

CHAPTER 11: FLUID STATICS

11.2 DENSITY

1. *Gold is sold by the troy ounce (31.103 g). What is the volume of 1 troy ounce of pure gold?*

Solution From Table 11.1: $\rho_{\text{Au}} = 19.32 \text{ g/cm}^3$, so using the equation $\rho = \frac{m}{V}$, we have:

$$V = \frac{m}{\rho} = \frac{31.103 \text{ g}}{19.32 \text{ g/cm}^3} = \underline{1.610 \text{ cm}^3}$$

6. *(a) A rectangular gasoline tank can hold 50.0 kg of gasoline when full. What is the depth of the tank if it is 0.500-m wide by 0.900-m long? (b) Discuss whether this gas tank has a reasonable volume for a passenger car.*

Solution

(a) From Table 11.1: $\rho_{\text{gas}} = 0.680 \times 10^3 \text{ kg/m}^3$, so using the equation $\rho = \frac{m}{V} = \frac{m}{\rho lw}$, the height is:
$$h = \frac{m}{\rho lw} = \frac{50.0 \text{ kg}}{(0.680 \times 10^3 \text{ kg/m}^3)(0.900 \text{ m})(0.500 \text{ m})} = \underline{0.163 \text{ m}}$$

- (b) The volume of this gasoline tank is 19.4 gallons, quite reasonably sized for a passenger car.

11.3 PRESSURE

12. *The pressure exerted by a phonograph needle on a record is surprisingly large. If the equivalent of 1.00 g is supported by a needle, the tip of which is a circle 0.200 mm in radius, what pressure is exerted on the record in N/m^2 ?*

Solution Using the equation $P = \frac{F}{A}$, we can solve for the pressure:

$$P = \frac{F}{A} = \frac{mg}{\pi r^2} = \frac{(1.00 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2)}{\pi (2.00 \times 10^{-4} \text{ m})^2} = 7.80 \times 10^4 \text{ Pa}$$

This pressure is approximately 585 mm Hg.

11.4 VARIATION OF PRESSURE WITH DEPTH IN A FLUID

18. *The aqueous humor in a person's eye is exerting a force of 0.300 N on the 1.10-cm² area of the cornea. (a) What pressure is this in mm Hg? (b) Is this value within the normal range for pressures in the eye?*

Solution (a) Using the equation $P = \frac{F}{A}$, we can solve for the pressure:

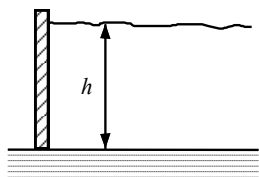
$$P = \frac{F}{A} = \frac{0.300 \text{ N}}{1.10 \text{ cm}^2} \times \left(\frac{100 \text{ cm}}{1 \text{ m}} \right)^2 = 2.73 \times 10^3 \text{ Pa} \times \frac{1 \text{ mm Hg}}{133.3 \text{ Pa}} = \underline{20.5 \text{ mm Hg}}$$

(b) From [Table 11.5](#), we see that the range of pressures in the eye is 12-24 mm Hg, so the result in part (a) is within that range.

23. *Show that the total force on a rectangular dam due to the water behind it increases with the square of the water depth. In particular, show that this force is given by $F = \rho g h^2 L / 2$, where ρ is the density of water, h is its depth at the dam, and L is the length of the dam. You may assume the face of the dam is vertical. (Hint: Calculate the average pressure exerted and multiply this by the area in contact with the water. See [Figure 11.42](#).)*

Solution The average pressure on a dam is given by the equation $\bar{P} = \frac{h}{2} \rho g$, where $\frac{h}{2}$ is the

average height of the water behind the dam. Then, the force on the dam is found using the equation $P = \frac{F}{A}$, so that $F = \bar{P}A = \left(\frac{h}{2}\rho g\right)(hL)$, or $F = \frac{\rho gh^2 L}{2}$. Thus, the average force on a rectangular dam increases with the square of the depth.



11.5 PASCAL'S PRINCIPLE

27. A certain hydraulic system is designed to exert a force 100 times as large as the one put into it. (a) What must be the ratio of the area of the slave cylinder to the area of the master cylinder? (b) What must be the ratio of their diameters? (c) By what factor is the distance through which the output force moves reduced relative to the distance through which the input force moves? Assume no losses to friction.

