CHAPTER 12: FLUID DYNAMICS AND ITS BIOLOGICAL AND MEDICAL APPLICATIONS

12.1 FLOW RATE AND ITS RELATION TO VELOCITY

1.   *What is the average flow rate in $\mathrm{cm^3/s}$ of gasoline to the engine of a car traveling at 100 km/h if it averages 10.0 km/L?*

Solution   We are given the speed of the car and a gas mileage, giving us a volume consumed per time, so the equation $Q = \dfrac{V}{t}$ is the formula we want to use to calculate the average flow rate:

$$Q = \frac{V}{t} = \frac{\text{speed}}{\text{gas mileage}} = \frac{100 \ \text{km/h}}{10.0 \ \text{km/L}} \times \frac{1000 \, \text{cm}^3}{1 \, \text{L}} \times \frac{1 \, \text{H}}{3600 \, \text{s}} = \underline{2.78 \ \text{cm}^3/\text{s}}$$

14.   *Prove that the speed of an incompressible fluid through a constriction, such as in a Venturi tube, increases by a factor equal to the square of the factor by which the diameter decreases. (The converse applies for flow out of a constriction into a larger-diameter region.)*

Solution   If the fluid is incompressible, then the flow rate through both sides will be equal: $Q = A_1 v_1 = A_2 v_2$. Writing the areas in terms of the diameter of the tube gives:

$$\pi \frac{d_1^2}{4} v_1 = \pi \frac{d_2^2}{4} v_2 \Rightarrow v_2 = v_1 \left(d_1^2 / d_2^2 \right) = \underline{v_1 \left(d_1 / d_2 \right)^2}$$

Therefore, the velocity through section 2 equals the velocity through section 1 times the square of the ratio of the diameters of section 1 and section 2.

12.2 BERNOULLI'S EQUATION

21.   *Every few years, winds in Boulder, Colorado, attain sustained speeds of 45.0 m/s (about 100 mi/h) when the jet stream descends during early spring. Approximately what is the force due to the Bernoulli effect on a roof having an area of $220\,\text{m}^2$? Typical air density in Boulder is $1.14\,\text{kg/m}^3$, and the corresponding atmospheric pressure is $8.89\times10^4\,\text{N/m}^2$. (Bernoulli's principle as stated in the text assumes laminar flow. Using the principle here produces only an approximate result, because there is significant turbulence.)*

Solution   Ignoring turbulence, we can use Bernoulli's equation:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \text{ ,where the heights are the same: } h_1 = h_2$$

because we are concerned about above and below a thin roof. The velocity inside the house is zero, so $v_1 = 0.0\,\text{m/s}$, while the speed outside the house is $v_2 = 45.0\,\text{m/s}$.

The difference in pressures, $P_1 - P_2$, can then be found: $P_1 - P_2 = \frac{1}{2}\rho v_2^2$. Now, we can

relate the change in pressure to the force on the roof, using the Equation $F = (P_1 - P_2)A$, because we know the area of the roof $\left(A = 200\,\text{m}^2\right)$:

$$F = (P_1 - P_2)A = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)A$$

and substituting in the values gives:

$$F = \frac{1}{2}\left(1.14\,\text{kg/m}^3\right)\left[(45\,\text{m/s})^2 - (0.0\,\text{m/s})^2\right]\left(220\,\text{m}^2\right) = \underline{2.54\times10^5\,\text{N}}$$

This extremely large force is the reason you should leave windows open in your home when there are tornados or heavy windstorms in the area: otherwise your roof will pop off!

12.3 THE MOST GENERAL APPLICATIONS OF BERNOULLI'S EQUATION

27.   *The left ventricle of a resting adult's heart pumps blood at a flow rate of $83.0\,\text{cm}^3/\text{s}$,*
         increasing its pressure by 110 mm Hg, its speed from zero to 30.0 cm/s, and its height
         by 5.00 cm. (All numbers are averaged over the entire heartbeat.) Calculate the total
         power output of the left ventricle. Note that most of the power is used to increase
         blood pressure.

Solution   Using the equation for power in fluid flow, we can calculate the power output by the
           left ventricle during the heartbeat:

$$\text{power} = \left(P + \frac{1}{2}\rho v^2 + \rho g h \right) Q, \text{where}$$

$$P = 110 \text{ mm Hg} \times \frac{133\,\text{N/m}^2}{1.0\,\text{mm Hg}} = 1.463 \times 10^4\ \text{N/m}^2,$$

$$\frac{1}{2}\rho v^2 = \frac{1}{2}\left(1.05 \times 10^3\ \text{kg/m}^3\right)\left(0.300\ \text{m/s}\right)^2 = 47.25\ \text{N/m}^2, \text{and}$$

$$\rho g h = \left(1.05 \times 10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left(0.0500\ \text{m}\right) = 514.5\ \text{N/m}^2, \text{giving}:$$

