CHAPTER 17: PHYSICS OF HEARING

17.2 SPEED OF SOUND, FREQUENCY, AND WAVELENGTH

1. When poked by a spear, an operatic soprano lets out a 1200-Hz shriek. What is its wavelength if the speed of sound is 345 m/s?

Solution

Use the equation \( v_w = f\lambda \), where \( f = 1200 \text{ Hz} \) and \( v_w = 345 \text{ m/s} \):

\[
\lambda = \frac{v_w}{f} = \frac{345 \text{ m/s}}{1200 \text{ Hz}} = 0.288 \text{ m}
\]

7. Dolphins make sounds in air and water. What is the ratio of the wavelength of a sound in air to its wavelength in seawater? Assume air temperature is 20.0°C.

Solution

The wavelengths of sounds in air and water are different because the speed of sound is different in air and water. We know \( v_{\text{seawater}} = 1540 \text{ m/s} \) (from Table 17.1) and \( v_{\text{air}} = 343 \text{ m/s} \) at 20.0°C from Problem 17.5, so from the equation \( v_w = f\lambda \) we know \( v_{\text{seawater}} = f\lambda_{\text{seawater}} \) and \( v_{\text{air}} = f\lambda_{\text{air}} \), so we can determine the ratio of the wavelengths:

\[
\frac{v_{\text{air}}}{v_{\text{seawater}}} = \frac{\lambda_{\text{air}}}{\lambda_{\text{seawater}}} \Rightarrow \frac{\lambda_{\text{air}}}{\lambda_{\text{seawater}}} = \frac{343 \text{ m/s}}{1540 \text{ m/s}} = 0.223
\]

17.3 SOUND INTENSITY AND SOUND LEVEL

13. The warning tag on a lawn mower states that it produces noise at a level of 91.0 dB. What is this in watts per meter squared?
Solution

\[ \beta = 10 \log_{10} \left( \frac{I}{I_0} \right), \] where \( I_0 = 10^{-12} \text{ W/m}^2 \), so that \( I = I_0 10^{\beta/10} \), and

\[ I = (1.00 \times 10^{-12} \text{ W/m}^2) 10^{910/100} = 1.26 \times 10^{-3} \text{ W/m}^2. \]

(To calculate an exponent that is not an integer, use the \( x^y \)-key on your calculator.)

21. People with good hearing can perceive sounds as low in level as \(-8.00 \text{ dB}\) at a frequency of \(3000 \text{ Hz}\). What is the intensity of this sound in watts per meter squared?

Solution

\[ \beta = 10 \log \left( \frac{I}{I_0} \right), \] so that

\[ I = I_0 10^{\beta/10} = (1.00 \times 10^{-12} \text{ W/m}^2) 10^{-8.00/10.0} = 1.58 \times 10^{-13} \text{ W/m}^2. \]

27. (a) Ear trumpets were never very common, but they did aid people with hearing losses by gathering sound over a large area and concentrating it on the smaller area of the eardrum. What decibel increase does an ear trumpet produce if its sound gathering area is \(900 \text{ cm}^2\) and the area of the eardrum is \(0.500 \text{ cm}^2\), but the trumpet only has an efficiency of \(5.00\%\) in transmitting the sound to the eardrum? (b) Comment on the usefulness of the decibel increase found in part (a).

Solution

(a) Using the equation \( I = \frac{P}{A} \), we see that for the same power, \( \frac{I_2}{I_1} = \frac{A_1}{A_2} \), so for a

5.00\% efficiency:

\[ \frac{I_e}{I_1} = \frac{A_1}{A_e} = \frac{(0.0500)(900 \text{ cm}^2)}{0.500 \text{ cm}^2} = 90. \]

Now, using the equation \( \beta(\text{dB}) = 10 \log_{10} \left( \frac{I}{I_0} \right) \), and remembering that

\[ \log A - \log B = \log \frac{A}{B}, \] we see that:
\[ \beta_c - \beta_i = 10 \log \left( \frac{I_c}{I_0} \right) - 10 \log \left( \frac{I_c}{I_t} \right) = 10 \log \left( \frac{I_c}{I_t} \right) = 10 \log(90) = 19.54 \text{ dB} = 19.5 \text{ dB} \]

(b) This increase of approximately 20 dB increases the sound of a normal conversation to that of a loud radio or classroom lecture (see Table 17.2). For someone who cannot hear at all, this will not be helpful, but for someone who is starting to lose their ability to hear, it may help. Unfortunately, ear trumpets are very cumbersome, so even though they could help, the inconvenience makes them rather impractical.