Solution (a) Using the equation $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ we see that the ratio of the areas becomes:

$$\frac{A_S}{A_M} = \frac{F_S}{F_M} = \frac{100}{1} = \underline{100}$$

(b) We know that the area goes as $\pi r^2 = \frac{\pi d^2}{4}$, so the ratio of the areas gives:

$$\frac{A_S}{A_M} = \frac{\pi r_S^2}{\pi r_M^2} = \frac{\pi (d_S/2)^2}{\pi (d_M/2)^2} = \frac{d_S^2}{d_M^2} = 100, \text{ so that } \frac{d_S}{d_M} = \sqrt{100} = \underline{10.0}$$

(c) Since the work input equals the work output, and work is proportional to force times distance, $F_i d_i = F_o d_o \Rightarrow \frac{d_o}{d_i} = \frac{F_i}{F_o} = \frac{1}{100}$.

This tells us that the distance through which the output force moves is reduced by

a factor of 100, relative to the distance through which the input force moves.

28. (a) Verify that work input equals work output for a hydraulic system assuming no losses to friction. Do this by showing that the distance the output force moves is reduced by the same factor that the output force is increased. Assume the volume of the fluid is constant. (b) What effect would friction within the fluid and between components in the system have on the output force? How would this depend on whether or not the fluid is moving?

Solution (a) If the input cylinder is moved a distance d_i , it displaces a volume of fluid V , where the volume of fluid displaced must be the same for the input as the

$$\text{output: } V = d_i A_i = d_o A_o \Rightarrow d_o = d_i \left(\frac{A_i}{A_o} \right)$$

Now, using the equation $\frac{F_1}{A_1} = \frac{F_2}{A_2}$, we can write the ratio of the areas in terms of

$$\text{the ratio of the forces: } \frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_o = F_i \left(\frac{A_o}{A_i} \right).$$

Finally, writing the output in terms of force and distance gives:

$$W_o = F_o d_o = \left(\frac{F_i A_o}{A_i} \right) \left(\frac{d_i A_i}{A_o} \right) = F_i d_i = W_i.$$

In other words, the work output equals the work input for a hydraulic system.

- (b) If the system is not moving, the fraction would not play a role. With friction, we know there are losses, so that $W_o = W_i - W_f$; therefore, the work output is less than the work input. In other words, with friction, you need to push harder on the input piston than was calculated. Note: the volume of fluid is still conserved.

11.7 ARCHIMEDES' PRINCIPLE

40. *Bird bones have air pockets in them to reduce their weight—this also gives them an average density significantly less than that of the bones of other animals. Suppose an ornithologist weighs a bird bone in air and in water and finds its mass is 45.0 g and its apparent mass when submerged is 3.60 g (the bone is watertight). (a) What mass of water is displaced? (b) What is the volume of the bone? (c) What is its average density?*

Solution (a) The apparent mass loss is equal to the mass of the fluid displaced, so the mass of the fluid displaced is just the difference the mass of the bone and its apparent mass: $m_{\text{displaced}} = 45.0 \text{ g} - 3.60 \text{ g} = \underline{41.4 \text{ g}}$

(b) Using Archimedes' Principle, we know that that volume of water displaced equals the volume of the bone; we see that $V_b = V_w = \frac{m_w}{\rho_w} = \frac{41.4 \text{ g}}{1.00 \text{ g/cm}^3} = \underline{41.4 \text{ cm}^3}$

(c) Using the following equation, we can calculate the average density of the bone:

$$\bar{\rho}_o = \frac{m_b}{V_b} = \frac{45.0 \text{ g}}{41.4 \text{ cm}^3} = \underline{1.09 \text{ g/cm}^3}$$

This is clearly not the density of the bone everywhere. The air pockets will have a density of approximately $1.29 \times 10^{-3} \text{ g/cm}^3$, while the bone will be substantially denser.

