify that the units of a volt-ampere are watts, as implied by the equation $P = IV$.*

Solution Starting with the equation $P = IV$, we can get an expression for a watt in terms of current and voltage: $[P] = \text{W}$, $[IV] = \text{A.V} = (\text{C/s})(\text{J/C}) = \text{J/s} = \text{W}$, so that a watt is equal to an ampere - volt.

55. *A cauterizer, used to stop bleeding in surgery, puts out 2.00 mA at 15.0 kV. (a) What is its power output? (b) What is the resistance of the path?*

Solution (a) Using the equation $P = IV$, we can determine the rms power given the current and the voltage:

$$P = IV = (2.00\times10^{-3}\ \text{A})(15.0\times10^3\ \text{V}) = \underline{30.0\ \text{W}}$$

(b) Now, using the equation $I = \frac{V}{R}$, we can solve for the resistance, without using the result from part (a): $I = \frac{V}{R} \Rightarrow R = \frac{V}{I} = \frac{(1.50\times10^4\ \text{V})}{2.00\times10^{-3}\ \text{A}} = 7.50\times10^6\ \Omega = \underline{7.50\ \text{M}\Omega}$

Note, this assume the cauterizer obeys Ohm's law, which will be true for ohmic materials like good conductors.

59. ***Integrated Concepts*** *Cold vaporizers pass a current through water, evaporating it with only a small increase in temperature. One such home device is rated at 3.50 A and utilizes 120 V AC with 95.0% efficiency. (a) What is the vaporization rate in grams per minute? (b) How much water must you put into the vaporizer for 8.00 h of overnight operation? (See Figure 20.42.)*

Solution

(a) From the equation $P = IV$, we can determine the power generated by the vaporizer. $P = IV = (3.50\ \text{A})(120\ \text{V}) = 420\ \text{J/s} = 0.420\ \text{kJ/s}$ and since the vaporizer has an efficiency of 95.0%, the heat that is capable of vaporizing the water is $Q = (0.950)\ Pt$. This heat vaporizes the water according to the equation $Q = mL_\text{v}$, where $L_\text{v} = 2256\ \text{kJ/kg}$, from Table 14.2, so that $(0.950)\ Pt = mL_\text{v}$, or

$$m = \frac{(0.950)Pt}{L_\text{v}} = \frac{(0.950)(0.420\ \text{kJ/s})(60.0\ \text{s})}{2256\ \text{kJ/kg}} = 0.0106\ \text{kg} \Rightarrow \underline{10.6\ \text{g/min}}$$

(b) If the vaporizer is to run for 8.00 hours , we need to calculate the mass of the water by converting units:

$$m_\text{required} = (10.6\ \text{g/min})(8.0\ \text{h})\left(\frac{60\ \text{min}}{1\ \text{h}}\right) = 5.09\times10^3\ \text{g} = \underline{5.09\ \text{kg}}$$

In other words, making use of Table 11.1 to get the density of water, it requires $5.09\ \text{kg}\times\frac{\text{m}^3}{10^3\ \text{kg}}\times\frac{\text{L}}{10^{-3}\ \text{m}^3} = \underline{5.09\ \text{L}}$ of water to run overnight.

65. ***Integrated Concepts*** *A light-rail commuter train draws 630 A of 650-V DC electricity when accelerating. (a) What is its power consumption rate in kilowatts? (b) How long does it take to reach 20.0 m/s starting from rest if its loaded mass is* $5.30\times10^4\ \text{kg}$*, assuming 95.0% efficiency and constant power? (c) Find its average acceleration. (d) Discuss how the acceleration you found for the light-rail train compares to what might be typical for an automobile.*

Solution

(a) Using the equation $P = IV$, we can determine the power generated:

$$P = IV = (630\ \text{A})(650\ \text{V}) = 4.10\times10^5\ \text{W} = 410\ \text{kW}$$

(b) Since the efficiency is 95.0%, the effective power is $P_{\text{effective}} = (0.950)P = 389.0\ \text{kW}$. Then we can calculate the work done by the train: $W = (P_{\text{eff}})t$. Setting that equal to the change in kinetic energy gives us an expression for the time it takes to reach 20.0 m/s from rest:

$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0{}^2 = (P_{\text{eff}})t$, so that

$$t = \frac{(1/2)mv^2 - (1/2)mv_0{}^2}{P_{\text{eff}}} = \frac{0.5(5.30\times10^4\ \text{kg})(20.0\ \text{m/s})^2}{3.890\times10^5\ \text{W}} = 27.25\ \text{s} = \underline{27.3\ \text{s}}$$