### 17.4 Doppler Effect and Sonic Booms

#### 33. A spectator at a parade receives an 888-Hz tone from an oncoming trumpeter who is playing an 880-Hz note. At what speed is the musician approaching if the speed of sound is 338 m/s?

**Solution**

We can use the equation \( f_{\text{obs}} = f_s \frac{v_w}{v_w - v_s} \) (with the minus sign because the source is approaching) to determine the speed of the musician (the source), given \( f_{\text{obs}} = 888 \text{ Hz}, f_s = 880 \text{ Hz}, \) and \( v_w = 338 \text{ m/s} \):

\[
v_s = \frac{v_w (f_{\text{obs}} - f_s)}{f_{\text{obs}}} = \frac{(338 \text{ m/s})(888 \text{ Hz} - 880 \text{ Hz})}{888 \text{ Hz}} = 3.05 \text{ m/s}
\]

### 17.5 Sound Interference and Resonance: Standing Waves in Air Columns

#### 39. What beat frequencies will be present: (a) If the musical notes A and C are played together (frequencies of 220 and 264 Hz)? (b) If D and F are played together (frequencies of 297 and 352 Hz)? (c) If all four are played together?
Solution

(a) Using the equation \( f_B = |f_1 - f_2| \): \( f_{B,A&C} = |f_1 - f_2| = |264 \text{ Hz} - 220 \text{ Hz}| = 44 \text{ Hz} \)

(b) \( f_{B,D,F} = |f_1 - f_2| = |352 \text{ Hz} - 297 \text{ Hz}| = 55 \text{ Hz} \)

(c) We get beats from every combination of frequencies, so in addition to the two beats above, we also have:

\[
\begin{align*}
  f_{B,F&A} & = 352 \text{ Hz} - 220 \text{ Hz} = 132 \text{ Hz}; \\
  f_{B,F&C} & = 352 \text{ Hz} - 264 \text{ Hz} = 88 \text{ Hz}; \\
  f_{B,D&C} & = 297 \text{ Hz} - 264 \text{ Hz} = 33 \text{ Hz}; \\
  f_{B,D,A} & = 297 \text{ Hz} - 220 \text{ Hz} = 77 \text{ Hz}
\end{align*}
\]

45. **How long must a flute be in order to have a fundamental frequency of 262 Hz (this frequency corresponds to middle C on the evenly tempered chromatic scale) on a day when air temperature is 20.0°C? It is open at both ends.**

Solution

We know that the frequency for a tube open at both ends is:

\[ f_n = n \left( \frac{v}{2L} \right) \text{ for } n = 1,2,3... \]

If the fundamental frequency \((n = 1)\) is \( f_i = 262 \text{ Hz} \), we can determine the length:

\[ f_i = \frac{v_w}{2L} \Rightarrow L = \frac{v_w}{2f_i} \]

We need to determine the speed of sound, from the equation

\[ v_w = (331 \text{ m/s}) \sqrt{\frac{T(K)}{273 \text{ K}}} \], since we are told the air temperature:

\[ v_w = (331 \text{ m/s}) \sqrt{\frac{293 \text{ K}}{273 \text{ K}}} = (331 \text{ m/s}) \sqrt{\frac{293}{273}} = 342.9 \text{ m/s}. \]

Therefore, \( L = \frac{342.9 \text{ m/s}}{2(262 \text{ Hz})} = 0.654 \text{ m} = 65.4 \text{ cm} \).
51. Calculate the first overtone in an ear canal, which resonates like a 2.40-cm-long tube closed at one end, by taking air temperature to be 37.0°C. Is the ear particularly sensitive to such a frequency? (The resonances of the ear canal are complicated by its nonuniform shape, which we shall ignore.)

Solution

First, we need to determine the speed of sound at 37.0°C, using the equation

\[ v_w = (331 \text{ m/s}) \sqrt{\frac{T (\text{K})}{273 \text{ K}}} = (331 \text{ m/s}) \sqrt{\frac{310 \text{ K}}{273 \text{ K}}} = 352.7 \text{ m/s}. \]

Next, for tubes closed at one end: \( f_n = n \left( \frac{v_w}{4L} \right) \) for \( n = 1, 3, 5... \), we can determine the frequency of the first overtone \( (n = 3) \)

\[ f_3 = 3 \frac{352.7 \text{ m/s}}{4(0.0240 \text{ m})} = 1.10 \times 10^4 \text{ Hz} = 11.0 \text{ kHz}. \]

The ear is not particularly sensitive to this frequency, so we don’t hear overtones due to the ear canal.

17.6 HEARING

57. What are the closest frequencies to 500 Hz that an average person can clearly distinguish as being different in frequency from 500 Hz? The sounds are not present simultaneously.