$$\text{power} = \left(1.463 \times 10^4\ \text{N/m}^2 + 47.25\ \text{N/m}^2 + 514.5\ \text{N/m}^2\right)\left(83.0\ \text{cm}^3/\text{s}\right)\frac{10^{-6}\ \text{m}^3}{\text{cm}^3}$$

$$= \underline{1.26\ \text{W}}$$

12.4 VISCOSITY AND LAMINAR FLOW; POISEUILLE'S LAW

35.   *The arterioles (small arteries) leading to an organ, constrict in order to decrease flow*
         to the organ. To shut down an organ, blood flow is reduced naturally to 1.00% of its
         original value. By what factor did the radii of the arterioles constrict? Penguins do this
         when they stand on ice to reduce the blood flow to their feet.

Solution   If the flow rate is reduced to 1.00% of its original value, then

$$Q_2 = \frac{\Delta P \pi r_2^4}{8\eta L_2} = 0.0100 Q_1 = 0.0100 \frac{\Delta P \pi r_1^4}{8\eta L_1}.$$ Since the length of the arterioles is kept

constant and the pressure difference is kept constant, we can get a relationship
between the radii: $r_2^4 = 0.0100 r_1^4 \Rightarrow r_2 = \left(0.0100\right)^{1/4} r_1 = \underline{0.316 r_1}$

The radius is reduced to 31.6% of the original radius to reduce the flow rate to 1.00% of its original value.

43.   *Example 12.8 dealt with the flow of saline solution in an IV system. (a) Verify that a pressure of $1.62 \times 10^4 \, \text{N/m}^2$ is created at a depth of 1.61 m in a saline solution, assuming its density to be that of sea water. (b) Calculate the new flow rate if the height of the saline solution is decreased to 1.50 m. (c) At what height would the direction of flow be reversed? (This reversal can be a problem when patients stand up.)*

Solution   (a) We can calculate the pressure using the equation $P_2 = \rho h g$ where the height is 1.61 m and the density is that of seawater:

$$P_2 = \rho h g = (1025 \, \text{kg/m}^3)(1.61 \, \text{m})(9.80 \, \text{m/s}^2) = \underline{1.62 \times 10^4 \, \text{N/m}^2}$$

(b) If the pressure is decreased to 1.50 m, we can use the equation $Q = \dfrac{(P_2 - P_1)\pi r^4}{8\eta l}$

to determine the new flow rate: $Q = \dfrac{(P_2 - P_1)\pi r^4}{8\eta l}$. We use

$l = 0.0250 \, \text{m}, \ r = 0.150 \times 10^{-3} \, \text{m}, \eta = 1.005 \times 10^{-3} \, \text{N·s/m}^2$, and
$P_1 = 1.066 \times 10^3 \, \text{N/m}^2$.

Using the equation $P_2 = \rho h g$, we can find the pressure due to a depth of 1.50 m:

$$P_2' = (1025 \, \text{kg/m}^3)(1.50 \, \text{m})(9.80 \, \text{m/s}^2) = 1.507 \times 10^4 \, \text{N/m}^2.$$

So substituting into the equation $Q = \dfrac{(P_2 - P_1)\pi r^4}{8\eta l}$ gives:

$$Q = \frac{\left[(1.507 \times 10^4 \, \text{N/m}^2 - 1.066 \times 10^3 \, \text{N/m}^2)\pi (0.150 \times 10^{-3} \, \text{m})^4\right]}{8(1.005 \times 10^{-3} \, \text{N·s/m}^2)(0.0250 \, \text{m})}$$

$$= 1.11 \times 10^{-7} \, \text{m}^3/\text{s} = \underline{0.111 \, \text{cm}^3/\text{s}}$$

(c) The flow rate will be zero (and become negative) when the pressure in the IV is equal to (or less than) the pressure in the patient's vein:

$$P_r = \rho h g \Rightarrow h = \frac{P_r}{\rho g} = \frac{1.066 \times 10^3 \, \text{N/m}^2}{(1025 \, \text{kg/m}^3)(9.80 \, \text{m/s}^2)} = 0.106 \, \text{m} = \underline{10.6 \, \text{cm}}$$

12.5 THE ONSET OF TURBULENCE

51. *Verify that the flow of oil is laminar (barely) for an oil gusher that shoots crude oil 25.0 m into the air through a pipe with a 0.100-m diameter. The vertical pipe is 50 m long. Take the density of the oil to be 900 kg/m^3 and its viscosity to be $1.00 \, (\text{N/m}^2) \cdot \text{s}$ (or $1.00 \, \text{Pa} \cdot \text{s}$).*

Solution   We will use the equation $N_R = \dfrac{2\rho vr}{\eta}$ to determine the Reynolds number, so we must determine the velocity of the oil. Since the oil rises to 25.0 m,

$v^2 = v_0^2 - 2gy$, where $v = 0$ m/s, $y = 25.0$ m, so

$v_0 = \sqrt{2gy} = \sqrt{2(9.80 \text{ m/s}^2)(25.0 \text{ m})} = 22.136$ m/s

Now, we can use the equation $N_R = \dfrac{2\rho vr}{\eta}$:

$$N_R = \frac{2(900 \text{ kg/m}^3)(22.136 \text{ m/s})(0.0500 \text{ m})}{1.00 \, (\text{N/m}^2) \cdot \text{s}} = \underline{1.99 \times 10^3} < 2000$$

Since $N_R = 2000$ is the approximate upper value for laminar flow. So the flow of oil is laminar (barely).

