ANSWERS - AP Physics Multiple Choice Practice - Gravitation

	Solution	Answer
1.	Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the	В
	object being orbited. Notice that satellite mass does not affect orbital speed. The smallest radius of orbit will be the fastest satellite.	
2.	As a satellite moves farther away, it slows down, also decreasing its angular momentum and kinetic energy. The total energy remains the same in the absence of resistive or thrust forces. The potential energy becomes less negative, which is an increase.	Е
3.	With different masses, g would have a different value, but the physical characteristics of the objects would not be affected.	D
4.	$g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the	Α
	mass of the planet and inversely proportional to the distance from the center of the planet squared. $M \times 2 = g \times 2$ and $r \times 2 = g \div 4$, so the net effect is the person's weight is divided by 2	
5.	$g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the	D
	mass of the planet and inversely proportional to the distance from the center of the planet squared. $M \div 10 = g \div 10$ and $r \div 2 = g \times 4$, so the net effect is $g \times 4/10$	
6.	Circular orbit = constant r, combined with constant speed gives constant angular momentum (mvr). As it is a circular orbit, the force is centripetal, points toward the center and is always perpendicular to the displacement of the satellite therefore does no work.	Е
7.	The gravitational force on an object <i>is</i> the weight, and is proportional to the mass. In the same circular orbit, it is only the mass of the body being orbited and the radius of the orbit that contributes to the orbital speed and acceleration.	С
8.	Newton's third law	E
9.	$g = \frac{GM}{r^2}$	D
10.	Force is inversely proportional to distance between the centers squared. $R \times 4 = F \div 16$	E
11.	$g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the	В
	mass of the planet and inversely proportional to the distance from the center of the planet squared. $M \times 2 = g \times 2$ and $r \times 2 = g \div 4$, so the net effect is the person's weight is divided by 2	
12.	A planet of the same size and twice the mass of Earth will have twice the acceleration due to gravity. The period of a mass on a spring has no dependence on g, while the period of a pendulum is inversely proportional to g.	Е
13.	Orbital speed is found from setting $\frac{GMm}{2} = \frac{mv^2}{2}$ which gives $v = \sqrt{\frac{GM}{2}}$	С

28. Escape speed with the speed at which the kinetic energy of the satellite is exactly equal to the negative amount of potential energy within the satellite/mass system. That is

$$\frac{1}{2}mv^2 = \left| -\frac{GMm}{r} \right|$$
 which gives the escape speed $v_e = \sqrt{\frac{2GM}{r}}$

- Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited. Also, $T = \frac{2\pi r}{v}$. Since the mass is divided by 2, v is divided by $\sqrt{2}$
- 30. $g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 7 = g × 7 and r × 2 = g ÷ 4, so the net effect is g × 7/4
- Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited. Notice that satellite mass does not affect orbital speed or period.
- 32. $g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M ÷ 5 = g ÷ 5 and r ÷ 2 = g × 4, so the net effect is g × 4/5
- 33. Part of the gravitational force acting on an object at the equator is providing the necessary centripetal force to move the object in a circle. If the rotation of the earth were to stop, this part of the gravitational force is no longer required and the "full" value of this force will hold the object to the Earth.
- 34. Standard orbital altitudes are not a large percentage of the radius of the Earth. The acceleration A due to gravity is only slightly smaller in orbit compared to the surface of the Earth.
- 35. $F = \frac{GMm}{r^2}$. F is proportional to each mass and inversely proportional to the distance between their centers squared. If each mass is doubled, F is quadrupled. If r is doubled F is quartered.
- 36. Since the acceleration due to gravity is less on the surface of the moon, to have the same gravitational force as a second object on the Earth requires the object on the Moon to have a larger mass.
- Satellites in orbit are freely falling objects with enough horizontal speed to keep from felling closer to the planet.
- 38. The mass of an object will not change based on its location. As one digs into a sphere of uniform density, the acceleration due to gravity (and the weight of the object) varies directly with distance from the center of the sphere.
- Combining $\frac{GMm}{r^2} = \frac{mv^2}{r}$ with $T = \frac{2\pi r}{v}$ gives the equation corresponding to Kepler's second law. The mass of the satellite cancels in these equations.

E

B

50. The energy of a circular orbit is

$$K + U = \frac{1}{2}mv^2 + (-\frac{Gmm}{r}) = \frac{1}{2}m\left(\sqrt{\frac{GM}{r}}\right)^2 + (-\frac{GMm}{r}) = -\frac{GMm}{2r}$$

The energy of an elliptical orbit is $-\frac{GMm}{2a}$ where a is the semimajor axis. If the speed is cut in

half we have
$$K + U = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + (-\frac{Gmm}{r}) = \frac{1}{2}m\left(\frac{1}{2}\sqrt{\frac{GM}{r}}\right)^2 + (-\frac{GMm}{r}) = -\frac{7GMm}{8r}$$

Setting
$$-\frac{7GMm}{8r} = -\frac{GMm}{2a}$$
 gives $a = (4/7)$ r

The distance to the planet from this point is r (the radius of the circular orbit and aphelion for the elliptical orbit). The opposite side of the ellipse is 2a away, or 8r/7, making the distance to the planet at perihelion 8r/7 - r = r/7