46. *(a) What is the density of a woman who floats in freshwater with 4.00% of her volume above the surface? This could be measured by placing her in a tank with marks on the side to measure how much water she displaces when floating and when held under water (briefly). (b) What percent of her volume is above the surface when she floats in seawater?*

Solution

(a) From the equation $\text{fraction submerged} = \frac{\bar{\rho}_{\text{obj}}}{\rho_{\text{fl}}}$, we see that:

$$\bar{\rho}_{\text{person}} = \rho_{\text{fresh water}} \times (\text{fraction submerged}) = (1.00 \times 10^3 \text{ kg/m}^3)(0.960) = \underline{960 \text{ kg/m}^3}$$

(b) The density of seawater is greater than that of fresh water, so she should float more.

$$\text{fraction submerged} = \frac{\bar{\rho}_{\text{person}}}{\rho_{\text{sea water}}} = \frac{960 \text{ kg/m}^3}{1025 \text{ kg/m}^3} = 0.9366.$$

Therefore, the percent of her volume above water is

$$\% \text{ above water} = (1.0000 - 0.9366) \times 100\% = \underline{6.34\%}$$

She does indeed float more in seawater.

50. *Scurrilous con artists have been known to represent gold-plated tungsten ingots as pure gold and sell them to the greedy at prices much below gold value but deservedly far above the cost of tungsten. With what accuracy must you be able to measure the mass of such an ingot in and out of water to tell that it is almost pure tungsten rather than pure gold?*

Solution To determine if the ingot is gold or tungsten, we need to calculate the percent difference between the two substances both out and in water. Then, the difference between these percent differences is the necessary accuracy that we must have in order to determine the substance we have. The percent difference is calculated by calculating the difference in a quantity and dividing that by the value for gold.

Out of water: Using the difference in density, the percent difference is then:

$$\%_{\text{out}} = \frac{\rho_{\text{g}} - \rho_{\text{t}}}{\rho_{\text{g}}} \times 100\% = \frac{19.32 \text{ g/cm}^3 - 19.30 \text{ g/cm}^3}{19.32 \text{ g/cm}^3} \times 100\% = \underline{0.1035\% \text{ in air}}$$

In water: Assume a 1.000 cm^3 nugget. Then the apparent mass loss is equal to that of the water displaced, i.e., 1.000 g. So, we can calculate the percent difference in the

mass loss by using the difference in masses:

$$\%_{\text{in}} = \frac{m'_g - m'_t}{m'_g} \times 100\% = \frac{18.32 \text{ g/cm}^3 - 18.30 \text{ g/cm}^3}{18.32 \text{ g/cm}^3} \times 100\% = \underline{0.1092\% \text{ in water}}$$

The difference between the required accuracies for the two methods is

$$0.1092\% - 0.1035\% = \underline{0.0057\%} = \underline{0.006\%}, \text{ so we need 5 digits of accuracy to}$$

determine the difference between gold and tungsten.

11.8 COHESION AND ADHESION IN LIQUIDS: SURFACE TENSION AND CAPILLARY ACTION

59. We stated in *Example 11.12* that a xylem tube is of radius $2.50 \times 10^{-5} \text{ m}$. Verify that such a tube raises sap less than a meter by finding h for it, making the same assumptions that sap's density is 1050 kg/m^3 , its contact angle is zero, and its surface tension is the same as that of water at 20.0°C .

Solution Use the equation $h = \frac{2\gamma \cos \theta}{\rho g r}$ to find the height to which capillary action will move sap through the xylem tube:

$$h = \frac{2\gamma \cos \theta}{\rho g r} = \frac{2(0.0728 \text{ N/m})(\cos 0^\circ)}{(1050 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(2.50 \times 10^{-5} \text{ m})} = \underline{0.566 \text{ m}}$$

65. When two soap bubbles touch, the larger is inflated by the smaller until they form a single bubble. (a) What is the gauge pressure inside a soap bubble with a 1.50-cm radius? (b) Inside a 4.00-cm-radius soap bubble? (c) Inside the single bubble they form if no air is lost when they touch?

Solution (a) Use the equation $P = \frac{4\gamma}{r}$ to find the gauge pressure inside a spherical soap

bubble of radius 1.50 cm: $P_1 = \frac{4\gamma}{r} = \frac{4(0.0370 \text{ N/m})}{(1.50 \times 10^{-2} \text{ m})} = \underline{9.87 \text{ N/m}^2}$

(b) Use $P = \frac{4\gamma}{r}$ to find the gauge pressure inside a spherical soap bubble of radius

4.00 cm: $P_2 = \frac{4\gamma}{r} = \frac{4(0.0370 \text{ N/m})}{(0.0400 \text{ m})} = \underline{3.70 \text{ N/m}^2}$

(c) If they form one bubble without losing any air, then the total volume remains

constant: $V = V_1 + V_2 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 = \frac{4}{3}\pi R^3$