59. *Gasoline is piped underground from refineries to major users. The flow rate is 3.00×10^{-2} m³/s (about 500 gal/min), the viscosity of gasoline is $1.00 \times 10^{-3} \, (\text{N/m}^2) \cdot \text{s}$, and its density is 680 kg/m^3. (a) What minimum diameter must the pipe have if the Reynolds number is to be less than 2000? (b) What pressure difference must be maintained along each kilometer of the pipe to maintain this flow rate?*

Solution   (a) We will use the equation $N_R = \dfrac{2\rho vr}{\eta}$, where $N_R = \dfrac{2\rho vr}{\eta} \leq 2000$, to find the minimum radius, which will give us the minimum diameter. First, we need to get an expression for the velocity, from the equation $v = \dfrac{Q}{A} = \dfrac{Q}{\pi r^2}$. Substituting gives:

$$\frac{2\rho(Q/\pi r^2)r}{\eta} = \frac{2\rho Q}{\pi \eta r} \le 2000 \text{ or } r \ge \frac{\rho Q}{\pi \eta(1000)},$$ so that the minimum diameter is

$$d \ge \frac{\rho Q}{500\pi\eta} = \frac{(680 \text{ kg/m}^3)(3.00\times 10^{-2} \text{ m}^3/\text{s})}{500\pi(1.00\times 10^{-3} \text{ N}\cdot\text{s/m}^2)} = \underline{13.0 \text{ m}}$$

(b) Using the equation $Q = \frac{\Delta P \pi r^4}{8\eta l}$, we can determine the pressure difference from the flow rate:

$$\Delta P = \frac{8\eta l Q}{\pi r^4} = \frac{8(1.00\times 10^{-3} \text{ N}\cdot\text{s/m}^2)(1000 \text{ m})(3.00\times 10^{-2} \text{ m}^3/\text{s})}{\pi(12.99 \text{ m})^4} = \underline{2.68\times 10^{-6} \text{ N/m}^2}$$

This pressure is equivalent to 2.65×10^{-11} atm, which is very small pressure!

12.7 MOLECULAR TRANSPORT PHENOMENA: DIFFUSION, OSMOSIS, AND RELATED PROCESSES

66.   *Suppose hydrogen and oxygen are diffusing through air. A small amount of each is released simultaneously. How much time passes before the hydrogen is 1.00 s ahead of the oxygen? Such differences in arrival times are used as an analytical tool in gas chromatography.*

Solution   From Table 12.2, we know $D_{H_2} = 6.4\times 10^{-5} \text{ m}^2/\text{s}$ and $D_{O_2} = 1.8\times 10^{-5} \text{ m}^2/\text{s}$. We want to use the equation $x_{rms} = \sqrt{2Dt}$, since that relates time to the distance traveled during diffusion. We have two equations: $x_{rms,O_2} = \sqrt{2D_{O_2}t_{O_2}}$ and $x_{rms,H_2} = \sqrt{2D_{H_2}t_{H_2}}$.

We want the distance traveled to be the same, so we can set the equations equal. The distance will be the same when the time difference between t_{H_2} and t_{O_2} is 1.00 s, so we can relate the two times: $t_{O_2} = t_{H_2} + 1.00 \text{ s}$.

Setting the two distance equations equal and squaring gives: $2D_{O_2}t_{O_2} = 2D_{H_2}t_{H_2}$ and substituting for oxygen time gives: $D_{O_2}(t_{H_2} + 1.00 \text{ s}) = D_{H_2}t_{H_2}$.

Solving for the hydrogen time gives:

$$t_{H_2} = \frac{D_{O_2}}{D_{H_2} - D_{O_2}} \times 1.00 \text{ s} = \frac{1.8\times 10^{-5} \text{ m}^2/\text{s}}{6.4\times 10^{-5} \text{ m}^2/\text{s} - 1.8\times 10^{-5} \text{ m}^2/\text{s}} \times 1.00 \text{ s} = \underline{0.391 \text{ s}}$$

The hydrogen will take 0.391 s to travel to the distance x, while the oxygen will take

1.391 s to travel the same distance. Therefore, the hydrogen will be 1.00 seconds ahead of the oxygen after 0.391 s.