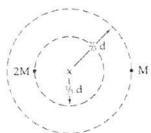
- 51. $U = -\frac{GMm}{r}$
- 52. The top of Pikes Peak is a very small fraction of the radius of the Earth. Moving to twice this elevation will barely change the value of g.
- Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited.
- 54. $F = \frac{GMm}{r^2}$. The masses of the proton and electron can be found in the table of constants (these masses do not need to be memorized)
- 55. $F = \frac{GMm}{r^2}$; If $r \div 2$, F × 4. If each mass is multiplied by 1.41, F is doubled (1.41×1.41)
- 56. $g = \Delta v/t = (31 \text{ m/s} 50 \text{ m/s})/(5 \text{ s}) = -3.8 \text{ m/s}^2$
- 57. mass is unchanged, weight is changed due to a change in the acceleration due to gravity
- Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited.
- 59. $g = v^2/r$ and $v = 2\pi r/T$

E

1977M3

a.
$$F_g = F_c$$
 gives $\frac{GMM}{(2R)^2} = \frac{Mv^2}{R}$. Solving for v gives $v = \frac{1}{2} \sqrt{\frac{GM}{R}}$

b.
$$E = PE + KE = -\frac{GMM}{2R} + 2\left(\frac{1}{2}Mv^2\right) = -\frac{GMM}{2R} + 2\left(\frac{1}{2}M\left(\frac{1}{2}\sqrt{\frac{GM}{R}}\right)^2\right) = -\frac{GM^2}{4R}$$



d.
$$F_{g2} = F_{g1} = F_{c}$$

$$\frac{(2M)v_2^2}{\frac{1}{3}d} = \frac{Mv_1^2}{\frac{2}{3}d}$$
 gives $v_2/v_1 = \frac{1}{2}$

1984M2

a.
$$F_g = F_c$$
 gives $\frac{GM_em}{(2R_e)^2} = \frac{mv^2}{2R_e}$ giving $v = \sqrt{\frac{GM_e}{2R_e}}$

b. conservation of momentum gives
$$(3m)v_0 - mv_0 = (4m)v'$$
 giving $v' = \frac{1}{2}v_0$

c.
$$E = PE + KE = -\frac{GM_e(4m)}{2R_e} + \left(\frac{1}{2}(4m)v^2\right) = -\frac{2GM_em}{R_e} + 2m\left(\frac{1}{2}\sqrt{\frac{GM_e}{2R_e}}\right)^2 = -\frac{7GM_em}{4R_e}$$

a.
$$E = PE + KE = -\frac{GMm}{r} + \frac{1}{2}mv^2 = -8.1 \times 10^9 \text{ J}$$

b.
$$L = mvr = 8.5 \times 10^{13} \text{ kg-m}^2/\text{s}$$

a.
$$E = PE + KE = -\frac{GMm}{R} + \frac{1}{2}mv^2 = -8.1 \times 10^9 \text{ J}$$

b. $L = mvr = 8.5 \times 10^{13} \text{ kg-m}^2/\text{s}$
c. Angular momentum is conserved so $mv_ar_a = mv_br_b$ giving $v_b = 2.4 \times 10^3 \text{ m/s}$

d.
$$F_g = F_c$$
 gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ and $v = \sqrt{\frac{GM}{R}} = 5.8 \times 10^3$ m/s

e. Escape occurs when E = PE + KE = 0 giving
$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = 0$$
 and $v_{esc} = \sqrt{\frac{2GM}{R}} = 8.2 \times 10^3$ m/s

a.
$$E = PE + KE = -\frac{GM_em}{a} + \frac{1}{2}mv_0^2$$

b.
$$L = mvr = mv_0a$$

Conservation of angular momentum gives $mv_0a = mv_bb$, or $v_b = v_0a/b$

d.
$$F_g = F_c$$
 gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ and $v = \sqrt{\frac{GM_e}{a}}$

d. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ and $v = \sqrt{\frac{GM_e}{a}}$ e. The work done is the change in energy of the satellite. Since the potential energy of the satellite is constant, the change in energy is the change in kinetic energy, or $W = \Delta KE = \frac{1}{2} m \left(\frac{GM_e}{a} - v_0^2 \right)$

2005M2

a.
$$F = \frac{GM_Sn}{R^2}$$

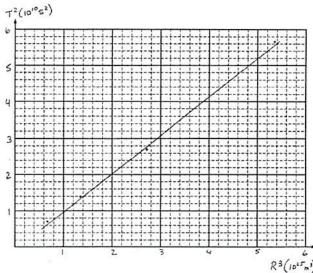
b.
$$F_g = F_c$$
 gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ and $v = \sqrt{\frac{GM}{R}} = \frac{2\pi R}{T}$ gives the desired equation $T = \sqrt{\frac{4\pi^2 R^3}{GM}}$

c. T^2 vs. R^3 will yield a straight line (let $y = T^2$ and $x = R^3$, we have the answer to b. as $y = \left(\frac{4\pi^2}{GM}\right)x$ where the quantity in parentheses is the slope of the line.

d.

Orbital Period, T (seconds)	Orbital Radius, R (meters)	T^2 (s ²)	R^3 (m ³)
8.14×10^{4}	1.85×10^{8}	0.663×10^{10}	0.633×10^{25}
1.18×10^{5}	2.38 × 10 ⁸	1.39×10^{10}	1.35×10^{25}
1.63×10^{5}	2.95×10 ⁸	2.66×10^{10}	2.57×10^{25}
2.37×10^{5}	3.77×10^{8}	5.62×10^{10}	5.36 × 10 ²⁵

e.



f. From part c. we have an expression for the slope of the line. Using the slope of the above line gives $M_S = 5.64 \times 10^{26}$ kg