Solution

We know that we can discriminate between two sounds if their frequencies differ by at least 0.3%, so the closest frequencies to 500 Hz that we can distinguish are \( f = (500 \text{ Hz})(1 \pm 0.003) = 498.5 \text{ Hz} \) and \( 501.5 \text{ Hz} \).
63. What is the approximate sound intensity level in decibels of a 600-Hz tone if it has a loudness of 20 phons? If it has a loudness of 70 phons?

Solution From Figure 17.36: a 600 Hz tone at a loudness of 20 phons has a sound level of about 23 dB, while a 600 Hz tone at a loudness of 70 phons has a sound level of about 70 dB.

69. A person has a hearing threshold 10 dB above normal at 100 Hz and 50 dB above normal at 4000 Hz. How much more intense must a 100-Hz tone be than a 4000-Hz tone if they are both barely audible to this person?

Solution From Figure 17.36, the 0 phons line is normal hearing. So, this person can barely hear a 100 Hz sound at 10 dB above normal, requiring a 47 dB sound level ($\beta_1$). For a 4000 Hz sound, this person requires 50 dB above normal, or a 43 dB sound level ($\beta_2$) to be audible. So, the 100 Hz tone must be 4 dB higher than the 4000 Hz sound. To calculate the difference in intensity, use the equation

$$\beta_1 - \beta_2 = 10 \log \left( \frac{I_1}{I_0} \right) - 10 \log \left( \frac{I_2}{I_0} \right) = 10 \log \left( \frac{I_1}{I_2} \right)$$

and convert the difference in decibels to a ratio of intensities. Substituting in the values from above gives:

$$10 \log \left( \frac{I_1}{I_2} \right) = 47 \text{ dB} - 43 \text{ dB} = 4 \text{ dB}, \text{ or } \frac{I_1}{I_2} = 10^{\frac{4}{10}} = 2.5$$

So the 100 Hz tone must be 2.5 times more intense than the 4000 Hz sound to be audible by this person.

17.7 ULTRASOUND

77. (a) Calculate the minimum frequency of ultrasound that will allow you to see details as small as 0.250 mm in human tissue. (b) What is the effective depth to which this sound is effective as a diagnostic probe?
Solution  
(a) From Table 17.1, the speed of sound in tissue is $v_w = 1540 \text{ m/s}$, so using $v_w = f\lambda$, we find the minimum frequency to resolve 0.250 mm details is:

$$f' = \frac{v_w}{\lambda} = \frac{1540 \text{ m/s}}{0.250 \times 10^{-3} \text{ m}} = 6.16 \times 10^6 \text{ Hz}$$

(b) We know that the accepted rule of thumb is that you can effectively scan to a depth of about $500\lambda$ into tissue, so the effective scan depth is:

$$500\lambda = 500\left(0.250 \times 10^{-3} \text{ m}\right) = 0.125 \text{ m} = 12.5 \text{ cm}$$

83. **A diagnostic ultrasound echo is reflected from moving blood and returns with a frequency 500 Hz higher than its original 2.00 MHz. What is the velocity of the blood? (Assume that the frequency of 2.00 MHz is accurate to seven significant figures and 500 Hz is accurate to three significant figures.)**

Solution  
This problem requires two steps: (1) determine the frequency the blood receives (which is the frequency that is reflected), then (2) determine the frequency that the scanner receives. At first, the blood is like a moving observer, and the equation

$$f_{\text{obs}} = f'_s \left( \frac{v_w + v_b}{v_w} \right)$$

gives the frequency it receives (with the plus sign used because the blood is approaching): $f'_b = f'_s \left( \frac{v_w + v_b}{v_w} \right)$ (where $v_b =$ blood velocity). Next, this frequency is reflected from the blood, which now acts as a moving source. The equation $f_{\text{obs}} = f'_s \left( \frac{v_w}{v_w + v_b} \right)$ (with the minus sign used because the blood is still approaching) gives the frequency received by the scanner:

$$f''_{\text{obs}} = f'_b \left( \frac{v_w}{v_w - v_b} \right) = f'_s \left( \frac{v_w + v_b}{v_w} \right) \left( \frac{v_w}{v_w - v_b} \right) = f'_s \left( \frac{v_w + v_b}{v_w - v_b} \right)$$

Solving for the speed of blood gives:

$$v_b = v_w \left( \frac{f''_{\text{obs}} - f'_s}{f''_{\text{obs}} + f'_s} \right) = \frac{(1540 \text{ m/s})(500 \text{ Hz})}{(2.00 \times 10^6 \text{ Hz} + 500 \text{ Hz}) + 2.00 \times 10^6 \text{ Hz}} = 0.192 \text{ m/s}$$
The blood’s speed is 19.2 cm/s.