Solving for the single bubble radius gives:

$$R = [r_1^3 + r_2^3]^{1/3} = [(0.0150 \text{ m})^3 + (0.0400 \text{ m})^3]^{1/3} = 0.0406 \text{ m}.$$

So we can calculate the gauge pressure for the single bubble using the equation

$$P = \frac{4\gamma}{r} = \frac{4(0.0370 \text{ N/m})}{0.0406 \text{ m}} = \underline{3.65 \text{ N/m}^2}$$

11.9 PRESSURES IN THE BODY

71. *Heroes in movies hide beneath water and breathe through a hollow reed (villains never catch on to this trick). In practice, you cannot inhale in this manner if your lungs are more than 60.0 cm below the surface. What is the maximum negative gauge pressure you can create in your lungs on dry land, assuming you can achieve $-3.00 \text{ cm H}_2\text{O}$ water pressure with your lungs 60.0 cm below the surface?*

Solution The negative gauge pressure that can be achieved is the sum of the pressure due to the water and the pressure in the lungs:

$$P = -3.00 \text{ cm H}_2\text{O} - (-60.0 \text{ cm H}_2\text{O}) = \underline{-63.0 \text{ cm H}_2\text{O}}$$

75. *Pressure in the spinal fluid is measured as shown in Figure 11.43. If the pressure in the spinal fluid is 10.0 mm Hg: (a) What is the reading of the water manometer in cm water? (b) What is the reading if the person sits up, placing the top of the fluid 60 cm above the tap? The fluid density is 1.05 g/mL.*

Solution (a) This part is a unit conversion problem:

$$P_0 = (10.0 \text{ mm Hg}) \left(\frac{133 \text{ N/m}^2}{1.0 \text{ mm Hg}} \right) \left(\frac{1.0 \text{ cm H}_2\text{O}}{98.1 \text{ N/m}^2} \right) = \underline{13.6 \text{ cm H}_2\text{O}}$$

(b) Solving this part in standard units, we know that

$$P = P_0 + \Delta P = P_0 + h\rho g, \text{ or}$$

$$P = 1330 \text{ N/m}^2 + (1.05 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.600 \text{ m}) = 7504 \text{ N/m}^2$$

$$\text{Then converting to cm water: } P = (7504 \text{ N/m}^2) \left(\frac{1.0 \text{ cm H}_2\text{O}}{98.1 \text{ N/m}^2} \right) = \underline{76.5 \text{ cm H}_2\text{O}}$$

82. *Calculate the pressure due to the ocean at the bottom of the Marianas Trench near the Philippines, given its depth is 11.0 km and assuming the density of sea water is constant all the way down. (b) Calculate the percent decrease in volume of sea water due to such a pressure, assuming its bulk modulus is the same as water and is constant. (c) What would be the percent increase in its density? Is the assumption of constant density valid? Will the actual pressure be greater or smaller than that calculated under this assumption?*

Solution (a) Using the equation $P = h\rho g$, we can calculate the pressure at a depth of 11.0 km:

$$\begin{aligned} P &= h\rho g = (11.0 \times 10^3 \text{ m})(1025 \text{ kg/m}^3)(9.80 \text{ m/s}^2) \\ &= 1.105 \times 10^8 \text{ N/m}^2 \times \frac{1 \text{ atm}}{1.013 \times 10^5 \text{ N/m}^2} = \underline{1.09 \times 10^3 \text{ atm}} \end{aligned}$$

(b) Using the following equation:

$$\frac{\Delta V}{V_0} = \frac{1}{B} \frac{F}{A} = \frac{P}{B} = \frac{1.105 \times 10^8 \text{ N/m}^2}{2.2 \times 10^9 \text{ N/m}^2} = 5.02 \times 10^{-2} = \underline{5.0\% \text{ decrease in volume.}}$$

(c) Using the equation $P = \frac{m}{V}$, we can get an expression for percent change in

$$\text{density: } \frac{\Delta \rho}{\rho} = \frac{m/(V_0 - \Delta V)}{m/V_0} = \frac{V_0}{V_0 - \Delta V} = \frac{1}{1 - (\Delta V/V_0)} = \frac{1}{1.00 - 5.02 \times 10^{-2}} = 1.053,$$

so that the percent increase in density is 5.3%. Therefore, the assumption of constant density is not strictly valid. The actual pressure would be greater, since the pressure is proportional to density.