(c) We recall that $v = v_0 + at$, so that $a = \frac{v}{t} = \frac{20.0\ \text{m/s}}{27.25\ \text{s}} = \underline{0.734\ \text{m/s}^2}$

(d) A typical automobile can go from 0 to 60 mph in 10 seconds, so that its acceleration is: $a = \frac{v}{t} = \frac{60\ \text{mi/hr}}{10\ \text{s}}\times\frac{1\ \text{hr}}{3600\ \text{s}}\times\frac{1609\ \text{m}}{\text{mi}} = 2.7\ \text{m/s}^2$

Thus, a light-rail train accelerates much slower than a car, but it can reach final speeds substantially faster than a car can sustain. So, typically light-rail tracks are very long and straight, to allow them to reach these faster final speeds without decelerating around sharp turns.

20.5 ALTERNATING CURRENT VERSUS DIRECT CURRENT

73. *Certain heavy industrial equipment uses AC power that has a peak voltage of 679 V. What is the rms voltage?*

Solution Using the equation $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$, we can determine the rms voltage, given the peak voltage: $V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{679\text{ V}}{\sqrt{2}} = \underline{480\text{ V}}$

79. *What is the peak power consumption of a 120-V AC microwave oven that draws 10.0 A?*

Solution Using the equation $P_{\text{ave}} = I_{\text{rms}}V_{\text{rms}}$, we can calculate the average power given the rms values for the current and voltage: $P_{\text{ave}} = I_{\text{rms}}V_{\text{rms}} = (10.0\text{ A})(120\text{ V}) = 1.20\text{ kW}$

Next, since the peak power is the peak current times the peak voltage:

$P_0 = I_0V_0 = 2(\frac{1}{2}I_0V_0) = 2P_{\text{ave}} = \underline{2.40\text{ kW}}$

83. *Find the time after $t = 0$ when the instantaneous voltage of 60-Hz AC first reaches the following values: (a) $V_0/2$ (b) V_0 (c) 0.*

Solution (a) From the equation $V = V_0 \sin 2\pi ft$, we know how the voltage changes with time for an alternating current (AC). So, if we want the voltage to be equal to $\frac{V_0}{2}$, we know that $\frac{V_0}{2} = V_0 \sin 2\pi ft$, so that: $\sin 2\pi ft = \frac{1}{2}$, or $t = \frac{\sin^{-1}(0.5)}{2\pi f}$. Since we have a frequency of 60 Hz, we can solve for the time that this first occurs (remembering to have your calculator in radians):

$$t = \frac{0.5236\text{ rad}}{2\pi\ (60\text{ Hz})} = 1.39\times10^{-3}\text{ s} = \underline{1.39\text{ ms}}$$

(b) Similarly, for $V = V_0 : V = V_0 \sin 2\pi ft = V_0$, so that

$$t = \frac{\sin^{-1} 1}{2\pi f} = \frac{\pi/2\text{ rad}}{2\pi(60\text{ Hz})} = 4.17\times10^{-3}\text{ s} = \underline{4.17\text{ ms}}$$

(c) Finally, for $V = 0$: $V = V_0 \sin 2\pi ft = 0$, so that $2\pi ft = 0, \pi, 2\pi, \ldots\ldots$, or for the first time after $t = 0$: $2\pi ft = \pi$, or $t = \dfrac{1}{2(60\text{ Hz})} = 8.33\times10^{-3}\text{s} = \underline{8.33\text{ ms}}$

20.6 ELECTRIC HAZARDS AND THE HUMAN BODY

89. *Foolishly trying to fish a burning piece of bread from a toaster with a metal butter knife, a man comes into contact with 120-V AC. He does not even feel it since, luckily, he is wearing rubber-soled shoes. What is the minimum resistance of the path the current follows through the person?*

Solution

From Table 20.3, we know that the threshold of sensation is $I = 1.00\text{ mA}$. The minimum resistance for the shock to not be felt will occur when I is equal to this value. So, using the equation $I = \dfrac{V}{R}$,we can determine the minimum resistance for 120 V AC current: $R = \dfrac{V}{I} = \dfrac{120\text{ V}}{1.00\times10^{-3}\text{ A}} = \underline{1.20\times10^{5}\ \Omega}$